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Stability Study of Logarithmic Double Autoregressive Model with Application

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ABSTRACT

In this research we suggest a new non-linear time series model known Logarithmi**c** Double Autoregressive model of order P, LogDAR(p) , this model is depends on Double Autoregressive model with Logarithmic function . By using a local linearization technique we find a stability Condition of anon-zero singular point of the model , and applying these Condition to an obtained models by modeling areal Data that represents a monthly Brent Crude oil prices at closing in dollars for period (1989-2021) for a several orders of suggested model.

Key words :Autoregressive Conditional Heteroscedastic ARCH, Double Autoregressive model DAR, Logarithmic Double Autoregressive LogDAR ,Local Linearization method, non-zero singular point ,stability of non-zero singular point, Conditional variance.

1.Introduction

A linear Autoregressive model of order p denoted by AR(P) has the formula

$$x_t + a_1 x_{t-1} + \cdots + a_p x_{t-p} = z_t \qquad \ldots(1.1)$$

Where $a_1, a_2, \ldots, a_p$ are constant ,and $z_t \sim \text{iid } N(0, \sigma_t^2)$

The equation (1.1) may be written more concisely by using Backward shift operator in the form

$\alpha(B)x_t = z_t$ where $\alpha(B) = 1 + a_1 z + a_2 z^2 + \cdots + a_p z^p$

The general solution of the model is given by

$x_t = f(t) + \alpha^{-1}(B) z_t$

Where f(t) is the complementary function form

$f(t) = A_1 \mu_1^t + A_2 \mu_2^t + \cdots + A_p \mu_p^t$,where $A_1, A_2, \ldots, A_p$ is an arbitrary constant and $\mu_1, \mu_2, \ldots \ldots, \mu_p$ is the roots of the characteristic equation $z^p + a_1 z^{p-1} + \cdots . + a_p = 0$

the autoregressive model AR(P) is a asymptotically stationary process if $\lim_{t\to\infty} f(t) = 0$

which implies that all the roots must lie inside the unit circle i,e $|\mu_i| < 1$ for i=1,2,….,p.[18]

In 1982 Engle 's presented family of non linear time series models named Autoregressive Conditional Heteroscedastic model or ARCH which is depend on a martingale difference series[2]. And Bollerslev's in 1986 generalized Autoregressive Conditional Heteroscedastic (GARCH) model [1]. In 2007 Ling proposed anew Autoregressive model is called a double Autoregressive model DAR .The double AR(P) is the Autoregressive AR(P) model with Autoregressive Conditional Heteroscedastic model ARCH [8]**.** Many variants of DAR models have been widely proposed and studied such as in 2016 Li, G., Zhu, Q., Liu, Z and Zhang, R. proposed threshold DAR [9] ,in 2017 Li, G., Zhu, Q., Liu, Z.,and Li,W.K. proposed the mixture DAR[10] , in 2018 Zhu and Ling studied Linear double autoregression [23] , in 2020 Jiang and Zhu proposed the augmented DAR [7].

The Double AR(P) model proposed by Ling has the form

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$$y_t = \sum_{i=1}^{p} \varphi_i y_{t-i} + \eta_t \sqrt{w + \sum_{i=1}^{p} \alpha_i\, y_{t-i}^2} \qquad \ldots(1.2)$$

Where w, α_i >0 , for $i = 1,2,\ldots,p$ are constants and $\{\eta_t\}$ is identically independent random variables normally distributed with mean 0 and unite variance , $\eta_t \sim \text{i.i.d } N(0,1)$, and

M={-P,…,0,1,2,…} , y_s is independent of $\{\eta_t : t \geq 1\}$ for $s \leq 0$. Let F_t to be the $\sigma -$field generated by $\{\eta_t, \ldots, \eta_1, y_0, \ldots, y_{-p}\}$ $t \in$ M,The conditional variance of y_t is

$\text{Var}\,(y_t \setminus F_{t-1}) = w + \sum_{i=1}^{p} \alpha_i\, y_{t-i}^2$

DAR(P) model in (1.2) is special case of mixed ARMA-ARCH models and the weakly stationary of the ARMA model required a geometrically ergodicity of a markov chain representation of the model [8].

The stability of anon -linear models deals with a different two categories, first is the general stability conditions deals with the geometrically ergodicity of an associated markov Chain with the given model. The second deals with finding a special condition in Terms of model parameters by using some approximation method to linearization anon -linear models ,such as local linearization method.[20]

The local linearization methods are used in many studies such, Mohammad and Salim in 2007 for find the Stability condition of the Logistic Autoregressive model LSTAR(P) model [15], Mohammad and ghannam in 2010 proposed a Cauchy Autoregressive model and the stability conditions of this model [12], Salim and Younis in 2012 studied the stability of non-linear Autoregressive models [19],Mohammad and Ghaffar in 2016 used it for study on stationarity of GARCH models [11],Mohammad and Mudhir in 2020 studied the stationarity of EGARCH(Q,P) models [13], and Noori and Mohammad 2021 studied the stationarity of GJR-GARCH(Q,P) models [16].

2.Prelimiaries

2.1 Some properties of ARCH and DAR

Autoregressive Conditional Heteroscedastic model or ARCH that depends on a martingale difference of $\{x_t\}$ that defines on a Conditional expectation of the first and second moments given that anon decreasing collection of $\sigma -$ field ,that is

$\ldots\ldots \subset F_{t-1} \subset F_t \subset F_{t+1} \subset \ldots\ldots$

Where $F_t = \sigma(x_t, x_{t-1}, \ldots\ldots, x_0)$ represent a $\sigma -$ field or Filter and a martingale series related to this filtering is a stochastic process $\{x_t\}$ such that the random variable x_t is F_t-measurable and $E(x_t \setminus F_{t-1})=0$,that is mean that x_t is orthogonal to all random variables in a filter F_{t-1} such that $x_s \subset F_s \subset F_{t-1} \subset F_t$ then $E(x_t x_s)=0$ for $s < t$, the Heteroscedasity (the variance is not constant and it depend on time) is a property known as conditional variance ,where $E({x_t}^2 \setminus F_{t-1}) = \sigma_t^2$ then the ARCH model determine or restricted the relation between z_t and σ_t as $x_t = \sigma_t\ z_t$ …(2.1)

Where $\{\sigma_t\}$ and $\{z_t\}$ is independent stochastic processes such that

1- σ_t is measurable with respect to F_{t-1} or $F_{t-1} -$ measurable

$2 - z_t \sim$ i.i.d $N(0,1)$

3- $\sigma_t > 0$

and the standard deviation σ_t known as the volatility of x_t ,under these conditions

$E(x_t) = E(\sigma_t\ z_t) = E(\sigma_t)\,E(z_t)=0$ and

$\text{Cov}\,(x_t, x_{t\pm k}) = E(z_t).E(\sigma_t\ z_{t\pm k}) = 0 \quad \forall\ k>0$ that is make $\{x_t\}$ a weak white noise process .

In order to solve the volatility problem R.Engle standard form of ARCH model where

$F_{t-1} = \sigma(x_s\ ; s<t)$ is a $\sigma -$ field generated by the past of stochastic processes $\{x_t\}$ and the standard ARCH model represents the squares of volatility or conditional variance as a linear function of the past of the series $\{x_t^2\}$.[1] ,[3]

And the ARCH model has the form

$x_t = \sigma_t\ z_t$

$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i\, x_{t-i}^2$ ...(2. 2)

Where $z_t \sim$ i.i.d $N(0,1)$ and $\alpha_0 > 0$, $\alpha_i \geq 0$ for i=1,2,….,q

$Var\ (x_t \setminus F_{t-1}) = E\ (x_t^2 \setminus F_{t-1}) = \alpha_0 + \sum_{i=1}^{q} \alpha_i\, x_{t-i}^2$

$Var\ (x_t) = \sigma_x^2 = \frac{\alpha_0}{1-\sum_{i=1}^{q}\alpha_i}$ then the unconditional variance σ_x^2 exists if $0 < \sum_{i=1}^{q} \alpha_i < 1$ (since $\alpha_0 > 0$)

$E\ (x_t^2) = \alpha_0 + \sum_{i=1}^{q} \alpha_i\, x_{t-i}^2 = \sigma_t^2$

Is the conditional variance . then the conditional variance depends on t but the unconditional variance does not depend on t. [4] ,[5],[21]

The double autoregressive model (DAR) that proposed by Ling (2007) by considering the double autoregressive ,first is a linear autoregressive and the second is Conditional Heteroscedastic autoregressive .

$y_t = \sum_{i=1}^{p} \varphi_i y_{t-i} + \eta_t \sqrt{w + \sum_{i=1}^{p} \alpha_i\, y_{t-i}^2}$ …(2.3)

Where w, α_i >0 are constant , and $\{\eta_t\}$ is independent random sequence , $\eta_t \sim$ i.i.d $N(0,1)$.

When $p = 1$ the necessary and sufficient condition of stationarity of DAR(1) models is $\varphi_1^2 + \alpha_1 < 1$,Figure 2.1 gives the regions on (φ_1, α_1) plane such that $\varphi_1^2 + \alpha_1 < 1$ (region B) and $\gamma_0 = E\ (\ln|\varphi_1 + \sqrt{\alpha_1}\,\gamma\,| < 0$ (region A) for the general case .by Jensen 's inequality and from the figure 2.1 we can see that the condition $\gamma < 0$ allows the case with some roots of $\varphi(z) = 1 - \sum_{i=1}^{p} \varphi_i z^i = 0$ on or outside the unit circle and the case with $E\ (y_t^2) \to \infty$.

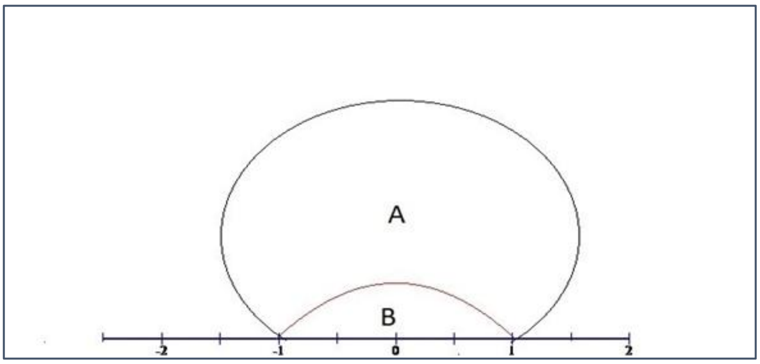

Figure 2.1 :$\varphi_1^2 + \alpha_1 < 1$ as $(\varphi_1 , \alpha_1) \in B$

The weakest sufficient conditions for $E\ (y_t^2) < \infty$,which is $\left(\sum_{i=1}^{p}|\varphi_i|\right)^2 + \sum_{i=1}^{p} \alpha_i < 1$

Except for the case with $p = 1$,this condition is also stronger than that for example when

$\alpha_1 = \alpha_2 = \ldots\ldots = \alpha_p = 0$ the necessary and sufficient condition for stationarity of the usually AR(P) model which is much weaker than $\sum_{i=1}^{p}|\varphi_i|) < 1$. [8]

2.1 Logarithmic Double Autoregressive model LogDAR(P)

The Logarithmic Double Autoregressive model defined by a stochastic process $\{y_t\}$ that satisfies stochastic difference equation.

$y_t = \sum_{i=1}^{p} \varphi_i y_{t-i} + \eta_t \sqrt{w + \sum_{i=1}^{p} \alpha_i \log y_{t-i}^2}$ ….(2.4)

Where$\{\alpha_i\}$, $\{\varphi_i\}$, $w > 0$ are model parameters $i = 1,2,\ldots,p$, $\eta_t \sim \text{i.i.d } N(0,1)$ for all t.

To find conditional variance of LogDAR(P) model by taking a conditional expectation with respect to the filter F_t by squaring both sides (2.4) , we get

$y_t^2 = \left(\sum_{i=1}^{p} \varphi_i y_{t-i}\right)^2 + {\eta_t}^2 (w + \sum_{i=1}^{p} \alpha_i \log y_{t-i}^2) + 2\eta_t \sum_{i=1}^{p} \varphi_i y_{t-i} \sqrt{w + \sum_{i=1}^{p} \alpha_i \log y_{t-i}^2}$

$y_t^2 = \sum_{i=1}^{p} {\varphi_i}^2 y_{t-i}^2 + 2 \sum_{i\neq j} \varphi_i \varphi_j\, y_{t-i} y_{t-j} + {\eta_t}^2\left(w + \sum_{i=1}^{p} \alpha_i \log y_{t-i}^2\right) + 2\eta_t \sum_{i=1}^{p} \varphi_i y_{t-i} \sqrt{w + \sum_{i=1}^{p} \alpha_i \log y_{t-i}^2}$

By Taking the conditional expectation with respect to afilter F_{t-1} ,then

$E(y_t^2 / F_{t-1}) = \sum_{i=1}^{p} {\varphi_i}^2 E(y_{t-i}^2 / F_{t-1}) + 2 \sum_{i\neq j} \varphi_i \varphi_j\, E y_{t-i} y_{t-j} / F_{t-1}) + E({\eta_t}^2) w + E({\eta_t}^2) \sum_{i=1}^{p} \alpha_i \log y_{t-i}^2 +$ 2 $\sum_{i=1}^{p} \varphi_i E(y_{t-i} / F_{t-1}) \sqrt{w + \sum_{i=1}^{p} \alpha_i \log E(y_{t-i}^2)}$

$\sigma_t^2 = \sum_{i=1}^{p} {\varphi_i}^2 \sigma_{t-i}^2 + 0 + w + \sum_{i=1}^{p} \alpha_i [\log E(1 + (y_{t-i}^2 - 1/F_{t-1}))] + 0$

By using expansion of *log(1+x)*

$\sigma_t^2 = \sum_{i=1}^{p} {\varphi_i}^2 \sigma_{t-i}^2 + w + (1) \sum_{i=1}^{p} \alpha_i E(y_{t-i}^2 - 1/F_{t-1})) - 0$

$\sigma_t^2 = \sum_{i=1}^{p} {\varphi_i}^2 \sigma_{t-i}^2 + w + \sum_{i=1}^{p} (\alpha_i \sigma_{t-i}^2 - \alpha_i)$

$\sigma_t^2 = \sum_{i=1}^{p} {\varphi_i}^2 \sigma_{t-i}^2 + \sum_{i=1}^{p} \alpha_i \sigma_{t-i}^2 + w - \sum_{i=1}^{p} \alpha_i$

$\sigma_t^2 = \sum_{i=1}^{p} ({\varphi_i}^2 + \alpha_i) \sigma_{t-i}^2 + w + \sum_{i=1}^{p} \alpha_i$

The conditional variance is constant if $\sigma_t = \sigma_{t-1} = \cdots = \sigma_{t-i} = \sigma_y$

${\sigma_y}^2 = \sum_{i=1}^{p} ({\varphi_i}^2 + \alpha_i)\, {\sigma_y}^2 + w + \sum_{i=1}^{p} \alpha_i$

${\sigma_y}^2 [1 - \sum_{i=1}^{p} ({\varphi_i}^2 + \alpha_i)] = w + \sum_{i=1}^{p} \alpha_i$

${\sigma_y}^2 = \sum_{i=1}^{p} ({\varphi_i}^2 + \alpha_i)\, {\sigma_y}^2 + w + \sum_{i=1}^{p} \alpha_i$

${\sigma_y}^2 [1 - \sum_{i=1}^{p} ({\varphi_i}^2 + \alpha_i)] = w + \sum_{i=1}^{p} \alpha_i$

${\sigma_y}^2 = \frac{w + \sum_{i=1}^{p} \alpha_i}{1 - \sum_{i=1}^{p} ({\varphi_i}^2 + \alpha_i)}$ ...(2.5)

since w>0 the unconditional variance ${\sigma_y}^2$ exists if $\frac{\sum_{i=1}^{p} \alpha_i}{1 - \sum_{i=1}^{p} ({\varphi_i}^2 + \alpha_i)} \geq 0$

Then the stationary condition of LogDAR(p) model required that the conditional variance σ_t^2 converges to the unconditional variance ${\sigma_y}^2$. See [22],[14]

LogDAR(p) model can be written as

$y_t = f(y_t, y_{t-1}, y_{t-2}, \ldots, y_{t-p}, \eta_t)$ …(2.6)

Where f: $R^{p+1} \rightarrow R^p$ be anon linear function .

Let ζ be anon zero singular point of the model that satisfies

$f(\zeta) = \zeta$ ...(2.7)

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To find the condition of stability of the LogDAR(p) by using a local linearization technique deal with approximate anon-linear model to linear model in order to study the stability condition of the non zero singular point of anon - linear model .

Then anon zero singular point can be found if we substate $y_{t-i}=\zeta$ for $i = 0,1,2,\dots,p$ and the model (2.4) became

$$\zeta^2 = \sum_{i=1}^{p} \varphi_i{}^2\zeta^2 + w + \sum_{i=1}^{p} \alpha_i \log\zeta^2$$

$$\zeta^2(1-\sum_{i=1}^{p} \varphi_i{}^2) = w + \sum_{i=1}^{p} \alpha_i \log\zeta^2 \qquad \dots(2.8)$$

$$\zeta^2(1-\sum_{i=1}^{p} \varphi_i{}^2) = w + \sum_{i=1}^{p} \alpha_i \log(1 +(\zeta^2 - 1))$$

By using expansion of log (1+x)

$$\zeta^2(1 - \sum_{i=1}^{p} \varphi_i{}^2)=w+\sum_{i=1}^{p} \alpha_i\, \zeta^2+\sum_{i=1}^{p} \alpha_i$$

$$\zeta^2(1 - \sum_{i=1}^{p} \varphi_i{}^2-\sum_{i=1}^{p} \alpha_i)=w+\sum_{i=1}^{p} \alpha_i$$

$$\zeta^2 = \frac{w+\sum_{i=1}^{p} \alpha_i}{1-\sum_{i=1}^{p}(\varphi_i{}^2+\alpha_i)} \qquad \dots(2.9)$$

$$\zeta = \pm\sqrt{\frac{w+\sum_{i=1}^{p} \alpha_i}{1-\sum_{i=1}^{p}(\varphi_i{}^2+\alpha_i)}}, \quad w\geq 0 \qquad \dots(2.10)$$

the non-zero singular point is exist and real if $\frac{\sum_{i=1}^{p} \alpha_i}{1-\sum_{i=1}^{p}(\varphi_i{}^2+\alpha_i)} \geq 0$

The following proposition show stability conditions for the model (2.4) by dependence on parameters and constants of the model.

Proposition 2.1 : The LogDAR(P) is asymptotically stationary if all the roots of the polynomial

$$\lambda^P - \sum_{i=1}^{p} h_i\ \lambda^{P-i}=0 \qquad \dots(2.11)$$

$$h_i = (\varphi_i{}^2 + \frac{\alpha_i}{k})\, i = 1,2,\dots,p \quad , \text{ where } k=\frac{w+\sum_{i=1}^{p} \alpha_i}{1-\sum_{i=1}^{p}(\varphi_i{}^2+\alpha_i)}$$

Lies inside the unit circle.

Proof: Let

$$y_t = \sum_{i=1}^{p} \varphi_i y_{t-i} + \eta_t\sqrt{w + \sum_{i=1}^{p} \alpha_i \log y_{t-i}^2}$$

by squaring both sides , we get

$$y_t^2 = \left(\sum_{i=1}^{p} \varphi_i y_{t-i}\right)^2 + \eta_t{}^2(w + \sum_{i=1}^{p} \alpha_i \log y_{t-i}^2)+ 2\eta_t \sum_{i=1}^{p} \varphi_i y_{t-i} \sqrt{w + \sum_{i=1}^{p} \alpha_i \log y_{t-i}^2}$$

$$y_t^2 = \sum_{i=1}^{p} \varphi_i{}^2\, y_{t-i}^2 + 2\sum_{i\neq j} \varphi_i\varphi_j\, y_{t-i}y_{t-j} + \eta_t{}^2\left(w + \sum_{i=1}^{p} \alpha_i \log y_{t-i}^2\right) + 2\eta_t \sum_{i=1}^{p} \varphi_i y_{t-i} \sqrt{w + \sum_{i=1}^{p} \alpha_i \log y_{t-i}^2}$$

By take the mathematical expectation

$$E(y_t^2)= \sum_{i=1}^{p} \varphi_i{}^2\, E(y_{t-i}^2) + 2\sum_{i\neq j} \varphi_i\varphi_j\, Ey_{t-i}y_{t-j})+ E(\eta_t{}^2)w+ E(\eta_t{}^2) \sum_{i=1}^{p} \alpha_i \log y_{t-i}^2+ 2\sum_{i=1}^{p} \varphi_i E(y_{t-i})\sqrt{w + \sum_{i=1}^{p} \alpha_i \log(y_{t-i}^2)}$$

Since $E(y_{t-i}y_{t-j}) = 0$ for $i \neq j$, $E(\eta_t) = 0$ and $E(\eta_t{}^2)=1$

$$y_t^2 = \sum_{i=1}^{p} \varphi_i{}^2\, (y_{t-i}^2) + w + \sum_{i=1}^{p} \alpha_i \log y_{t-i}^2$$

The local linearization technique consist of linearization the model near the nonzero singular point or in sufficiently small neighborhood around ζ ,that is mean by using a variational equation $y_{t-i}= \zeta + \zeta_{t-i}$ …(2.12)

for $i = 0,1,2, \dots, p$ such that the radius ζ_t sufficiently small such that $|\zeta_t|^n \to 0$ for $n \geq 2$.

By substuting equation (2.12) in a model (2.4) we get

$$(\zeta + \zeta_t)^2 = \sum_{i=1}^{p} \varphi_i^2 (\zeta + \zeta_{t-i})^2 + w + \sum_{i=1}^{p} \alpha_i \log(\zeta + \zeta_{t-i})^2$$

$\zeta^2 + 2\zeta\zeta_t + {\zeta_t}^2 = \sum_{i=1}^{p} \varphi_i^2 \zeta^2 + 2\sum_{i=1}^{p} \varphi_i^2 \zeta\zeta_{t-i} + \sum_{i=1}^{p} \varphi_i^2 {\zeta_{t-i}}^2$+w+$\sum_{i=1}^{p} \alpha_i \log(\zeta^2 + 2\zeta\zeta_{t-i} + {\zeta_{t-i}}^2)$ (since ${\zeta_{t-i}}^2 \to 0$) ,then

$$\zeta^2 + 2\zeta\zeta_t = \sum_{i=1}^{p} \varphi_i^2 \zeta^2 + 2\sum_{i=1}^{p} \varphi_i^2 \zeta\zeta_{t-i} + w + \sum_{i=1}^{p} \alpha_i \log(\zeta^2 + 2\zeta\zeta_{t-i}) \qquad \dots(2.13)$$

but $\log(\zeta^2 + 2\zeta\zeta_{t-i}) = \log\zeta^2 (1 + 2\frac{\zeta_{t-i}}{\zeta}) = \log\zeta^2 + \log(1 + 2\frac{\zeta_{t-i}}{\zeta})$,then

$$\zeta^2 + 2\zeta\zeta_t = \sum_{i=1}^{p} \varphi_i^2 \zeta^2 + 2\sum_{i=1}^{p} \varphi_i^2 \zeta\zeta_{t-i} + w + \sum_{i=1}^{p} \alpha_i [\log\zeta^2 + \log(1 + 2\frac{\zeta_{t-i}}{\zeta})]$$

By using Taylor expansion

$$\log\left(1 + 2\frac{\zeta_{t-i}}{\zeta}\right) = 2\frac{\zeta_{t-i}}{\zeta} + 4\frac{{\zeta_{t-i}}^2}{\zeta^2} + \dots$$

$$= 2\frac{\zeta_{t-i}}{\zeta} \quad (\text{since } {\zeta_{t-i}}^2 \to 0)$$

Substitute it in (2.13) ;we get

$$\zeta^2 + 2\zeta\zeta_t = \sum_{i=1}^{p} \varphi_i^2 \zeta^2 + 2\sum_{i=1}^{p} \varphi_i^2 \zeta\zeta_{t-i} + w + \sum_{i=1}^{p} \alpha_i \log\zeta^2 + 2\sum_{i=1}^{p} \alpha_i \frac{\zeta_{t-i}}{\zeta}$$

$$\zeta^2 + 2\zeta\zeta_t = \sum_{i=1}^{p} \varphi_i^2 \zeta^2 + 2\zeta\sum_{i=1}^{p} \varphi_i^2 \zeta_{t-i} + w + \sum_{i=1}^{p} \alpha_i \log\zeta^2 + \frac{2}{\zeta}\sum_{i=1}^{p} \alpha_i \zeta_{t-i}$$

$$\zeta^2(1 - \sum_{i=1}^{p} \varphi_i^2) + 2\zeta\zeta_t = 2\zeta\sum_{i=1}^{p} \varphi_i^2 \zeta_{t-i} + \frac{2}{\zeta}\sum_{i=1}^{p} \alpha_i \zeta_{t-i} + w + \sum_{i=1}^{p} \alpha_i \log\zeta^2$$

$$\zeta^2[(1 - \sum_{i=1}^{p} \varphi_i^2) - \sum_{i=1}^{p} \alpha_i \log\zeta^2] + 2\zeta\zeta_t = 2\zeta\sum_{i=1}^{p} \varphi_i^2 \zeta_{t-i} + \frac{2}{\zeta}\sum_{i=1}^{p} \alpha_i \zeta_{t-i} + w$$

But from equation (2.8)

$$W + 2\zeta\zeta_t = 2\zeta\sum_{i=1}^{p} \varphi_i^2 \zeta_{t-i} + \frac{2}{\zeta}\sum_{i=1}^{p} \alpha_i \zeta_{t-i} + w$$

$$2\zeta\zeta_t = 2\zeta\sum_{i=1}^{p} \varphi_i^2 \zeta_{t-i} + \frac{2}{\zeta}\sum_{i=1}^{p} \alpha_i \zeta_{t-i}$$

divided both sides by 2ζ,we get

$$\zeta_t = \sum_{i=1}^{p} (\varphi_i^2 + \frac{\alpha_i}{k}) \zeta_{t-i} \qquad \dots(2.14)$$

where $k = \frac{w + \sum_{i=1}^{p} \alpha_i}{1 - \sum_{i=1}^{p} (\varphi_i^2 + \alpha_i)}$, $w>0$

Which is difference equation of order p and its converge to zero (asymptotically stationary) iff all the roots of the characteristic equation

$$\lambda^p - \sum_{i=1}^{p} h_i \lambda^{p-i} = 0 \text{ where } \quad h_i = \left(\varphi_i^2 + \frac{\alpha_i}{k}\right) \quad k = \frac{w + \sum_{i=1}^{p} \alpha_i}{1 - \sum_{i=1}^{p} (\varphi_i^2 + \alpha_i)}$$

lies inside the unit circle so the model is stable if $|\lambda_i| < 1$ for all $i = 1,2, \dots, p$.

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3-Application

3.1 Data Description

we apply the stability condition of LogDAR(P) model to the data that represents a the Monthly mean of Brent Crude oil prices at closing in Dollars for the period from January 1989 to December 2021 by 396 observations get up from the website of (http://sa.investing.com/commodities/brent-oil-historical-data).

3.2 Data Analysis

First: we enter data to create a time series and we plot it , and figure 3.1 represent the time series plot of monthly mean of Historical contract data for Brent Crude oil closed from January 1989 to December 2021.

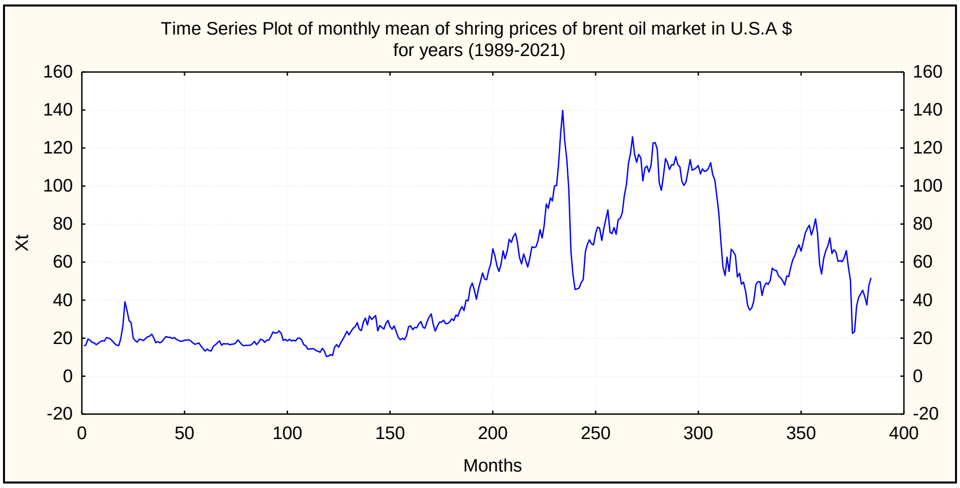


Figure 3.1: time series plot of monthly of Historical contract data for Brent Crude oil closed from (Jan 1989 to Dec 2021).

Then we transform the original series to the Returns series ,the transformation was used defined as the formula: $r_t = \log\frac{p_t}{p_{t-1}}$

Where r_t represent the Returns series and p_t, p_{t-1} represent the observed data at t,t-1 respectively, we used in MATLAB for this conversion is $r_r = \text{price2ret}(p_t)$, and figure 3.2 represent the Returns series . It contain volatility and fluctuation at some values. For this reason LogDAR are useful model to analyze the fluctuation s that accompany these phenomena ,the figure 3.3 represents the autocorrelation and partial autocorrelation function of the data number study and volatility at some values are appear out of the conference interval with boundaries $\pm\frac{1.96}{\sqrt{N}}$ where N is the simple size [13]

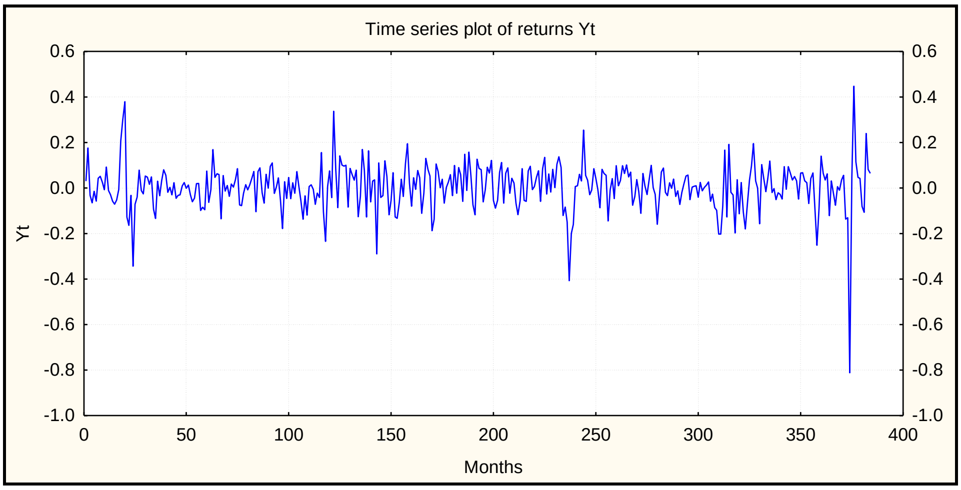


figure 3.2: represent the Returns series of monthly mean of Historical contract data for Brent Crude oil closed

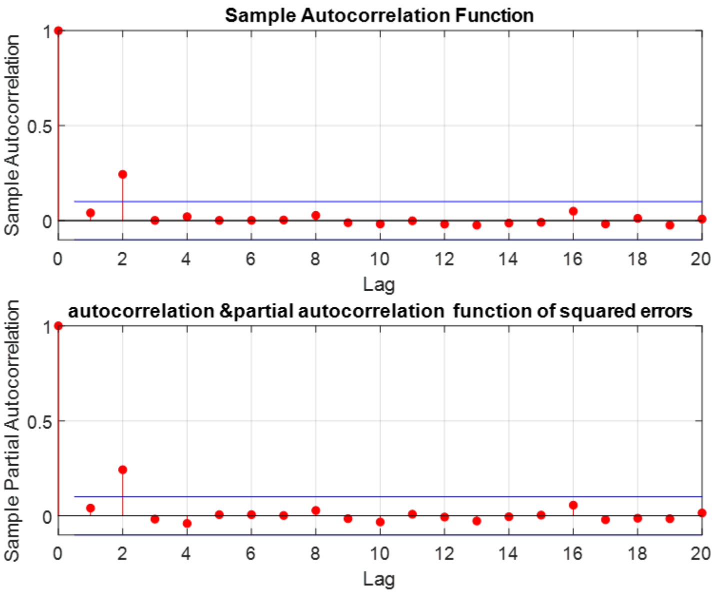
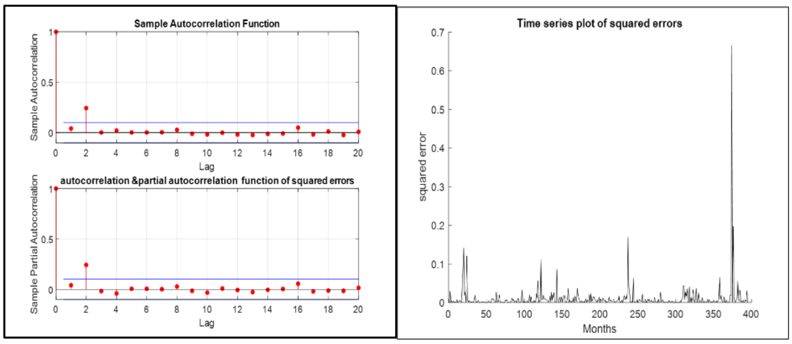


Figure 3.3 : the autocorrelation and partial autocorrelation function

Second step, it is the procedure to detect the presence of the effect of the heteroscedasticity variance by finding the series of Error squares for the returns series shown by the relationship $e_t = (\gamma_t - \overline{\gamma_t})^2$ where $\overline{\gamma_t}$ is the returns mean, and then drawing the autocorrelation and partial autocorrelation functions shown in the figure 3.4.



(b) (a)

Figure3.4: (a) time series plot of square Errors Return , (b) autocorrelation and partial autocorrelation function

The autocorrelation and partial autocorrelation function Shows that some correlation values lies outside confidence interval.

Third step, there are many tests used to detect the presence of heteroscedasticity among the most famous and the most used Ljung-Box test which includes null hypothesis H_0 against the alternative hypothesis H_1 such that

H_0: $\rho_0 = \rho_1 = \rho_2 = \rho_3 = \ldots\ldots = \rho_m$

H_1: $H_i \neq 0$ i=1,2,3,….,m

Such that n is simple size ,k is lag , $(\widehat{\sigma_k{}^2})$ squares of the autocorrelation coefficients of the residual series. That statistic Q(m) is chi-square distribution is distributed with a degree of freedom below an acceptable morale level α then hypothesis H_0 refuse if it $Q(m) > \chi^2_\alpha$.And in software then refused H_0 it is expressed as logical value h=1 ,that decision to refuse H_0 if it was $p \leq \propto$ that often takes 0.05.

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The table 3.1 shows values of h,p for 10 lags ,and what gives the decision rejecting the null hypothesis H_0 which means that there are non zero correlation in the series of residual squares squares ,and there fore there is aneffect of heteroscedasticity of variance in the series returns .

Table3.1: results a Ljung-box test on the returns series

Lags	h-value	p-value	Q-test	Critical value
Lag 1	1	0.0005	12.2191	3.8415
Lag 2	1	0.0011	13.6064	5.9915
Lag 3	1	0.0023	14.4673	7.8147
Lag 4	1	0.0003	21.3047	9.4877
Lag 5	1	0.0003	23.6038	11.0705
Lag 6	1	0.0005	24.0643	12.5916
Lag 7	1	0.0008	24.7616	14.0671
Lag 8	1	0.0012	25.5841	15.5073
Lag 9	1	0.0024	25.5843	16.9190
Lag 10	1	0.0025	27.1506	18.3070

3.3 Modeling and creating the LogDAR(P) model

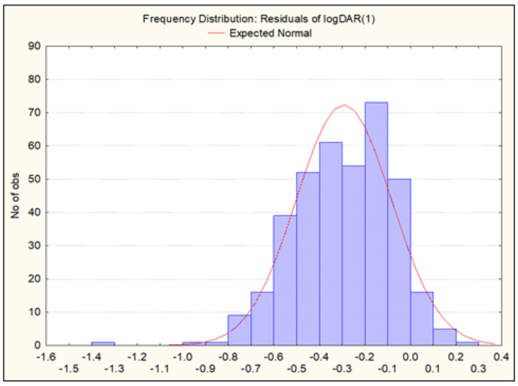
We will apply LogDAR(P) model to data series and monitor and verify that expectation conditional variance approaches the value of unconditional variance ,and estimate parameters , take the appropriate and then find singular and trajectory drawing .The program used the statistic to estimate parameters, and used (MATLAB R2021a) to creating and programming the time series and draw the orbit .

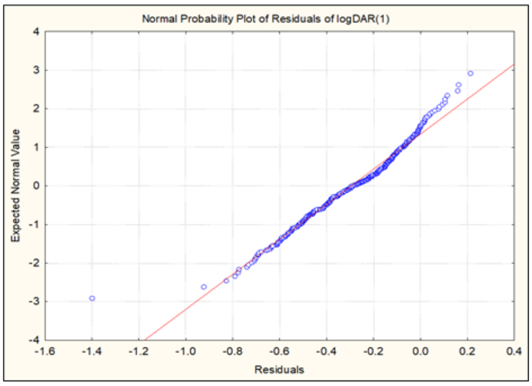
We study the stability case of six order of the model LogDAR(P) and take LogDAR(1) example of stable model and take LogDAR(6) example of unstable model.

Starting by LogDAR(1) ,we obtain the following model

$$y_t = 0.299374y_{t-1}+\eta_t\sqrt{0.499451 + 0.021029\log y_{t-1}^2} \quad ...(3.1)$$

$\eta_t \sim \mathrm{i.i.d}\ N(0,1)$, the unconditional variance is $\sigma_y^2 = 0.0467 > 0$,AIC(1)= -1152.19 , BIC(1)= -1140.38, the two-dimensional plot of model (3.1) has shown in figure 3.5 with normal probability plot of residuals.





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(b) **(a)**

Figure 3.5: (a) the normal probability plot of residuals and (b) plot of frequency distribution of LogDAR(1) model

And we calculate the non zero singular point of the model (3.1) by using equation (2.10) and we obtain then non zero singular point $\zeta = 0.7233$.

By using Proposition 2.1 the characteristic equation was write as

$$\lambda - 0.1298 = 0 \qquad \ldots(3.2)$$

The root of the equation is $\lambda_1 = 0.1298$ we obtain that the model has stable nonzero singular point.

To check this result we plot the trajectories of this model as the figure 3.6 which shows that the model is asymptotically stationary

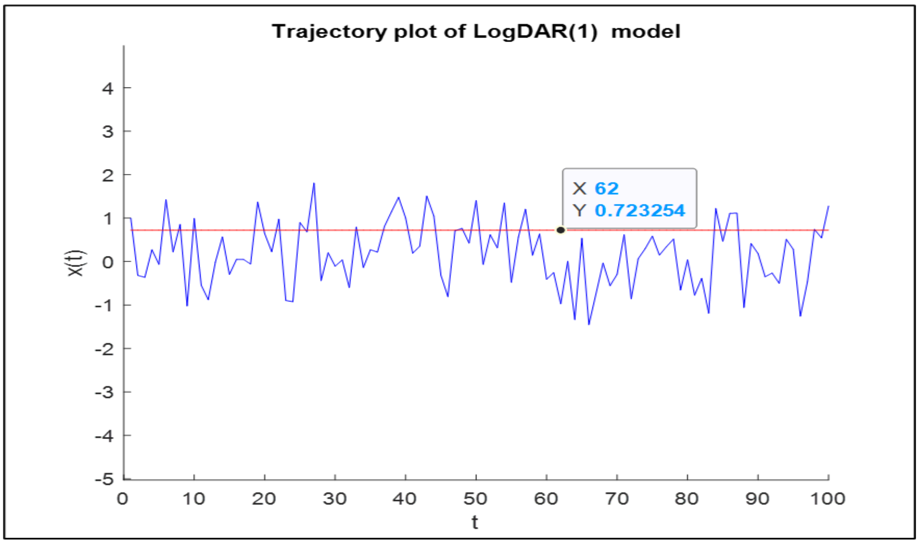

figure 3.6: the plot of the trajectory of LogDAR(1) model

Similarly, we obtain the following LogDAR(6) model

$$y_t = 0.240732 y_{t-1} + \eta_t \sqrt{5.421095 - 2.139887 \log y_{t-1}^2}$$

$$+ \ 0.216032 y_{t-2} + \eta_t \sqrt{5.421095 + 0.223390 \log y_{t-2}^2}$$

$$+ \ 0.378899 y_{t-3} + \eta_t \sqrt{5.421095 - 0.172187 \log y_{t-3}^2}$$

$$+ \ 0.159778 y_{t-4} + \eta_t \sqrt{5.421095 + 0.190210 \log y_{t-4}^2}$$

$$+ \ 0.177550 y_{t-5} + \eta_t \sqrt{5.421095 + 0.118944 \log y_{t-5}^2}$$

$$+ \ 0.129905 y_{t-6} + \eta_t \sqrt{5.421095 - 0.457812 \log y_{t-6}^2} \qquad \ldots(3.3)$$

$\eta_t \sim \text{i.i.d } N(0,1)$, the unconditional variance is $\sigma_y^2 = 0.0401 > 0$,AIC(6)= -1157.62, BIC(6)= -1106.61,

the normal probability plot of frequency Distribution of the model (3.3) is shows in figure 3.7

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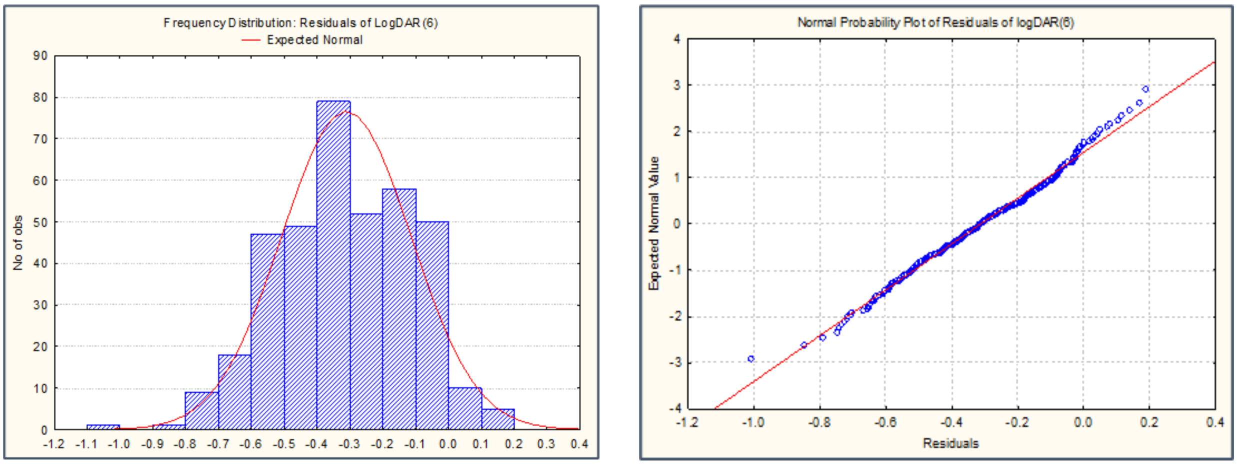

(b) (a)

Figure 3.7: (a) the normal probability plot of residuals and (b) plot of frequency distribution of LogDAR(6) model

And we calculate the non zero singular point of the model (3.3) by using equation(2.10) and we obtain then non zero singular point $\zeta = 1.0450$.

By using Proposition 2.1 the characteristic equation was write as

$$\lambda^6+(1.9016067223)\,\lambda^5-(0.2512347388)\,\lambda^4+(0.0141123409)\,\lambda^3-(0.1997074469)\,\lambda^2$$
$$-(0.1404445858)\lambda+(0.4023568514)=0 \qquad \ldots(3.4)$$

The roots of the equation is $\lambda_1 = 1.7330 + 0.0000i$, $\lambda_2 = -0.6715 + 0.0000i$,

$\lambda_3 = 0.6657 + 0.4216i$, $\lambda_4 = 0.6657 - 0.4216i$,

λ_5 $-0.2458 + 0.7045i$, $\lambda_6 = -0.2458 - 0.7045i$,

we obtain that the model has unstable nonzero singular point because $|\lambda_1| > 1$.

To check this result we plot the trajectories of this model as the figure 3.8 which shows that the model LogDAR(6) is non -stationary.

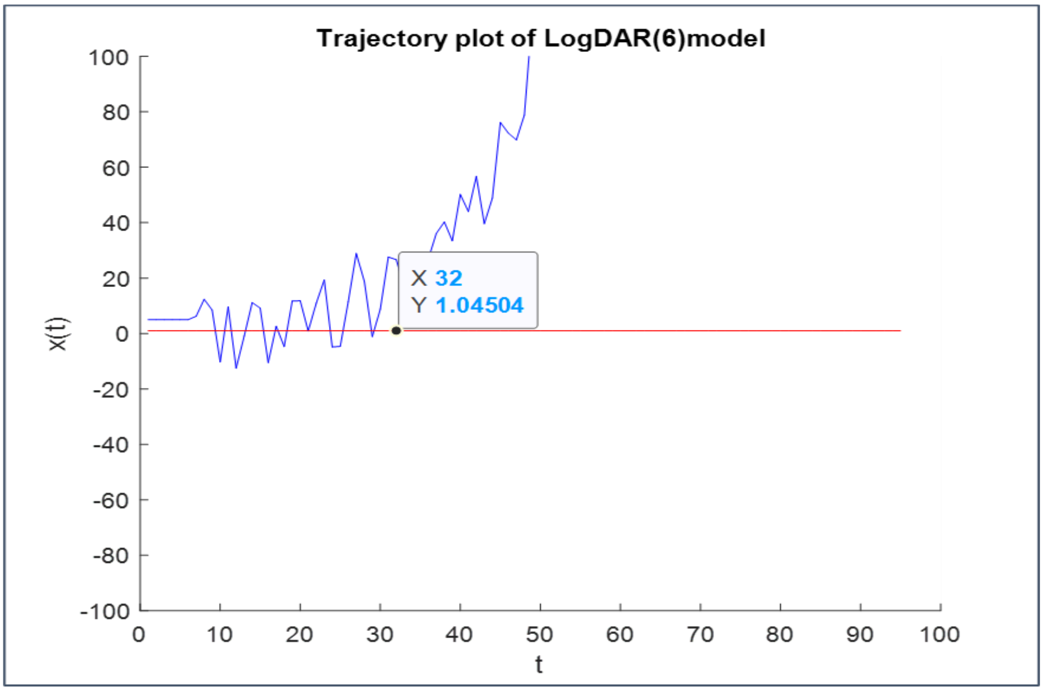

figure 3.8 : the plot of the trajectory of LogDAR(6) model

we make table of six order for the LogDAR(P) model and choose a best order for the model by using AIC and BIC information standards

Table 3.2 : the stability states ,AIC and BIC of deferent order for LogDAR model

Model	ζ	AIC(P)	BIC(P)	Stability states
LogDAR(1)	0.7233	-1152.19	-1140.38	Stable
LogDAR(2)	1.7477	-321.65	-301.96	Stable
LogDAR(3)	1.7153	-244.07	-216.51	Stable
LogDAR(4)	2.0112	-240.43	-204.99	Stable
LogDAR(5)	1.9945	-120.97	-77.66	Stable
LogDAR(6)	1.0450	-1157.62	-1106.61	Un stable

From table 3.2 we get the best model with less value of AIC and BIC the model is logDAR(1) and we previously introduced.

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دراسة استقرارية نموذج الإنحدار الذاتي اللوغارتمي المزدوج مع التطبيق

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الملخص

في هذا البحث تم اقتراح نموذج لا خطي في السلاسل الزمنية والذي يسمى نموذج الإنحدار الذاتي اللوغارتمي المزدوج مع التطبيق من الرتبة p والذي يرمز له اختصاراً بـــ LogDAR(p) وهذا النموذج يعتمد على النموذج DAR . والدالة اللوغارتيمية وقد تم ايجاد شروط استقراريه النموذج المقترح باستخدام تقنية التقريب بالخطية المحلية للنقطة الثابتة غير الصفرية للنموذج . وكذلك تم تطبيق هذه الشروط على النماذج المستحصلة من نمذجة بيانات حقيقية تمثل المعدل الشهري لأسعار الاغلاق بالدولار لعقود نفط خام برنت لمسنوات (1989-2021) وتم اثبات احتوائها على التطاير.