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Exploration on Bulk Arrival Queues with Heptagonal and Decagonal Fuzzy Parameters

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ABSTRACT

Bulk arrival queuing models throw enormous functionality in situations where customers are waiting for long durations to attain service, recovers the problem in separating to multiple phases, estimates productivity measures, consumes time, reduces loss, enhances optimality and tackles effectively congestions. The normal queues stretched to fuzzy queues have multifaceted nourishment in all fields. This paper reviews the non-linear parametric programming approach to the K-phase fuzzy queues with its membership functions adopting heptagonal and decagonal fuzzy numbers with numerical illustrations and graphs.

Keywords: Fuzzy Queuing, membership function, Mixed integer non-linear programming.

I. INTRODUCTION

Waiting queues are an everyday occurrence, productive mechanism, impacting people for all basic chores and necessities which can take the form of devices waiting to be fixed, trucks in line to be delivered, or planes lined up on a runway waiting for going to take off etc., The three vital ingredients of a queuing process are arrivals, service facilities, and the actual waiting line which has relevance and theoretical structure in real life overcrowding scenarios like production process, computer and telecommunication systems, service and distribution systems etc., Fuzzy queues can accommodate the ambiguities of real-world human language and logic in relevant to practical context than the classical ones to widen applicability in measurement technology, telecommunications, software development, has enormous compatibility in all ventures such as connectivity infrastructures, commodities, commercial centers, more economical, lowers the cost, has flexibility and enhances adaptability in all fields, optimizes inputs depending on the scope demanded and anticipates extempore frameworks for excellent future prognostication.

MINLP signifies optimization problems with continuous and discrete parameters, incorporates combinatorial and non-linear optimal computational construction constraints, which are generated with a structural frame that describes production. Ranking mechanisms are particularly substantial to handle imprecise data strategy for defuzzification. Many researchers have produced their eminent investigations like Chaudry M.L.,[4], Kao. C, C.C. Li, S.P. Chen [3], Shih-Pin Chen [8], Sharma R.,[7], Ramya S and Jeba Presitha [6], Sivaraman Geetha, Bharathi Ramesh Kumar and Sankara Murugesan [9], Anushya, B., Ramaand, B. and Sudha, L.[1] on the conceptual aspects for phase services,bulk arrival queues, Erlangian and parametric programming with distinct types of fuzzy parameters. This journal projects the interval level of optimality of K-phase queuing system with the construction of inverse membership functions for heptagonal and decagonal fuzzy parameters and the range of the performance indicators are calculated using RBT technique.

II. BASIC CONCEPTIONS

1. Heptagonal Fuzzy Number: The membership function of a $HPTAFN(h_{hepta1}, h_{hepta2}, h_{hepta3}, h_{hepta4}, h_{hepta5}, h_{hepta6}, h_{hepta7})$ is,

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$$\phi_{HPTA}(x)=\begin{cases}0 & ,x<h_{hepta1}\\ \frac{1}{2}\left[\frac{x-h_{hepta1}}{h_{hepta2}-h_{hepta1}}\right] & ,h_{hepta1}\le x\le h_{hepta2}\\ \frac{1}{2} & ,h_{hepta2}\le x\le h_{hepta3}\\ \frac{1}{2}+\frac{1}{2}\left[\frac{x-h_{hepta3}}{h_{hepta4}-h_{hepta3}}\right] & ,h_{hepta3}\le x\le h_{hepta4}\\ \frac{1}{2}+\frac{1}{2}\left[\frac{h_{hepta5}-x}{h_{hepta5}-h_{hepta4}}\right] & ,h_{hepta4}\le x\le h_{hepta5}\\ \frac{1}{2} & ,h_{hepta5}\le x\le h_{hepta6}\\ \frac{1}{2}\left[\frac{h_{hepta7}-x}{h_{hepta7}-h_{hepta6}}\right] & ,h_{hepta6}\le x\le h_{hepta7}\\ 0 & ,x\ge h_{hepta7}\end{cases}$$

2. **Decagonal Fuzzy Number:** The membership function of a $DECAFN(h_{deca1},h_{deca2},h_{deca3},h_{deca4},h_{deca5},h_{deca6},h_{deca7},h_{deca8},h_{deca9},h_{deca10})$ is,

$$\boldsymbol{\phi_{DECA}(x)}=\begin{cases}\frac{1}{4}\left(\frac{x-h_{deca1}}{h_{deca2}-h_{deca1}}\right) & ,h_{deca1}\le x\le h_{deca2}\\ \frac{1}{4}+\frac{1}{4}\left(\frac{x-h_{deca2}}{h_{deca3}-h_{deca2}}\right) & ,h_{deca2}\le x\le h_{deca3}\\ \frac{1}{2}+\frac{1}{4}\left(\frac{x-h_{deca3}}{h_{deca4}-h_{deca3}}\right) & ,h_{deca3}\le x\le h_{deca4}\\ \frac{3}{4}+\frac{1}{4}\left(\frac{x-h_{deca4}}{h_{deca5}-h_{deca4}}\right) & ,h_{deca4}\le x\le h_{deca5}\\ 1 & ,h_{deca5}\le x\le h_{deca6}\\ 1-\frac{1}{4}\left(\frac{x-h_{deca6}}{h_{deca7}-h_{deca6}}\right) & ,h_{deca6}\le x\le h_{deca7}\\ \frac{3}{4}-\frac{1}{4}\left(\frac{x-h_{deca7}}{h_{deca8}-h_{deca7}}\right) & ,h_{deca7}\le x\le h_{deca8}\\ \frac{1}{2}-\frac{1}{4}\left(\frac{x-h_{deca8}}{h_{deca9}-h_{deca8}}\right) & ,h_{deca8}\le x\le h_{deca9}\\ \frac{1}{4}\left(\frac{h_{deca10}-x}{h_{deca10}-h_{deca9}}\right) & ,h_{deca9}\le x\le h_{deca10}\\ 0 & ,otherwise\end{cases}$$

III. FUZZY QUEUES WITH K-PHASE INFINITE CAPACITY MODEL

Consider fuzzy queues with k-phase infinite capacity in which the customer's entry follows Poisson process and service time has Erlang type k-phases queuing network in heptagonal and decagonal fuzzy numbers and structuring its membership functions with performance measures opting Zadeh's extension principle as

$$\mu_{w_q}(z)=\sup\min\left\{\left[\mu_{\tilde{\lambda}_{pm}}(x),\mu_{\tilde{\mu}_{erg}}(y)\right], where\ z=P(x,y)=\frac{6}{10}\left(\frac{\tilde{\lambda}_{pn}}{\tilde{\mu}_{erg}(\tilde{\mu}_{erg}-\tilde{\lambda}_{pn})}\right)\right\}$$

$$\mu_{L_q}(z)=\sup min\left\{\left[\mu_{\tilde{\lambda}_{pn}}(x),\mu_{\tilde{\mu}_{erg}}(y)\right], where\ z=P(x,y)=\tilde{\lambda}_{pn}\cdot W_q\right\}$$

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$$\mu_W(z) = \sup min\left\{\left[\mu_{\tilde{\lambda}_{pn}}(x), \mu_{\tilde{\mu}_{erg}}(y)\right], where\ z = P(x,y) = W_q + \frac{1}{\tilde{\mu}_{erg}}\right\}$$

$$\mu_L(z) = \sup min\left\{\left[\mu_{\tilde{\lambda}_{pn}}(x), \mu_{\tilde{\mu}_{erg}}(y)\right], where\ z = P(x,y) = \tilde{\lambda}_{pn}\cdot W\right\}$$

The notion is to entrench the mathematical programming interface with different levels of α for estimation of the performance measures $L,\ L_q,\ W,\ W_q$.

Solution Procedure

To re-express the membership function system characteristic in a usual form we adopt Zadeh's approach based on the alpha cuts.

The alpha level sets of the input and service process is expressed as

$$\tilde{\lambda}_{pn}(\alpha) = \left[x_\alpha^{lower}, x_\alpha^{upper}\right] = \left[\min\left\{x / \mu_{\lambda_{pn}}(x) \ge \alpha\right\}, \max\left\{x / \mu_{\lambda_{pn}}(x) \ge \alpha\right\}\right]$$

$$\tilde{\mu}_{erg}(\alpha) = \left[y_\alpha^{lower}, y_\alpha^{upper}\right] = \left[\min\left\{y / \mu_{\mu_{erg}}(y) \ge \alpha\right\}, \max\left\{y / \mu_{\mu_{erg}}(y) \ge \alpha\right\}\right]$$

denotes the queue constraints in different possibility levels of intervals, which indicates the sets of movable boundaries forming nested structures for expressing the relation between ordinary and fuzzy sets. $FM/FE_K/1$ can be visualized as a family of normal queues $M/E_K/1$.

By convexity of fuzzy numbers, the bounds of α -cut fuzzy interval obtained as

$$x_\alpha^{lower} = \min\left(\mu_{\tilde{\lambda}_{pn}}^{-1}(\alpha)\right); \qquad x_\alpha^{upper} = \max\left(\mu_{\tilde{\lambda}_{pn}}^{-1}(\alpha)\right)$$

$$y_\alpha^{lower} = \min\left(\mu_{\tilde{\mu}_{erg}}^{-1}(\alpha)\right); \qquad y_\alpha^{upper} = \max\left(\mu_{\tilde{\mu}_{erg}}^{-1}(\alpha)\right)$$

$\mu_{\bar{L}_q}(z)$, $\mu_{\bar{W}_q}(z)$, $\mu_W(z)$, $\mu_L(z)$ is the minimum of $\mu_{\bar{\lambda}}(x)$ and $\mu_{\bar{\mu}}(y)$.

To tackle the membership value, we need either of the two cases:

(i) $\mu_{\bar{\lambda}}(x) = \alpha$ and $\mu_{\bar{\mu}}(y) \ge \alpha$; (ii) $\mu_{\bar{\lambda}}(x) \ge \alpha$ and $\mu_{\bar{\mu}}(y) = \alpha$

such that $L_q(z)$, $W_q(z)$, $\mu_W(z)$, $\mu_L(z)$ to satisfy that $\mu_{L_q}(z) = \alpha$; $\mu_{W_q}(z) = \alpha$; $\mu_W(z) = \alpha$; $\mu_L(z) = \alpha$. This can be accomplished through PNLP technique.

The lower and upper bounds for the measures are

$$L_{q_{\bar{\alpha}}}^{lower}(z) = \min\frac{6}{10}\left(\frac{\tilde{\lambda}_{pn}}{\tilde{\mu}_{erg}(\tilde{\mu}_{erg} - \tilde{\lambda}_{pn})}\right); \qquad L_{q_{\bar{\alpha}}}^{upper}(z) = \max\frac{6}{10}\left(\frac{\tilde{\lambda}_{pn}}{\tilde{\mu}_{erg}(\tilde{\mu}_{erg} - \tilde{\lambda}_{pn})}\right)$$

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$$L_{q_{\tilde{\alpha}}}^{lower}(z) = \min(\tilde{\lambda}_{pn} \cdot W_q) \quad ; \quad L_{q_{\tilde{\alpha}}}^{lower}(z) = \max(\tilde{\lambda}_{pn} \cdot W_q)$$

$$L_{q_{\tilde{\alpha}}}^{lower}(z) = \min\left\{W_q + \frac{1}{\tilde{\mu}_{erg}}\right\} \quad ; \quad L_{q_{\tilde{\alpha}}}^{lower}(z) = \max\left\{W_q + \frac{1}{\tilde{\mu}_{erg}}\right\}$$

$$L_{q_{\tilde{\alpha}}}^{lower}(z) = \min(\overline{\lambda}_{pn} \cdot W_q) \quad ; \quad L_{q_{\tilde{\alpha}}}^{lower}(z) = \max(\tilde{\lambda}_{pn} \cdot W_q)$$

such that $x \in \left[x_{\alpha}^{lower}, x_{\alpha}^{upper}\right]$; $y \in \left[y_{\alpha}^{lower}, y_{\alpha}^{upper}\right]$; where $x \in \lambda(\alpha)$ and $y \in \mu(\alpha)$.

These pair of mathematical programs help in systematic study of optimality when the bounds of the interval vary in $[0,1]$ which fall into the category of PNLP. If $L_q(z)$,

$W_q(z)$, $\mu_W(z)$, $\mu_L(z)$ are invertible with respect to α, then the left and right shape

functions are $\left[L(z), R(z)\right] = L_{q_\alpha}^{-1}$; $\left[L(z), R(z)\right] = W_{q_\alpha}^{-1}$; $\left[L(z), R(z)\right] = L_{\alpha}^{-1}$ and $\left[L(z), R(z)\right] = W_{\alpha}^{-1}$ obtained from the membership functions as:

$$\mu_{L_q}(z), \mu_{W_q}(z), \mu_W(z), \mu_L(z) = \begin{cases} LEFT(z) & , \ z_1 \le z \le z_2 \\ 1 & , \ z = z_1 \\ RIGHT(z), & z_2 \le z \le z_3 \end{cases}$$

In most cases, the supports of the interval cannot be solved numerically which does not yield closed form membership function. Here, we can find the numerical solutions to approximate the shapes of the measures. The fuzziness values are converted to crisp value using Robust Ranking Technique and we estimate the queue parameters.

IV. Numerical Illustration

Consider the centralized five parallel processing system in which the arrival rate occurs at different level of phases. The system manager wants to evaluate the performance measures of the system such as the expected number of customers in the queue and system, expected number of customers waiting in the queue and the system with *k* = 5 phases and the steady state condition is $\rho = \dfrac{\tilde{\lambda}_{pn}}{\tilde{\mu}_{erg}} < 1$.

(i) For Heptagonal Fuzzy Numbers

Consider the arrival and Service rates as heptagonal fuzzy numbers with

$\lambda_{HEPTA} = [1, 2, 3, 4, 5, 6, 7]$ and $\mu_{HEPTA} = [11, 12, 13, 14, 15, 16, 17]$

The interval of confidence at possibility level are $\left[3\alpha + 1, 7 - 3\alpha\right]$ and $\left[3\alpha + 11, 17 - 3\alpha\right]$

(i) **Average Length of the queue**

The membership function is calculated as

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$$\phi_{L_{HEPTAq}}(z) = \begin{cases} \dfrac{(750z+18) \pm \sqrt{72900z^2+174960z}}{180z-54} & , \dfrac{3}{1360} \le z \le \dfrac{12}{175} \\ 1 & , \dfrac{12}{175} \le z \le \dfrac{3}{10} \\ \dfrac{-(390z+126) \pm \sqrt{72900z^2+174960z}}{180z-54} & , \dfrac{3}{10} \le z \le \dfrac{147}{220} \end{cases}$$

(ii) Average waiting time in the queues

The membership function is obtained as

$$\phi_{W_{HEPTAq}}(z) = \begin{cases} \dfrac{(750z+9) \pm \sqrt{72900z^2+14580z+81}}{180z} & , \dfrac{3}{1360} \le z \le \dfrac{3}{175} \\ 1 & , \dfrac{3}{175} \le z \le \dfrac{3}{175} \\ \dfrac{-(390z+9) \pm \sqrt{72900z^2+14580z+81}}{180z} & , \dfrac{3}{175} \le z \le \dfrac{21}{220} \end{cases}$$

(iii) Average number of customers in the system

The membership function is obtained as,

$$\phi_{L_{HEPTA}}(z) = \begin{cases} \dfrac{(750z+228) \pm \sqrt{72900z^2+29160z+72900}}{180z+126} & , \dfrac{83}{1360} \le z \le \dfrac{62}{175} \\ 1 & , \dfrac{62}{175} \le z \le \dfrac{62}{175} \\ \dfrac{-(390z-24) \pm \sqrt{72900z^2+29160z+72900}}{180z+186} & , \dfrac{62}{175} \le z \le \dfrac{287}{220} \end{cases}$$

(iv) Average waiting time of customers in the system

The membership function is determined as

$$\phi_{W_{HEPTA}}(z) = \begin{cases} \dfrac{-(21-750z) \pm \sqrt{72900z^2-1620z+441}}{180z} & , \dfrac{83}{1360} \le z \le \dfrac{31}{350} \\ 1 & , \dfrac{31}{350} \le z \le \dfrac{31}{350} \\ \dfrac{-(390z-21) \pm \sqrt{72900z^2-1620z+441}}{180z} & , \dfrac{31}{350} \le z \le \dfrac{41}{220} \end{cases}$$

(ii) For Decagonal Fuzzy Numbers

Assume the entry and exit rate of customers as decagonal Fuzzy numbers with

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$\lambda_{DECA} = [3,4,5,6,7,9,10,11,12,13]$ and $\mu_{DECA} = [11,12,13,14,15,17,18,19,20,21]$

The possibility level of α-cuts: $\lambda_{DECA} = [4\alpha+3, 13-4\alpha]$; $\mu_{DECA} = [4\alpha+11, 21-4\alpha]$

i) Average length of the queue

$$l_{L_{DECAq}}(\alpha) = \;= \frac{48\alpha^2+72\alpha+27}{160\alpha^2-1200\alpha+1890}\,; U_{L_{DECAq}}(\alpha) = \frac{48\alpha^2-312\alpha+507}{160\alpha^2+400\alpha-110}$$

The membership function is estimated as

$$\phi_{L_{DECAq}}(z) = \begin{cases} \dfrac{(1200z+72)\pm\sqrt{230400z^2+552960z}}{320z-96}, & \dfrac{1}{70} \le z \le \dfrac{147}{850} \\ 1, & \dfrac{147}{850} \le z \le \dfrac{3}{10} \\ \dfrac{-(400z+312)\pm\sqrt{230400z^2+552960z}}{320z-96}, & \dfrac{3}{10} \le z \le \dfrac{27}{50} \end{cases}$$

(ii) Average waiting time in the queue

$$l_{W_{DECAq}} = \frac{12\alpha+9}{160\alpha^2-1200\alpha+1890}\,; U_{W_{DECAq}}(\alpha) = \frac{39-12\alpha}{160\alpha^2+400\alpha-110}$$

The membership function is derived as,

$$\phi_{W_{DECAq}}(z) = \begin{cases} \dfrac{(1200z+12)\pm\sqrt{230400z^2+34560z+144}}{320z}, & \dfrac{1}{210} \le z \le \dfrac{21}{850} \\ 1, & \dfrac{21}{850} \le z \le \dfrac{3}{50} \\ \dfrac{-(400z+12)\pm\sqrt{230400z^2+34560z+144}}{320z}, & \dfrac{3}{50} \le z \le \dfrac{39}{110} \end{cases}$$

(iii) Average number of customers in the system

$$l_{L_{DECA}} = \frac{-112\alpha^2+312\alpha+297}{160\alpha^2-1200\alpha+1890}\,; U_{L_{DECA}} = \frac{-112\alpha^2+248\alpha+377}{160\alpha^2+400\alpha-110}$$

The membership function is obtained as

$$\phi_{L_{DECA}}(z)=\begin{cases}\dfrac{(1200z+312)\pm\sqrt{230400z^2+92160z+230400}}{320z+224}, & \dfrac{11}{50}\le z\le\dfrac{497}{850}\\ 1, & \dfrac{497}{850}\le z\le\dfrac{7}{10}\\ \dfrac{-(400z-248)\pm\sqrt{230400z^2+92160z+230400}}{320z+224}, & \dfrac{7}{10}\le z\le\dfrac{57}{50}\end{cases}$$

(iv) Average waiting time of customers in the system

$$l_{W_{DECA}}(z)=\frac{99-28\alpha}{160\alpha^2-1200\alpha+1890};\ U_{W_{DECA}}=\frac{28\alpha+29}{160\alpha^2+400\alpha-110}$$

The membership function is obtained as,

$$\phi_{W_{DECAq}}(z)=\begin{cases}\dfrac{-(28-1200z)\pm\sqrt{230400z^2-3840z+784}}{320z}, & \dfrac{11}{210}\le z\le\dfrac{71}{850}\\ 1, & \dfrac{71}{850}\le z\le\dfrac{19}{150}\\ \dfrac{-(400z-28)\pm\sqrt{230400z^2-3840z+784}}{320z}, & \dfrac{19}{150}\le z\le\dfrac{29}{110}\end{cases}$$

The Performance Measures of Heptagonal Fuzzy Numbers are calculated

α	$\boldsymbol{l_x^\alpha}$	$\boldsymbol{{}^\alpha U_x}$	$\boldsymbol{\overset{\alpha}{l}_y}$	$\boldsymbol{u_y^\alpha}$	$l_{L_{HEPTAq}}(\alpha)$	$U_{L_{HEPTAq}}(\alpha)$	$l_{W_{HEPTAq}}(\alpha)$	$U_{W_{HEPTAq}}(\alpha)$	$l_{L_{HEPTA}}(\alpha)$	$u_{L_{HEPTA}}(\alpha)$	$l_{W_{HEPTA}}(\alpha)$	$u_{W_{HEPTA}}(\alpha)$
0	1	7	11	17	0.002	0.66	0.002	0.095	0.061	1.304	0.061	0.186
0.1	1.3	6.7	11.3	16.7	0.003	0.52	0.003	0.077	0.081	1.111	0.062	0.165
0.2	1.6	6.4	11.6	16.4	0.006	0.41	0.004	0.063	0.103	0.959	0.064	0.149
0.3	1.9	6.1	11.9	16.1	0.009	0.32	0.005	0.053	0.127	0.836	0.067	0.137
0.4	2.2	5.8	12.2	15.8	0.013	0.26	0.006	0.044	0.152	0.734	0.069	0.126
0.5	2.5	5.5	12.5	15.5	0.018	0.20	0.007	0.037	0.179	0.647	0.071	0.117
0.6	2.8	5.2	12.8	15.2	0.024	0.16	0.008	0.0321	0.209	0.573	0.074	0.110
0.7	3.1	4.9	13.1	14.9	0.033	0.13	0.010	0.027	0.240	0.508	0.077	0.103
0.8	3.4	4.6	13.4	14.6	0.042	0.10	0.012	0.023	0.275	0.450	0.080	0.098
0.9	3.7	4.3	13.7	14.3	0.054	0.08	0.014	0.020	0.312	0.400	0.084	0.093
1	4	4	14	14	0.068	0.06	0.017	0.017	0.354	0.354	0.088	0.088

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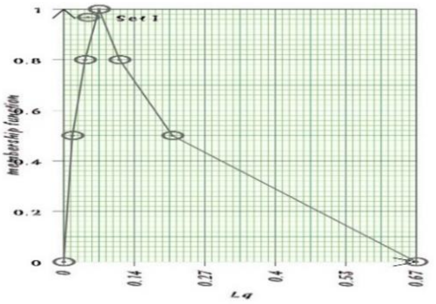

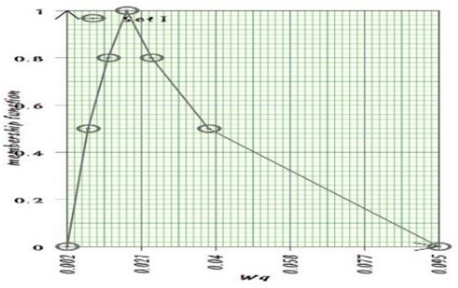

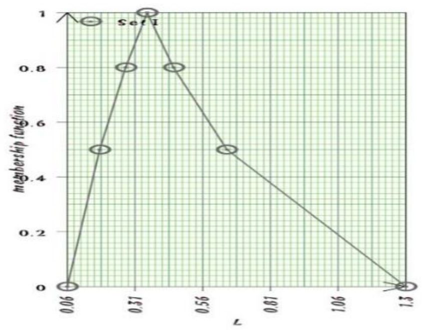

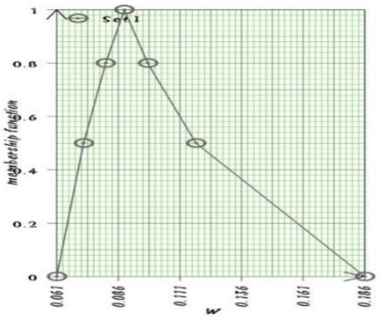

Alpha cuts of the Performance Measures in Heptagonal

α	$R\left[L^{\alpha}_{HEPTAq}\right]$	$R\left[W^{\alpha}_{HEPTAq}\right]$	$R\left[L^{\alpha}_{HEPTA}\right]$	$R\left[W^{\alpha}_{HEPTA}\right]$
0	0.33519	0.04883	0.68279	0.12370
0.1	0.26105	0.04018	0.59644	0.11437
0.2	0.20687	0.03381	0.53152	0.10740
0.3	0.16647	0.02901	0.48179	0.10208
0.4	0.13601	0.02536	0.44371	0.09799
0.5	0.11302	0.02256	0.41367	0.09484
0.6	0.09586	0.02049	0.39110	0.09245
0.7	0.08345	0.01897	0.37450	0.09071
0.8	0.07504	0.01794	0.36310	0.08950
0.9	0.07017	0.01734	0.35648	0.08881
1	0.06857	0.01714	0.35429	0.08857

Performance Measures of Decagonal Fuzzy Numbers

α	l_x^{α}	u_x^{α}	l_y^{α}	u_y^{α}	$l_{L_{DECAq}}(\alpha)$	$U_{L_{DECAq}}(\alpha)$	$l_{W_{DECAq}}(\alpha)$	$U_{W_{DECAq}}(\alpha)$	$l_{L_{DECA}}(\alpha)$	$u_{L_{DECA}}(\alpha)$	$l_{W_{DECA}}(\alpha)$	$u_{W_{DECA}}(\alpha)$
0	3	13	11	21	0.014	4.609	0.004	0.35	0.157	3.427	0.052	0.263
0.1	3.4	12.6	11.4	20.6	0.019	6.963	0.005	0.55	0.184	6.127	0.054	0.486
0.2	3.8	12.2	11.8	20.2	0.026	18.92	0.006	1.55	0.214	20.49	0.056	1.679
0.3	4.2	11.8	12.2	19.8	0.034	17.11	0.008	1.451	0.246	16.12	0.059	1.365
0.4	4.6	11.4	12.6	19.4	0.044	5.157	0.009	0.452	0.281	5.83	0.061	0.511

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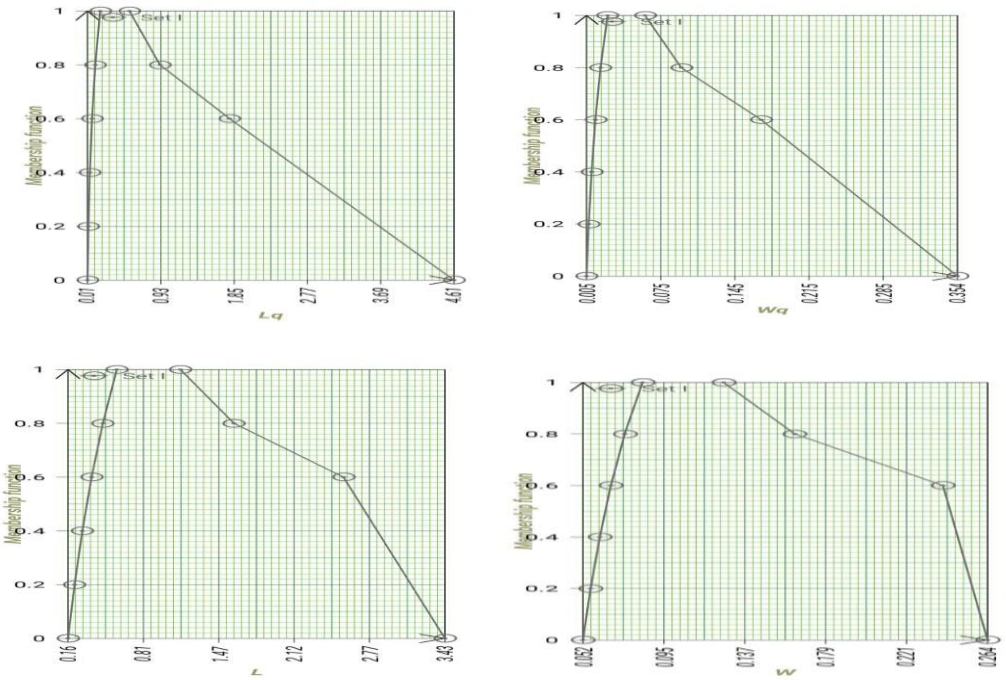
0.5	5	11	13	19	0.056	2.792	0.011	0.254	0.319	3.55	0.064	0.323
0.6	5.4	10.6	13.4	18.6	0.071	1.796	0.013	0.169	0.362	2.547	0.066	0.24
0.7	5.8	10.2	13.8	18.2	0.089	1.256	0.015	0.123	0.408	1.972	0.07	0.193
0.8	6.2	9.8	14.2	17.8	0.111	0.922	0.018	0.094	0.46	1.597	0.074	0.163
0.9	6.6	9.4	14.6	17.4	0.139	0.698	0.021	0.074	0.518	1.332	0.079	0.142
1	7	9	15	17	0.172	0.540	0.024	0.06	0.581	1.132	0.084	0.126



Alpha cuts of the Performance Measures in Decagonal

α	$R(L_{DECAq}^{\alpha})$	$R(W_{DECAq}^{\alpha})$	$R(L_{DECA}^{\alpha})$	$R(W_{DECA}^{\alpha})$
0	2.29741	0.17487	1.63507	0.1056
0.1	3.4718	0.27342	2.97100	0.2160
0.2	9.44709	0.77206	10.1385	0.8116
0.3	8.57697	0.72948	8.17648	0.71181
0.4	2.60066	0.23101	3.0560	0.2863
0.5	1.42436	0.13256	1.9380	0.1936
0.6	0.93400	0.09135	1.4543	0.1536
0.7	0.67298	0.06930	1.1899	0.1318
0.8	0.51700	0.05606	1.0286	0.1186
0.9	0.41870	0.04768	0.9250	0.1101
1	0.35647	0.04236	0.8565	0.1047

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V. INTERPRETATION

In heptagonal fuzzy numbers, the interval Optimization, if α = 0 is $\overline{L_q} = [0.00221, 0.668181]$, $\overline{W_q} = [0.00221, 0.09545]$, $\overline{L}$ = [0.06103, 1.30455], $\overline{W}$ =[0.06103, 0.18636] are calculated. The degree of certainty for

α = 1, is obtained as, $\overline{L_q} = 0.06857$, $\overline{W_q} = 0.01714$, $\overline{L}$ =0.3543, $\overline{W}$ =0.0886. The fuzziness values are transformed as crisp values implementing Robust ranking function and the values obtained are $\overline{L_q} = 0.06857$, $W_q = 0.01714$. L=0.35429, W=0.8857. In decagonal fuzzy numbers, the interval optimization, if α = 0 then, $\overline{W_q}$ = [0.00476, 0.3545], $\overline{L}$ = [0.15714, 3.42727], $\overline{W}$ = [0.05238, 0.26364]. The degree of certainty for α=1 is obtained as, $\overline{L_q} = [0.17294, 0.54]$, $\overline{W_q} = [0.02471, 0.06]$, $\overline{L} = [0.5806, 1.13245]$, $\overline{W} = [0.08353, 0.1258]$.The fuzziness values are transformed as crisp values implementing robust ranking function the value are $\overline{L_q} = 0.03565$, $\overline{W_q} = 0.0424$, $\overline{L} = 0.8565$, $\overline{W} = 0.1047$.

VI. CONCLUSION

An optimal *K*-policy with $FM / FE_5 / 1$ queue with general server setup time was investigated and the performance indicators are pertinent to the service status of the customers acquiring service. The average number of customers in the system, in queue and waiting time in the queue, in the system for fuzzy queue has been discussed with numerical example. Further the fuzzy problem has been converted into crisp problem by using Robust Ranking Technique. In real scenarios, most of the data are imprecise by nature which can be solved adopting fuzzy measures. This research work is of much vitality for system operation and production purposes which involves several phases of service and has compatibility.

VII. REFERENCES

[1] Anushya, B., Ramaand, B. and Sudha, L. (2019), "Transportation Problem using Intuitionistic Decagonal Fuzzy Numbers", IJRAR – International Journal of Research and Analytical Reviews, E-ISSN: 2348-1269, Print ISSN: 2349-5138, Volume 6, Issue 1, pp. 271-277.

[2] Chen S.P., (2014), "Parametric nonlinear programming for analyzing fuzzy queues with finite capacity, European Journal of Operation Research, 157, pp. 429-438.

[3] Chiang Kao, Chang Chung Li, Shih Pin Chen (1999), "Parametric Programming to the analysis of fuzzy queues", Fuzzy sets and Systems, Volume 107, Issue 1, pp. 93-100.

[4] Chaudhry M.L., Templeton J.G.C., (1983), "A First Course in Bulk Oueues", John Wiley & Sons, New York.

[5] Mohammed Shapique, A. (2016), "Fuzzy queue with Erlang Service model using DSW Algorithm", International Journal of Engineering Sciences & Research Technology, ISSN: 2277-9655, pp. 50-54.

[6] Ramya, S. and Jeba Presitha (2019), "Solving an unbalanced fuzzy transportation problem using a heptagonal fuzzy number by Robust Ranking Method", International Review of Pure and Applied Mathematics, Volume 15, pp. 99-105.

[7] Sharma. R., (2014), Mathematical analysis of queue with phase service, Advances in Operations Research, pp. 1-19.

[8] Shih-Pin Chen (2006), "A bulk arrival queuing model with fuzzy parameters and varying batch sizes", Applied Mathematical modelling 30, pp. 920-929.

[9] Sivaraman Geetha, Bharathi Ramesh Kumar and Sankara Murugesan (2017), "Predictor Analysis on Non- parametric Bulk Arrival Fuzzy Queueing System", Global Journal Technology and Optimization, Vol.8, Issue 1, pp. 1-5.

[10] Zadeh L.A., (1978), "Fuzzy sets as a basis for a theory of possibility", Fuzzy sets and systems 01: pp. 3-28