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A Solution Procedure for Fully Fuzzy Linear Fractional Model with Ranking Functions

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Abstract: This work is about a type of fully fuzzy fractional linear programming problem (FFFLPP) in which all of the objective function (OF) and constraint coefficients and decision variables (DV) are fuzzy numbers (FN). To tackle these issues, a strategy based on the three ranking function method (TRFM) is offered. We employ the simplex method (SM) to deal with constraints and the ranking function method (RFM) of fuzzy numbers (FN) to rank fuzzy objective functions in order to arrive at an optimal solution (OS). It is proposed a computational approach for obtaining an optimal solution (OS).Finally, to demonstrate the proposed method, a numerical example is provided. The results show that when using the ranking function method (RFM) to transform fully fuzzy linear programming problem (FFFLPP) to linear programming problem (LPP) and solved by using simplex method (SM) to obtain the optimal solution (OS), the proposed strategy provides a superior optimal solution (OS).

Keywords: fuzzy set (FS), Pentagonal fuzzy number (PFN), ranking function (RF), linear programming problems (LPP), fully fuzzy fractional linear programming problems (FFFLPP).

1. Introduction:

The fractional programming problem (FPP) is a decision-making challenge that arises while attempting to maximize a ratio that is constrained. The problem of linear fractional programming is now widely used in a variety of real-world applications, including production planning, financial services, health care, and all engineering fields. Charnes and Cooper [1] have proposed that the linear fractional programming problem (LFPP) can be optimized. The coefficients of the objective function, as well as the restrictions and resources, are expected to be correct in such issues. However, the coefficients are not exact in practice due to measurement mistakes, changes in market conditions, or other uncontrolled issues (climate, traffic, customers etc.). In this case, it is highly usual for decision makers (DMs) to be hesitant to estimate their desired level of objective function as well as the problem parameters. In such situations, a decision-making must deal with doubt and hesitancy. The intuitionistic fuzzy linear fractional programming problem can be used to simulate these situations effectively. Many researchers have looked into fuzzy linear fractional programming [2-6]. D. Sahoo et al. [7] have studied multi-objective linear fractional programming problem with pentagonal intuitionistic fuzzy numbers. N. Safae [8] proposed a new method for solving FFLFP has been developed. The authors [9] proposed a new multi – objective linear programming approach for solving FFFLPP. They also use ranking functions to study the objective function values with fuzzy numbers [10 - 14]. The goal of this study is to address a type of fuzzy linear fractional programming problem in which all variables are pentagonal FN. In this research, we investigate the problem using a simplex technique after transforming the problem to crisp values using three ranking algorithms.

The remain parts of this paper were orchestrated as follow: some basic definitions with respect to the pentagonal fuzzy numbers are introduced in Section 2. we provide FFFLPP was contained in Section 3. The explained the Fully Fuzzy Linear Programming (FFLP) Problem with Pentagonal Fuzzy Numbers in section 4. We presents are ranking function of fuzzy number in section 5.

Shows the algorithm in Section 6. A numerical example is provided for illustration in Section 7. Conclusion is presented in Section 8.

2.Preliminaries:

We covered the basic concepts of FS and pentagonal FN in this section, which were quite beneficial in this work.

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2.1 Definition (FS): [15]

A FS $\tilde{\mathcal{U}}$ is defined by $\tilde{\mathcal{U}}$= {(g, $\tilde{\mathcal{U}}$(g)): g ∈$\mathcal{S}$, $\mu_{\tilde{\mathcal{U}}}$ (g)∈[0,1] }. In the pair (g,̃(g)), the first member g belongs to the classical set $\mathcal{S}$, while the second element $\mu_{\tilde{\mathcal{U}}}$ (g) belongs to the interval [0,1], which is known as Membership function.

2.2 Definition (PFN): [16]

A FN $\tilde{\mathcal{U}}$ is a PFN defined by $\tilde{\mathcal{U}} = (f, h, i, j, k)$, where f, h, i, j, k are real numbers and the membership function $\mu_{\tilde{\mathcal{U}}}$ (g) is given by

$$\mu_{\tilde{\mathcal{U}}}(g) = \begin{cases} 0 & for \quad g < f \\ \frac{1}{2}\frac{(g-f)}{(h-f)} & for \quad f \le g \le h \\ \frac{1}{2}+\frac{1}{2}\frac{(g-h)}{(i-h)} & for \quad h \le g \le i \\ 1 & for \quad g = i \\ \frac{1}{2}+\frac{1}{2}\frac{(j-g)}{(j-i)} & for \quad i \le g \le j \\ \frac{1}{2}\frac{(k-g)}{(k-j)} & for \quad j \le g \le k \\ 0 & g > k \end{cases}$$

3.FFFLPP:[17]

A FFFLPP with PFN can be defined as:

$$Max\ \tilde{\tilde{\mathfrak{J}}} = \frac{\tilde{\check{c}}\,\tilde{h} + \check{e}}{d\,\tilde{h} + \tilde{\check{k}}}$$

Subject to

$$\check{\tilde{D}}\,\tilde{h} \le \tilde{E}$$

$$\tilde{h} \ge 0$$

Where $\tilde{E} \in \mathcal{Y}(\mathfrak{R})^{t}$, $\tilde{h} \in \mathcal{Y}(\mathfrak{R})^{r}$, $\check{\tilde{D}} \in \mathcal{Y}(\mathfrak{R})^{t*r}$ and $\tilde{\check{c}}$, d , $\check{e}$, $\tilde{\check{k}} \in \mathcal{Y}(\mathfrak{R})^{r}$.

4. FFLPP with PFN is defined as [18] :

$$Max\ \tilde{\tilde{\mathfrak{J}}} = \tilde{\check{c}}\,\tilde{h}$$

Subject to

$$\check{\tilde{D}}\,\tilde{h} \le \tilde{E}$$

$$\tilde{h} \ge 0$$

Where $\tilde{E} \in \mathcal{Y}(\mathfrak{R})^{t}$, $\tilde{h} \in \mathcal{Y}(\mathfrak{R})^{r}$, $\check{\tilde{D}} \in \mathcal{Y}(\mathfrak{R})^{t*r}$ and $\tilde{\check{c}} \in \mathcal{Y}(\mathfrak{R})^{r}$.

5. Ranking Functions[19]:

Defining a ranking function $\mathcal{F}(\mathcal{H})$ is also an effective way of arranging the items of $\mathcal{H}\ (\tilde{A}_P) : \mathcal{F}(\mathcal{H}) \to \mathcal{H}$ is a function that converts each FN into a real line with a natural order.

We define orders on $\mathcal{F}(\mathcal{H})$ by:

$\tilde{A}_P \ge \tilde{B}_P$ if and only if $\mathcal{H}\ (\tilde{A}_P) \ge \mathcal{H}\ (\tilde{B}_P)$

$\tilde{A}_P \le \tilde{B}_P$ if and only if $\mathcal{H}\ (\tilde{A}_P) \le \mathcal{H}\ (\tilde{B}_P)$

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$\tilde{A}_P = \tilde{B}_P$ if and only if $\mathcal{H}(\tilde{A}_P) = \mathcal{H}(\tilde{B}_P)$

6. Algorithm for solution FFLFPP with PFN:

In this section, we'll show you how to solve a FLFPP using the procedures below:

Step 1: Consider the LFPP in which the objective function and constraints have pentagonal fully fuzzy parameters.

Step 2: Convert the FFLFPP to the corresponding FFLPP, utilize Complementary technique.

Step 3: Using three RF, convert a FFLPP to a crisp linear programming (CLP) problem.

Step 4: To find the best solution to problems, compare three RF with PFN.

Step 5: Using the simplex approach and the Win QSB program, solve the LPP to find the OS, which is the most efficient solution.

7. Numerical Example:

Consider the following FFLFPP

$$Max\, \mathfrak{I} = \frac{(17,18,8,10,12) w_1 + (16,19,6,9,11)\, w_2}{(10,12,5,7,9)\, w_1 + (9,10,3,4,6)\, w_2}$$

subject to

$(4,5,3,4,6) w_1 + (5,8,3,4,6)\, w_2 \leq (7,14,4,5,7)$

$(0,1,2,3,5) w_1 + (3,5,7,9,11)\, w_2 \leq (11,19,6,7,9)$

$w_1 , w_2 \geq 0.$

Convert the FFLFPP to the FFLPP, utilize Complementary technique

$Max\, \mathfrak{I}_1 = (17,18,8,10,12) w_1 + (16,19,6,9,11)\, w_2$

subject to

$(4,5,3,4,6) w_1 + (5,8,3,4,6)\, w_2 \leq (7,14,4,5,7)$

$(0,1,2,3,5) w_1 + (3,5,7,9,11)\, w_2 \leq 11,19,6,7,9)$

$w_1 , w_2 \geq 0.$

$Min\, \mathfrak{I}_2 = (10,12,5,7,9)\, w_1 + (9,10,3,4,6)\, w_2$

subject to

$(4,5,3,4,6) w_1 + (5,8,3,4,6)\, w_2 \leq (7,14,4,5,7)$

$(0,1,2,3,5) w_1 + (3,5,7,9,11)\, w_2 \leq 11,19,6,7,9)$

$w_1 , w_2 \geq 0.$

$Max\, \mathfrak{I}^* = (5,8,3,3,1) w_1 + (6,10,3,5,5)\, w_2$

subject to

$(4,5,3,4,6) w_1 + (5,8,3,4,6)\, w_2 \leq (7,14,4,5,7)$

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$(0,1,2,3,5)w_1 + (3,5,7,9,11)\, w_2 \leq 11,19,6,7,9)$

$w_1, w_2 \geq 0.$

The issue of PLP problem is converted into crisp linear problem (CLP) by using, first ranking function [20] $\mathfrak{Z}(\tilde{\vartheta}) = \frac{f+h+2i+j+k}{6}$, and the problem is as follows:

$$\mathfrak{Z}(\tilde{\vartheta}) = \frac{f+h+2i+j+k}{6} \quad \ldots\ldots (1)$$

$Max\ \mathfrak{I}^* = 3.833\, w_1 + 5.333\, w_2$

subject to

$4.166\, w_1 + 4.833\, w_2 \leq 6.833$

$2.166\, w_1 + 7\, w_2 \leq 9.666$

$w_1, w_2 \geq 0.$

Using Win QSB, we can solve the crisp LP problem and obtain the best solution.

$w_1 = 0.0597$, $w_2 = 1.3624$, $Max\ \mathfrak{I}^* = 7.4943$ are the solutions.

The issue of PLP problem is converted into CLP by using, second ranking function [20] $\mathfrak{Z}(\tilde{\vartheta}) = \frac{f+h+i+j+k}{5}$, and the problem is as follows:

$$\mathfrak{Z}(\tilde{\vartheta}) = \frac{f+h+i+j+k}{5} \quad \ldots\ldots (2)$$

$Max\ \mathfrak{I}^* = 4\, x_1 + 5.8\, x_2$

subject to

$4.4\, w_1 + 5.2\, w_2 \leq 7.4$

$2.2\, w_1 + 7\, w_2 \leq 10.4$

$w_1, w_2 \geq 0.$

Using Win QSB, we can solve the CLP problem and obtain the best solution.

$w_1 = 0$, $w_2 = 1.4231$, $Max\ \mathfrak{I}^* = 8.2538$ are the solutions.

The issue of PLP problem is converted into C LP problem by using, three ranking function [21] $\mathfrak{Z}(\tilde{\vartheta}) = \frac{2f+3h+2i+3j+2k}{4}$, and the problem is as follows:

$$\mathfrak{Z}(\tilde{\vartheta}) = \frac{2f+3h+2i+3j+2k}{4} \quad \ldots. (3)$$

$Max\ \mathfrak{I}^* = 12.75\, w_1 + 18.25\, w_2$

subject to

$13.25\, w_1 + 16\, w_2 \leq 23.25$

$6.5\, w_1 + 21\, w_2 \leq 32.5$

$w_1, w_2 \geq 0.$

Using Win QSB, we can solve the crisp LP problem and obtain the best solution.

$w_1 = 0$, $w_2 = 1.4531$, $Max\ \mathfrak{J}^* = 26.5195$ are the answers.

Let's look at the following table that compares three ranking methods, and it's evident that our, three ranking function is always maximizing the result.

No.	Ranking Function	Transform ranking function	optimal solution
1	Ranking Function I	$Max\ \mathfrak{J}^* = 3.833\ w_1 + 5.333\ w_2$	$w_1 = 0.0597$, $w_2 = 1.3624$, $Max\ \mathfrak{J}^* = 7.4943$
2	Ranking Function II	$Max\ \mathfrak{J}^* = 4\ w_1 + 5.8\ w_2$	$w_1 = 0$, $w_2 = 1.4231$, $Max\ \mathfrak{J}^* = 8.2538$
3	Ranking Function III	$Max\ \mathfrak{J}^* = 12.75\ w_1 + 18.25\ w_2$	$w_1 = 0$, $w_2 = 1.4531$, $Max\ \mathfrak{J}^* = 26.5195$

8. Conclusion

We solved a pentagonal linear fractional programming problem in this article. We utilized three different ranking functions to convert pentagonal numbers to crisp numbers. We solve the problem using the simplex approach after utilizing this ranking algorithm to transform the problem to crisp values.

Reference

[1] Charnes, A. and Cooper, W.W. (1962). Programming with linear fractional functions, Naval Research Logistics Quart ., Vol.9, pp. 181-186.

[2] Sapan Kumar Das , S.A. Edalatpanah , Jatindra Kumar Dash, An Intelligent Dual Simplex Method to Solve Triangular Neutrosophic Linear Fractional Programming Problem, Neutrosophic Sets and Systems, Vol. 36, 2020.

[3] Majed G. Alharbi and Hamiden Abd El- Wahed Khalifa , On Solutions of Fully Fuzzy Linear Fractional Programming Problems Using Close Interval Approximation for Normalized Heptagonal Fuzzy Numbers , Applied Mathematics & Information Sciences An International Journal, 15, No. 4, 471-477 (2021).

[4] Mitlif R J, An Efficient Algorithm for Fuzzy Linear Fractional Programming Problems via Ranking Function, Baghdad Science Journal, Vol. 19, No. 1, 2022, p: 71-76.

[5] Mitlif R J, Computation the Optimal Solution of Octagonal Fuzzy Numbers, Journal of Al-Qadisiyah for Computer Science and Mathematics, Vol.12, No.4, 2020, pp: 71–78.

[6] Rasha Jalal Mitlif , Solving fuzzy fractional linear programming problems by ranking function methods , JOURNAL OF COLLEGE OF EDUCATION, 2016,.NO 1., pp 93-108.

[7] D. Sahoo , A.K. Tripathy , J.K. Pati , Study on multi-objective linear fractional programming problem involving pentagonal intuitionistic fuzzy number, Results in Control and Optimization, 6 , 2022.

[8] N. Safaei , A new method for solving fully fuzzy linear fractional

programming with a triangular fuzzy numbers, App. Math. and Comp. Intel., Vol. 3(1) (2014) 273–281.

[9] Sapan Kumar Das1, Tarni Mandal and S.A. Edalatpanah, A NEW APPROACH FOR SOLVING FULLY FUZZY LINEAR FRACTIONAL

PROGRAMMING PROBLEMS USING THE MULTI-OBJECTIVE LINEAR PROGRAMMING, RAIRO-Oper. Res, 51 (2017) 285–297.

[10] Rasha Jalal Mitlif , A New Method for Solving Fully Fuzzy Multi-Objective Linear Programming Problems , Iraqi Journal of Science, 2016, Vol. 57, No.3C, pp:2307-2311.

[11] Moumita Deb and P. K. De , Optimal Solution of a Fully Fuzzy Linear Fractional Programming Problem by Using Graded Mean Integration Representation Method , Applications and Applied Mathematics: An International Journal (AAM) , Vol. 10, Issue 1 , 2015, 571-587.

[12] Mitlif R J, Fatema Ahmad Sadiq, Finding the Critical Path Method for Fuzzy Network with Development Ranking Function, Journal of Al-Qadisiyah for Computer Science and Mathematics, Vol. 13, No. 3 , 2021 , pp : 98–106.

[13] Mitlif R J, Ranking Function Application for Optimal Solution of Fractional Programming Problem, Al-Qadisiyah Journal Of Pure Science, Vol.25 , No.1, 2020, p:27-35.

[14] I H Hussein and R J Mitlif, Ranking Function to Solve a Fuzzy Multiple Objective Function, Baghdad Science Journal, Vol. 18, No. 1, 2021, p:144-148.

[15] Al. Nachammai , Solving Fuzzy Linear Fractional Programming Problem Using Metric Distance Ranking, Applied Mathematical Sciences, Vol. 6, 2012, no. 26, 1275 – 1285.

[16] D. Stephen Dinagar and M. Mohamed Jeyavuthin, Fully Fuzzy Integer Linear Programming Problems Under Robust Ranking Techniques, Int. J. Math. And Appl., 6(3)(2018), 19- 25.

[17] Amir Sabir Majeed , Solving Linear Fractional Programming Problem in symmetric trapezoidal fuzzy environment, Journal of University of Garmian, 6 (3), 2019 , 368-374.

[18] Manuel Arana Jiménez, Carmen Sánchez Gil , On generating the set of nondominated solutions of a linear programming problem with parameterized fuzzy numbers , Journal of Global Optimization , 2019.

[19] S. Kumar-Das , A New Method for Solving Fuzzy Linear Fractional Programming Problem with New Ranking Function , International Journal of Research in Industrial Engineering, Vol. 8, No. 4 (2019) 384–393.

[20] D. Stephen Dinagar and M. Mohamed Jeyavuthin , Distinct Methods for Solving Fully Fuzzy Linear Programming Problems with Pentagonal Fuzzy Numbers , Journal of Computer and Mathematical Sciences, Vol.10(6),1253-1260 , 2019.

[21] Ponnivalavan. K. and Pathinathan. T. , Ranking of a Pentagonal Fuzzy Number and Its Applications, Journal of Computer and Mathematical Sciences,Vol.6(11),571-584, 2015.