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A New Notion of Closed Sets in Intuitionistic Topological Space

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ABSTRACT

In this paper is to introduce a new concept of $I\alpha_g^{\wedge}$-closed set in intuitionistic topological spaces and investigate some of their basic properties and give characterizations for these spaces. We also study the relationship between $I\alpha_g^{\wedge}$-closed sets and the other intuitionistic closed sets are also discussed.

Keywords: $I\alpha_g^{\wedge}$-closed, $I\alpha_g^{\wedge}$-open, $I\alpha_g^{\wedge}$-closure and $I\alpha_g^{\wedge}$-interior.

1 INTRODUCTION

The concept of intuitionistic sets in topological spaces was first introduced by Coker[2] in 1996. He also introduced the concept of intuitionistic points and investigated some fundamental properties of closed sets in intuitionistic topological spaces. In 2009 J .Younis Yaseen and G.Asmaa Raouf [5] has given some results in intuitionistic generalized closed sets in intuitionistic topological spaces. K.Meena and V.P.Anuja,[7] was introduced the concept of alpha ^ generalized closed sets in topological spaces. The purpose of this paper is to develop alpha ^ generalized closed sets in intuitionistic topological spaces and discuss some properties related to $I\alpha_g^{\wedge}$-closed set in intuitionistic topological spaces.

2 PRELIMINARIES

Definition 2.1 [2]. Let $\mathcal{H}$ be a non-empty set. An intuitionistic set (IS for short) $\mathcal{A}$ is an object having the form $\mathcal{A} =< \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2 >$ Where $, \mathcal{A}_2$ are subsets of $\mathcal{H}$ satisfying $\mathcal{A}_1 \cap \mathcal{A}_2 = \varphi$. The set $\mathcal{A}_1$ is called the set of members of $\mathcal{A}$, while $\mathcal{A}_2$ is called set of non members of $\mathcal{A}$.

Definition 2.2 [2]: Let $\mathcal{H}$ be a non-empty set and $\mathcal{A}$ and $\mathfrak{B}$ are intuitionistic set in the form $\mathcal{A} =< \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2 >$, $\mathfrak{B} =< \mathcal{H}, \mathfrak{B}_1, \mathfrak{B}_2 >$ respectively. Then

a) $\mathcal{A} \subseteq \mathfrak{B}$ iff $\mathcal{A}_1 \subseteq \mathfrak{B}_1$ and $\mathcal{A}_2 \supseteq \mathfrak{B}_2$

b) $\mathcal{A} = \mathfrak{B}$ iff $\mathcal{A} \subseteq \mathfrak{B}$ and $\mathfrak{B} \subseteq \mathcal{A}$

c) $\mathcal{A}^C = < \mathcal{H}, \mathcal{A}_2, \mathcal{A}_1 >$

d) $\mathcal{A} - \mathfrak{B} = \mathcal{A} \cap \mathfrak{B}^C$

e) $\varphi = < \mathcal{H}, \varphi, \mathcal{H} >, \mathcal{H} =< \mathcal{H}, \mathcal{H}, \varphi >$

f) $\mathcal{A} \cup \mathfrak{B} = < \mathcal{H}, \mathcal{A}_1 \cup \mathfrak{B}_1, \mathcal{A}_2 \cap \mathfrak{B}_2 >$

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g) $\mathcal{A} \cap \mathfrak{B} = < \mathcal{H}, \mathcal{A}_1 \cap \mathfrak{B}_1, \mathcal{A}_2 \cup \mathfrak{B}_2 >$.

Furthermore, let $\{\mathcal{A}_\alpha / \alpha \epsilon$ J$\}$ be an arbitrary family of intuitionistic sets in $\mathcal{H}$, where $\mathcal{A}_\alpha =< \mathcal{H}, \mathcal{A}_\alpha^{(1)}, \mathcal{A}_\alpha^{(2)} >$. Then

(i) $\cap \mathcal{A}_\alpha == < \mathcal{H}, \cap \mathcal{A}_\alpha^{(1)}, \cup \mathcal{A}_\alpha^{(2)} >$

(j) $\cup \mathcal{A}_\alpha =< \mathcal{H}, \cup \mathcal{A}_\alpha^{(1)}, \cap \mathcal{A}_\alpha^{(2)} >$.

Definition 2.3 [2]: An intuitionistic topology is (for short IT) on a non-empty set $\mathcal{H}$ is a family $I\tau$ of IS's in $\mathcal{H}$ satisfying following axioms.

1) $\varphi, \mathcal{H} \epsilon I\tau$

2) $G_1 \cap G_2 \epsilon I\tau$, for any $G_1, G_2 \epsilon I \tau$

$\cup G_\alpha \epsilon I\tau$ for any arbitrary family $\{G_i : G_\alpha / \alpha \epsilon$ J$\}$ where $(\mathcal{H}, I\tau)$ is called an **intuitionistic topological space** (for short ***I*TS($\mathcal{H}$)**) and any intuitionistic set is called an **intuitionistic open set** (for short ***I*OS**) in $\mathcal{H}$. The complement $\mathcal{A}^C$ of an ***I*OS** of $\mathcal{A}$ is called an **intuitionistic closed set** (for short ***I*CS**) in $\mathcal{H}$.

Definition 2.4 [2]: Let $(\mathcal{H}, I\tau)$ be an intuitionistic topological space (for short ITS($\mathcal{H}$)) and $\mathcal{A} =< \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2 >$ be an IS in $\mathcal{H}$. Then the interior and closure of $\mathcal{A}$ are defined by

1) $I\text{cl}(\mathcal{A}) = \cap \{K : K$ is an ICS in $\mathcal{H}$ and $\mathcal{A} \subseteq K\}$

2) $I\text{int}(\mathcal{A}) = \cup \{G : G$ is an IOS in $\mathcal{H}$ and $G \subseteq \mathcal{A}\}$.

It can be shown that $I\text{cl}(\mathcal{A})$ is an ICS and $I\text{int}(\mathcal{A})$ is an IOS in $\mathcal{H}$ and $\mathcal{A}$ is an ICS in $\mathcal{H}$ iff $I\text{cl}(\mathcal{A}) = \mathcal{A}$ and $\mathcal{A}$ is an I_{ntu}OS in $\mathcal{H}$ iff $I\text{int}(\mathcal{A}) = \mathcal{A}$.

Definition 2.5 [2]: Let $(\mathcal{H}, I\tau)$ be an ITS($\mathcal{H}$). An intuitionistic set $\mathcal{A}$ of $\mathcal{H}$ is said to be

(i) Intuitionistic semi-open if $\mathcal{A} \subseteq I\text{cl}(I\text{int}(\mathcal{A}))$.

(ii) Intuitionistic pre-open if $\mathcal{A} \subseteq I\text{int}(I\text{cl}(\mathcal{A}))$.

(iii) Intuitionistic α-open if $\mathcal{A} \subseteq I\text{int}(I\text{cl}(I\text{int}(\mathcal{A})))$.

(iv) Intuitionistic β-open if $\mathcal{A} \subseteq I\text{cl}(I\text{int}(I\text{cl}(\mathcal{A})))$.

(v) Intuitionistic regular-open if $\mathcal{A} = I\text{int}(I\text{cl}(\mathcal{A}))$.

The family of all intuitionistic semi-open, intuitionistic pre-open, intuitionistic α-open, intuitionistic β-open and intuitionistic regular-open sets of $(\mathcal{H}, I\tau)$ are denoted by IS-OS, IP-OS, Iα-OS, Iβ-OS and Ir-OS respectively.

Lemma 2.6[2] Let $(\mathcal{H}, I\tau)$ be an ITS and $\mathcal{A}$ and $\mathfrak{B}$ be an intuitionistic subset of $\mathcal{H}$. Then the following hold.

(i) $I\text{cl}(\emptyset) = \emptyset$ and $I\text{cl}(\mathcal{H}) = \mathcal{H}$.

(ii) $\mathcal{A}$ is ICS if and only iff $\mathcal{A} = I\text{cl}(\mathcal{A})$.

(iii) $I\text{cl}(I\text{cl}(\mathcal{A})) = I\text{cl}(\mathcal{A})$.

(iv) $\mathcal{A} \subseteq \mathfrak{B}$ implies that $I\text{cl}(\mathcal{A}) \subseteq I\text{cl}(\mathfrak{B})$

(v) $I\text{cl}(\mathcal{A} \cap \mathfrak{B}) = I\text{cl}(\mathcal{A}) \cap I\text{cl}(\mathfrak{B})$.

(vi) $I\text{cl}(\mathcal{A} \cup \mathfrak{B}) = I\text{cl}(\mathcal{A}) \cup I\text{cl}(\mathfrak{B})$

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Lemma 2.7[2] Let $(\mathcal{H}, I\tau)$ be an ITS and $\mathcal{A}$ and $\mathfrak{B}$ be an intuitionistic subset of $\mathcal{H}$. Then the following hold.

(i) $I\text{int}(\emptyset) = \emptyset$ and $I\text{int}(\mathcal{H}) = \mathcal{H}$.

(ii) $\mathcal{A}$ is IO set if and only iff $\mathcal{A} = I\text{int}(\mathcal{A})$.

(iii) $I\text{int}(I\text{int}(\mathcal{A})) = I\text{int}(\mathcal{A})$.

(iv) $\mathcal{A} \subseteq \mathfrak{B}$ implies that $I\text{int}(\mathcal{A}) \subseteq I\text{int}(\mathfrak{B})$

(v) $I\text{int}(\mathcal{A} \cap \mathfrak{B}) = I\text{int}(\mathcal{A}) \cap I\text{int}(\mathfrak{B})$.

(vi) $I\text{int}(\mathcal{A} \cup \mathfrak{B}) = I\text{int}(\mathcal{A}) \cup I\text{int}(\mathfrak{B})$

Definition 2.8 [5]: A subset $\mathcal{A}$ of ITS $(\mathcal{H}, I\tau)$ is called an intuitionistic semi-generalized closed set (briefly Isg-C) if Iscl $(\mathcal{A}) \subseteq \mathcal{F}$ whenever $\mathcal{A} \subseteq \mathcal{F}$ and $\mathcal{F}$ is Is-O in $\mathcal{H}$.

Definition 2.9 [5]: A subset $\mathcal{A}$ of ITS$(\mathcal{H}, I\tau)$ is called an intuitionistic generalized semi-closed set (briefly Igs -C) if Iscl $(\mathcal{A}) \subseteq \mathcal{F}$ whenever $\mathcal{A} \subseteq \mathcal{F}$ and $\mathcal{F}$ is I-O in $\mathcal{H}$.

Definition 2.10 [5]: A subset $\mathcal{A}$ of ITS $(\mathcal{H}, I\tau)$ is called an intuitionistic generalized-CS (briefly Ig-C) if Icl $(\mathcal{A}) \subseteq \mathcal{F}$ whenever $\mathcal{A} \subseteq \mathcal{F}$ and $\mathcal{F}$ is I-O in $\mathcal{H}$.

Definition 2.11 [4]: A subset $\mathcal{A}$ of ITS $(\mathcal{H}, I\tau)$ is called an intuitionistic generalized star-CS (briefly I g^*-C) if Icl $(\mathcal{A})$ $\subseteq \mathcal{F}$ whenever $\mathcal{A} \subseteq \mathcal{F}$ and $\mathcal{F}$ is Ig-O in $\mathcal{H}$.

Definition 2.12 [3]: Let $(\mathcal{H}, I\tau)$ be an ITS and $\mathcal{A} =< X, \mathcal{A}_1, \mathcal{A}_2 >$ be an IS in $\mathcal{H}$. Then the semi interior and semi closure of $\mathcal{A}$ are defined by

1) Iscl $(\mathcal{A}) = \cap$ {K: K is an ISC in $\mathcal{H}$ and $\mathcal{A} \subseteq$ K}

2) Isint $(\mathcal{A}) = \cup$ {G: G is an ISO in $\mathcal{H}$ and G $\subseteq \mathcal{A}$}.

It can be established that Iscl $(\mathcal{A})$ is the smallest an ISC set contained in all ISC sets containing $\mathcal{A}$ and Isint $(\mathcal{A})$ is the largest ISO set contained in $\mathcal{A}$, $\mathcal{A}$ is an ISC set in $\mathcal{H}$ iff Iscl $(\mathcal{A}) = \mathcal{A}$ and $\mathcal{A}$ is an ISO in $\mathcal{H}$ iff Isint $(\mathcal{A}) = \mathcal{A}$.

Definition 2.13 [4]: A subset $\mathcal{A}$ of ITS $(\mathcal{H}, I\tau)$ is called an intuitionistic w-closed set (briefly Iw-C) if Icl $(\mathcal{A}) \subseteq \mathcal{F}$ whenever $\mathcal{A} \subseteq \mathcal{F}$ and $\mathcal{F}$ is Is-open in $\mathcal{H}$.

Definition 2.14 [4]: A subset $\mathcal{A}$ of ITS $(\mathcal{H}, I\tau)$ is called an intuitionistic generalized α-closed set (briefly Igα-C) if Iαcl $(\mathcal{A}) \subseteq \mathcal{F}$ whenever $\mathcal{A} \subseteq \mathcal{F}$ and $\mathcal{F}$ is I-open in $\mathcal{H}$.

3. INTUITIONISTIC $\alpha_g^\wedge$-CLOSED SETS

Definition 3.1: A subset $\mathcal{A}$ of $(\mathcal{H}, I\tau)$ is called an **intuitionistic alpha ^ generalized closed (**briefly ***$I\alpha_g^\wedge$-CS*)** if Igcl$(\mathcal{A}) \subseteq \mathcal{F}$, whenever $\mathcal{A} \subseteq \mathcal{F}$ and $\mathcal{F}$ is Iα-O in $\mathcal{H}$. We denote the family of all $I\alpha_g^\wedge$-*CS* in space $\mathcal{H}$ by $I\alpha_g^\wedge$-*CS* $(\mathcal{H}, I\tau)$.

Example 3.2: Let $\mathcal{H}$ ={s,t} and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2\}$ where $\mathcal{A}_1 =< \mathcal{H}, \varphi, \{t\} >$, $\mathcal{A}_2 =< \mathcal{H}, \{s\}, \varphi >$. Then intuitionistic generalized closed set (briefly $IgCS$) = $< \mathcal{H}, \{t\}, \varphi >, < \mathcal{H}, \varphi, \{s\} >, < \mathcal{H}, \{t\}, \{s\} >$. Then $I\alpha_g^\wedge$-*CS* $(\mathcal{H}, I\tau)$= $< \mathcal{H}, \{t\}, \varphi >, < \mathcal{H}, \varphi, \{s\} >, < \mathcal{H}, \{t\}, \{s\} >$.

Theorem 3.3: Every ICS of an ITS $(\mathcal{H}, I\tau)$ is $I\alpha_g^\wedge$-*CS*, but not conversely.

Proof: Let $\mathcal{A} \subseteq \mathcal{H}$ be an ICS and $\mathcal{A} \subseteq \mathcal{F}$ where $\mathcal{F}$ is Iα-O. Since $\mathcal{A}$ is ICS and every ICS is Ig- CS, Igcl$(\mathcal{A}) \subseteq Icl(\mathcal{A})$

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= $\mathcal{A} \subseteq \mathcal{F}$. Hence $\mathcal{A}$ is an $I\alpha_g^{\wedge}$- *CS*.

Example 3.4: Let $\mathcal{H}$ ={p,q} and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2\}$ where $\mathcal{A}_1 =< \mathcal{H}, \{q\}, \varphi >$, $\mathcal{A}_2 =< \mathcal{H}, \varphi, \varphi >$. Then intuitionistic subset $\{< \mathcal{H}, \{p\}, \{q\} >\}$ is an $I\alpha_g^{\wedge}$-*CS* but it is not an *ICS*.

Theorem 3.5: Every $I\alpha_g^{\wedge}$-*CS* is *I*gα –*CS*, but not conversely.

Proof: Let $\mathcal{A}$ be $I\alpha_g^{\wedge}$-C. Let $\mathcal{A} \subseteq \mathcal{F}$ and $\mathcal{F}$ is *I*-O. Since every *I*OS is *I*α-OS and $\mathcal{A}$ is $I\alpha_g^{\wedge}$-*CS*, *I*αcl($\mathcal{A}$) ⊆ $\mathcal{F}$, Hence $\mathcal{A}$ is *I*gα-*CS*.

Example 3.6: Let $X = \{l, m, n\}$ and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4, \mathcal{A}_5, \mathcal{A}_6\}$ where $\mathcal{A}_1 =< \mathcal{H}, \{l, n\}, \varphi >$, $\mathcal{A}_2 =< \mathcal{H}, \{l\}, \varphi >$,$\mathcal{A}_3 =< \mathcal{H}, \{l\}, \{m\} >$, $\mathcal{A}_4 =< \mathcal{H}, \varphi, \{m\} >$ $\mathcal{A}_5 =< \mathcal{H}, \varphi, \varphi >$ $\mathcal{A}_6 =< \mathcal{H}, \{n\}, \varphi >$. Here intuitionistic subset $\{< \mathcal{H}, \varphi, \varphi >, < \mathcal{H}, \varphi, \{l, m\} >$ is *I*gα- *CS* but it is not an $I\alpha_g^{\wedge}$-*CS*.

Theorem 3.7: Every *I*r-*CS* is $I\alpha_g^{\wedge}$- *CS*, but not conversely.

Proof: Let $\mathcal{A}$ be *I*r-*C* in ($\mathcal{H}$,τ). Let $\mathcal{A} \subseteq \mathcal{F}$ where $\mathcal{F}$ is an *I*α-O. Since $\mathcal{A}$ is *I*r-*C*, *I*rcl($\mathcal{A}$)= $\mathcal{A} \subseteq \mathcal{F}$. Every *I*r-*CS* is *I*-*C* and Every *ICS* is *I*g-*C*, then *I*gcl($\mathcal{A}$) ⊆ *I*rcl($\mathcal{A}$) ⊆ $\mathcal{F}$. Therefore *I*gcl($\mathcal{A}$) ⊆ $\mathcal{F}$ and hence $\mathcal{A}$ is $I\alpha_g^{\wedge}$-*CS*.

Example 3.8: Let $\mathcal{H}$ = {3,4} and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4\}$ where $\mathcal{A}_1 =< \mathcal{H}, \varphi, \{4\} >$, $\mathcal{A}_2 =< \mathcal{H}, \{3\}, \{4\}, >$, $\mathcal{A}_3 = < \mathcal{H}, \{3\}, \varphi >$, $\mathcal{A}_4 =< \mathcal{H}, \varphi, \varphi >$. Then intuitionistic subset $\{< \mathcal{H}, \varphi, \{4\} >\}$ are an $I\alpha_g^{\wedge}$-closed set but it is not a *I*r-CS

Theorem 3.9: Every *I*g*-*CS* is $I\alpha_g^{\wedge}$-C, but not conversely.

Proof: Let $\mathcal{A}$ be *I*g*-C in ($\mathcal{H}$,τ). Let $\mathcal{A} \subseteq \mathcal{F}$ where $\mathcal{F}$ is *I*α-O. Since every *I*α-OS is *I*g-O and $\mathcal{A}$ is *I*g*-C, *I*cl($\mathcal{A}$) ⊆ $\mathcal{F}$. Every *I*-*CS* is *I*g-C, then *I*gcl($\mathcal{A}$) ⊆ *I*cl(A) ⊆ $\mathcal{F}$. Hence $\mathcal{A}$ is $I\alpha_g^{\wedge}$-C.

Example 3.10: Let $\mathcal{H}$ = {f,g} and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2\}$ where $\mathcal{A}_1 =< \mathcal{H}, \varphi, \{f\} >$, $\mathcal{A}_2 =< \mathcal{H}, \{g\}, \{f\}, >$. Then intuitionistic subsets $\{< \mathcal{H}, \varphi, \{g\} >\}$ is an $I\alpha_g^{\wedge}$-*CS* but it is not a *I*g*-*CS*.

Theorem 3.11: Every *I*w-*CS* is $I\alpha_g^{\wedge}$-C, but not conversely.

Proof: Let $\mathcal{A}$ be a *I*w-CS. Let $\mathcal{A} \subseteq \mathcal{F}$ where $\mathcal{F}$ is *I*α-O. Since every *I*α-OS is *I*SC and $\mathcal{A}$ is *I*w-C, *I*cl($\mathcal{A}$)⊆ $\mathcal{F}$. Every *I*-*CS* is *I*g-C. Therefore *I*-gcl($\mathcal{A}$) ⊆ *I*-cl($\mathcal{A}$) ⊆ $\mathcal{F}$. Hence $\mathcal{A}$ is $I\alpha_g^{\wedge}$-C.

Example 3.12: Let $\mathcal{H} = \{e, i, j\}$ and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4, \mathcal{A}_5, \mathcal{A}_6\}$ where $\mathcal{A}_1 =< \mathcal{H}, \{e, j\}, \varphi >$,$\mathcal{A}_2 =< \mathcal{H}, \{e\}, \varphi >$,$\mathcal{A}_3 =< \mathcal{H}, \{e\}, \{i\} >$ $A_4 =< \mathcal{H}, \varphi, \{i\} >$ $\mathcal{A}_5 =< \mathcal{H}, \varphi, \varphi >$,$\mathcal{A}_6 =< \mathcal{H}, \{j\}, \varphi >$. Here intuitionistic subset $\{< \mathcal{H}, \{e, i\}, \{j\} >\}$ is $I\alpha_g^{\wedge}$- *CS* but it is not an *I*w-*CS*.

Remark 3.13: $I\alpha_g^{\wedge}$-*CS* and *Is*-*CS* are independent as shown by the following examples.

Example 3.14: Let $\mathcal{H}$ ={a,b} and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2\}$ where $\mathcal{A}_1 =< \mathcal{H}, \varphi, \{b\} >$, $\mathcal{A}_2 =< \mathcal{H}, \{a\}, \varphi >$. Here *I* subset $\{< \mathcal{H}, \varphi, \{b\} >\}$ is an *Is*-*CS* but not an $I\alpha_g^{\wedge}$-*CS*.

Example 3.15: Let $\mathcal{H}$ ={14,15} and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2\}$ where $\mathcal{A}_1 =< X, \varphi, \{14\} >$, $\mathcal{A}_2 =< X, \{15\}, \{14\}, >$. Then intuitionistic subset $\{< \mathcal{H}, \{15\}, \varphi >\}$ is an $I\alpha_g^{\wedge}$-*CS* but not an *Is*-*CS*.

Remark 3.16: $I\alpha_g^{\wedge}$-*CS* and *Ip*-*CS* are independent as shown by the following examples.

Example 3.17: Let $\mathcal{H} = \{d, r, c\}$ and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3\}$ where $\mathcal{A}_1 =< \mathcal{H}, \varphi, \{d, c\} >$, $\mathcal{A}_2 =< \mathcal{H}, \{d, r\}, \varphi >$,$\mathcal{A}_3 =< \mathcal{H}, \{d\}, \{c\} >$. Here intuitionistic subset $\{< \mathcal{H}, \varphi, \{r\} >\}$ is an *Ip*-C but not an $I\alpha_g^{\wedge}$-*CS*.

Example 3.18: Let $\mathcal{H}$ ={w,b} and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2\}$ where $\mathcal{A}_1 =< \mathcal{H}, \varphi, \{w\} >$, $\mathcal{A}_2 =<$

$\mathcal{H}, \{b\}, \{w\}, >$. Then I subset $\{< \mathcal{H}, \{b\}, \varphi >\}$ is an $I\alpha_g^{\wedge}$- *CS* but not an Ip- *CS*.

Remark 3.19: $I\alpha_g^{\wedge}$-*CS* and $I\alpha$-*CS* are independent as shown by the following examples.

Example 3.20: Let $\mathcal{H} = \{u, v, h\}$ and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4\}$ where $A_1 =< \mathcal{H}, \{u, h\}, \varphi >$, $\mathcal{A}_2 =< \mathcal{H}, \{u\}, \varphi >$, $\mathcal{A}_3 =< \mathcal{H}, \{u\}, \{v\} >$ $\mathcal{A}_4 =< \mathcal{H}, \varphi, \{v\} >$. Here intuitionistic subset $\{< \mathcal{H}, \varphi, \{u, v\} >\}$ is an $I\alpha$-C but not an $I\alpha_g^{\wedge}$- *CS*.

Example 3.21: Let $\mathcal{H}$ ={1,2} and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2\}$ where $\mathcal{A}_1 =< \mathcal{H}, \varphi, \{2\} >$, $\mathcal{A}_2 =< \mathcal{H}, \{1\}, \varphi >$. Then intuitionistic subsets $\{< \mathcal{H}, \{2\}, \{1\} >\}$ is an $I\alpha_g^{\wedge}$-CS but not an $I\alpha c$ set.

Remark 3.22: $I\alpha_g^{\wedge}$- *CS* and $I\beta$- *CS* are independent as shown by the following examples.

Example 3.23: Let $\mathcal{H} = \{6,7,8\}$ and the family $\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4\}$ where $\mathcal{A}_1 =< \mathcal{H}, \{6,8\}, \varphi >$, $\mathcal{A}_2 =< \mathcal{H}, \{6\}, \varphi >$,$\mathcal{A}_3 =< \mathcal{H}, \{6\}, \{7\} >$, $\mathcal{A}_4 =< \mathcal{H}, \varphi, \{7\} >$. Here intuitionistic subset $\{< \mathcal{H}, \varphi, \{7,8\}\}$ is an $I\beta$-C but not an $I\alpha_g^{\wedge}$-*CS*.

Example 3.24: Let $\mathcal{H}$ =$\{x, y, z\}$ and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3\}$ where $\mathcal{A}_1 =< \mathcal{H}, \{x, y\}, \{z\} >$, $\mathcal{A}_2 =< \mathcal{H}, \{x\}, \{y, z\} >$, $\mathcal{A}_3 =< \mathcal{H}, \{x, z\}, \{y\} >$. Then intuitionistic subset $\{< \mathcal{H}, \{x\}, \varphi >\}$is an $I\alpha_g^{\wedge}$- *CS* but not an $I\beta$- *CS*.

Remark 3.25: $I\alpha_g^{\wedge}$- *CS* and Isg- *CS* are independent as shown by the following examples.

Example 3.26: Let $\mathcal{H} = \{k, \ell, m\}$ and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4, \mathcal{A}_5, \mathcal{A}_6\}$ where $\mathcal{A}_1 =< \mathcal{H}, \{k, m\}, \varphi >$, $\mathcal{A}_2 =< \mathcal{H}, \{k\}, \varphi >$,$\mathcal{A}_3 =< \mathcal{H}, \{k\}, \{\ell\} >$, $\mathcal{A}_4 =< \mathcal{H}, \varphi, \{\ell\} >$ $\mathcal{A}_5 =< \mathcal{H}, \varphi, \varphi >$ $\mathcal{A}_6 =< \mathcal{H}, \{m\}, \varphi >$. Here intuitionistic subset $\{< \mathcal{H}, \varphi, \{\ell, m\} >\}$ is an Isg- *CS* but not an $I\alpha_g^{\wedge}$- *CS*.

Example 3.27: Let $X = \{\$, \#, @\}$ and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4, \mathcal{A}_5, \mathcal{A}_6\}$ where $\mathcal{A}_1 =< \mathcal{H}, \{\$, @\}, \varphi >$, $\mathcal{A}_2 =< \mathcal{H}, \{\$\}, \varphi >$, $\mathcal{A}_3 =< \mathcal{H}, \{\$\}, \{\#\} >$ $\mathcal{A}_4 =< \mathcal{H}, \varphi, \{\#\} >$, $\mathcal{A}_5 =< \mathcal{H}, \varphi, \varphi >$, $\mathcal{A}_6 =< \mathcal{H}, \{@\}, \varphi >$. Then intuitionistic subset $\{< \mathcal{H}, \{@\}, \{\#\} >\}$ is an $I\alpha_g^{\wedge}$- *CS* but not an Isg- *CS*.

Remark 3.28: $I\alpha_g^{\wedge}$- *CS* and Igs- *CS* are independent as shown by the following examples.

Example 3.29: Let $\mathcal{H} = \{3,4,5\}$ and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2\}$ where $\mathcal{A}_1 =< \mathcal{H}, \{3,5\}, \varphi >$, $\mathcal{A}_2 =< \mathcal{H}, \varphi, \{4,5\} >$. Here intuitionistic subset $\{< \mathcal{H}, \{3,5\}, \varphi, >\}$ is an Igs-C, but not an $I\alpha_g^{\wedge}$-*CS*. Here intuitionistic subset $\{< \mathcal{H}, \{4\}, \varphi >\}$ is an $I\alpha_g^{\wedge}$- *CS* but not an Igs-C.

Theorem 3.30: Let $\mathcal{A}$ be an $I\alpha_g^{\wedge}$- *CS* in a ITS of $\mathcal{H}$. Then Igcl($\mathcal{A}$)- $\mathcal{A}$ contains no non-empty Iα- *CS* in $\mathcal{H}$.

Proof: Let F be a Iα- *CS* such that F⊆ Igcl($\mathcal{A}$)- $\mathcal{A}$. Then F⊆ $\mathcal{H}$ - $\mathcal{A}$ implies $\mathcal{A} \subseteq \mathcal{H}$ -F. Since $\mathcal{A}$ is $I\alpha_g^{\wedge}$- *CS* and $\mathcal{H}$ -F is Iα-OS, then Igcl($\mathcal{A}$) $\subseteq \mathcal{H} - $ F. That is F⊆ $\mathcal{H}$ - Igcl($\mathcal{A}$). Hence F⊆(Igcl($\mathcal{A}$)-A) ∩ $\mathcal{H}$ - Igcl($\mathcal{A}$)= φ. Thus F= φ.

Remark 3.31: The converse of the above theorem need not be true, that means if Igcl($\mathcal{A}$)- $\mathcal{A}$ contains no non-empty Iα-*CS*.

Example 3.32: Let $\mathcal{H}$ ={8,9,10} and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3\}$ where $\mathcal{A}_1 =< \mathcal{H}, \{8,9\}, \{10\} >$, $\mathcal{A}_2 =< \mathcal{H}, \{8\}, \{9,10\} >$, $\mathcal{A}_3 =< \mathcal{H}, \{8,10\}, \{9\} >$. Let $\mathcal{A}$ = < $\mathcal{H}$, {10}, {9}> then Igcl($\mathcal{A}$)- $\mathcal{A}$ = < $\mathcal{H}$, φ,{10}>, it does not contains non-empty Iα- *CS* in $\mathcal{H}$. But $\mathcal{A}$ =< $\mathcal{H}$,{10},{9}> is not an $I\alpha_g^{\wedge}$- *CS*.

Corollary 3.33: Let $\mathcal{A}$ be an $I\alpha_g^{\wedge}$- *CS*. Then $\mathcal{A}$ is Iα-C if and only if Igcl($\mathcal{A}$)- $\mathcal{A}$ is I-C.

Proof: Let $\mathcal{A}$ be $I\alpha_g^{\wedge}$- *CS*. If $\mathcal{A}$ is Iα-C, then we have Igcl($\mathcal{A}$)- $\mathcal{A}$ = φ which is I *CS*. Conversely, let Igcl($\mathcal{A}$)- $\mathcal{A}$ is IC. Then, by theorem 3.30, Igcl($\mathcal{A}$)- $\mathcal{A}$ does not contain any non-empty I-C subset and since Igcl($\mathcal{A}$)- $\mathcal{A}$ is I-C subset of itself, then Igcl($\mathcal{A}$)- $\mathcal{A}$ = φ. This implies that Igcl($\mathcal{A}$) = $\mathcal{A}$ and so $\mathcal{A}$ is Iα- *CS*.

Remark 3.34: Union of any two $I\alpha_g^\wedge$- CS is $I\alpha_g^\wedge$-C.

Proof: Let $\mathcal{A}$ and $\mathfrak{B}$ are any two $I\alpha_g^\wedge$- CS. Let $\mathcal{A} \cup \mathfrak{B} \subseteq \mathcal{F}$ where $\mathcal{F}$ is $I\alpha$-O. Then $\mathcal{A} \subseteq \mathcal{F}$ and $\mathfrak{B} \subseteq \mathcal{F}$. Since $\mathcal{A}$ and $\mathfrak{B}$ are $I\alpha_g^\wedge$- CS, Igcl($\mathcal{A}$) $\subseteq \mathcal{F}$ and Igcl($\mathfrak{B}$) $\subseteq \mathcal{F}$. Then Igcl($\mathcal{A} \cup \mathfrak{B}$)= Igcl($\mathcal{A}$) $\cup$ Igcl($\mathfrak{B}$) $\subseteq \mathcal{F}$. Hence $\mathcal{A} \cup \mathfrak{B}$ is $I\alpha_g^\wedge$-CS.

Example 3.35: Let $\mathcal{H}$ ={12,13} and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4\}$ where $\mathcal{A}_1 =< \mathcal{H}, \varphi, \varphi >$, $\mathcal{A}_2 =< \mathcal{H}, \{12\}, \{13\} >$, $\mathcal{A}_3 =< \mathcal{H}, \{12\}, \varphi >$, $\mathcal{A}_4 =< \mathcal{H}, \varphi, \{13\} >$. Then intuitionistic subsets $< \mathcal{H}, \varphi, \{12\} >$ and $< \mathcal{H}, \{13\}, \{12\} >$ are $I\alpha_g^\wedge$-C and their union is also $I\alpha_g^\wedge$-C.

Remark 3.36: Intersection of any two $I\alpha_g^\wedge$-CS is $I\alpha_g^\wedge$- CS which is seen from the following example.

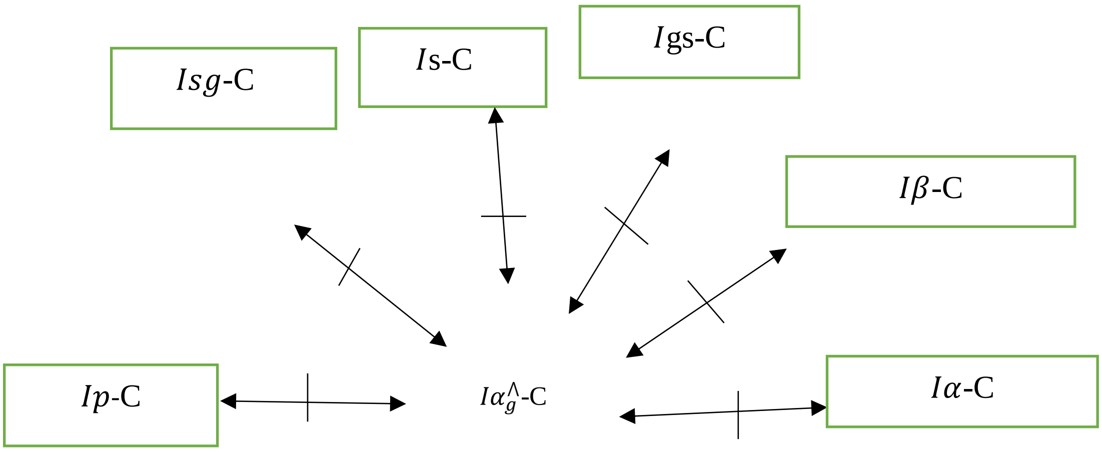
Example 3.37: : Let $\mathcal{H}$ ={p,q,c} and the family $I\tau = \{\varphi, \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3\}$ where $\mathcal{A}_1 =< \mathcal{H}, \varphi, \{p, c\} >$, $\mathcal{A}_2 =< \mathcal{H}, \{p\}, \{c\} >$, $\mathcal{A}_3 =< \mathcal{H}, \{p, q\}, \varphi >$. Then the intuitionistic subsets $< \mathcal{H}, \varphi, \{p, q\} > and < \mathcal{H}, \{p, c\}, \varphi >$ are $I\alpha_g^\wedge$-C and their intersection is also $I\alpha_g^\wedge$-C.

Remark 3.38: In an ITS ($\mathcal{H}, I\tau$) if Igcl($\mathcal{A}$)= $\mathcal{A}$ then $\mathcal{A}$ is $I\alpha_g^\wedge$-C.

Theorem 3.39: Let $\mathcal{A}$ be an $I\alpha_g^\wedge$-CS of an ITS ($\mathcal{H}, I\tau$) and $\mathcal{A} \subseteq \mathfrak{B} \subseteq I$gcl($\mathcal{A}$), then $\mathfrak{B}$ is $I\alpha_g^\wedge$-C in $\mathcal{H}$.

Proof: Let A be an $I\alpha_g^\wedge$- CS of an ITS ($\mathcal{H}, \tau$) and $\mathcal{A} \subseteq \mathfrak{B} \subseteq I$gcl($\mathcal{A}$). Let $\mathfrak{B} \subseteq \mathcal{F}$ where $\mathcal{F}$ be an $I\alpha$-O. Then $\mathcal{A} \subseteq \mathfrak{B}$ implies $\mathcal{A} \subseteq \mathcal{F}$ and since $\mathcal{A}$ is an $I\alpha_g^\wedge$-C, we have Igcl($\mathcal{A}$) $\subseteq \mathcal{F}$. Now $\mathfrak{B} \subseteq I$gcl($\mathcal{A}$) which implies Igcl($\mathfrak{B}$) $\subseteq I$gcl(Igcl($\mathcal{A}$))= Igcl($\mathcal{A}$) $\subseteq \mathcal{F}$. Hence $\mathfrak{B}$ is $I\alpha_g^\wedge$-C in $\mathcal{H}$.

Inter-Relationship 3.40:



Relationship 3.41: The above theorems, we have the following diagram.

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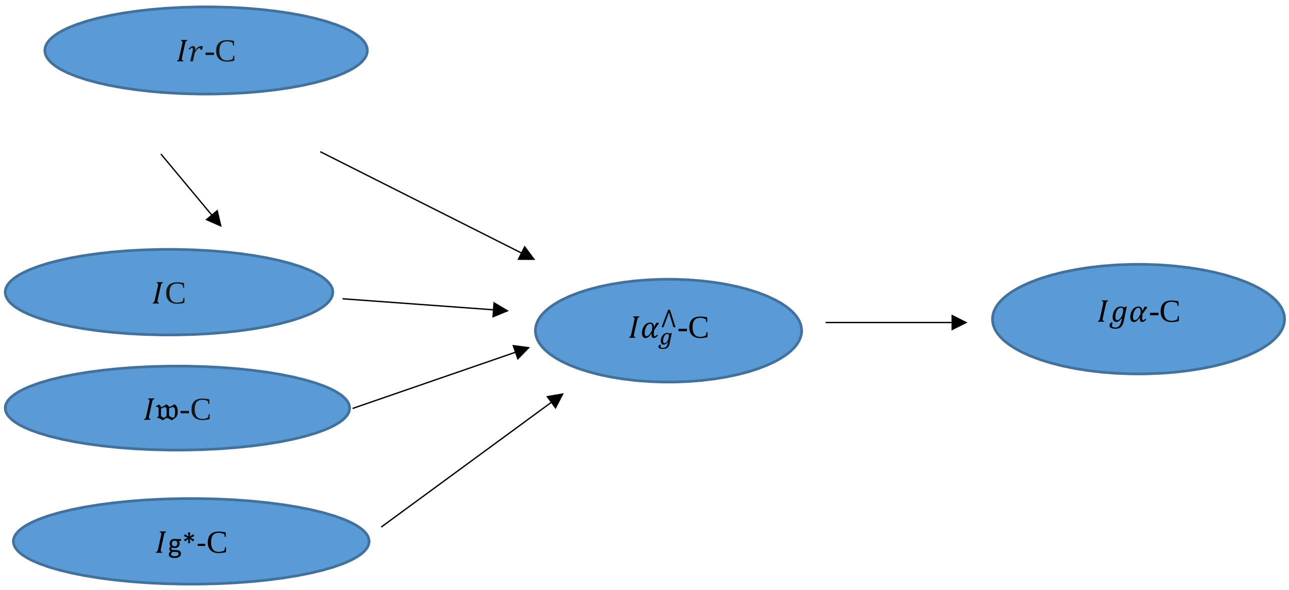

4. INTUITIONISTIC $\alpha_g^\wedge$-OPEN SETS

Definition 4.1: A subset $\mathcal{A}$ of (X, $I\tau$) is called an ***intuitionistic alpha ^ generalized open set*** (briefly ***$I\alpha_g^\wedge$-OS***) if $\mathcal{H} \setminus \mathcal{A}$ is $I\alpha_g^\wedge$-C. The collection of all $I\alpha_g^\wedge$-OS in $(\mathcal{H}, I\tau)$ is denoted by $I\alpha_g^\wedge O(\mathcal{H}, I\tau)$.

Theorem 4.2: Every I-OS of an ITS $(\mathcal{H}, I\tau)$ is $I\alpha_g^\wedge$-OS.

Theorem 4.3: Every $I\alpha_g^\wedge$-OS is Igα-O.

Theorem 4.4: Every Ig*-OS is $I\alpha_g^\wedge$-O.

Theorem 4.5: Every Iw-OS is $I\alpha_g^\wedge$-O.

Theorem 4.6: Every Ir-OS is $I\alpha_g^\wedge$-O.

Remark 4.7: $I\alpha_g^\wedge$-OS and Is-OP are independent.

Remark 4.8: $I\alpha_g^\wedge$-OS and Ip-OS are independent.

Remark 4.9: $I\alpha_g^\wedge$-OS and $I\alpha$-OS are independent.

Remark 4.10: $I\alpha_g^\wedge$-OS and $I\beta$-OS are independent.

Remark 4.11: $I\alpha_g^\wedge$-OS and Isg-OS are independent.

Remark 4.12: $I\alpha_g^\wedge$-OS and Igs-OS are independent.

Proof: The proof follows from theorem 3.2 to 3.29

Theorem 4.13: Let $\mathcal{A}$ be an intuitionistic subset of an ITS $(\mathcal{H}, I\tau)$ then $\mathcal{A}$ is $I\alpha_g^\wedge$-O if and only if $\mathcal{F} \subseteq I\text{gint}(\mathcal{A})$ whenever $\mathcal{F}$ is I-C and $\mathcal{F} \subseteq \mathcal{A}$.

Proof: Necessity: Let $\mathcal{A}$ be an $I\alpha_g^\wedge$-O in $\mathcal{H}$ and $\mathcal{F}$ be I-C in $\mathcal{H}$ such that $\mathcal{F} \subseteq \mathcal{A}$, then $\mathcal{F}^c$ is an I-O in $\mathcal{H}$ such that $\mathcal{A}^c \subseteq \mathcal{F}^c$, $\mathcal{A}^c$ is an $I\alpha_g^\wedge$-C so $I\text{gcl}(\mathcal{A}^c) \subseteq \mathcal{F}^c$ but $I\text{gcl}(\mathcal{A}^c) = (I\text{gint}(\mathcal{A}))^c \subseteq \mathcal{F}^c$ implies $\mathcal{F} \subseteq I\text{gint}(\mathcal{A})$.
Sufficiency: Let F be an I-O in $\mathcal{H}$ such that $\mathcal{A}^c \subseteq$F. Then F^c is I-C in $\mathcal{H}$ and $F^c \subseteq \mathcal{A}$. To Prove: $\mathcal{A}^c$ is an $I\alpha_g^\wedge$-C.

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Now $F^c \subseteq I$gint($\mathcal{A}$) which implies Igcl($\mathcal{A}^c$)= (Igint($\mathcal{A}$))^c ⊆F. Hence $\mathcal{A}^c$ is an $I\alpha_g^\wedge$-C which implies $\mathcal{A}$ is an $I\alpha_g^\wedge$-O in $\mathcal{H}$.

5. $I\alpha_g^\wedge$*-closure* and $I\alpha_g^\wedge$*-interior*

Definition 5.1: Let $\mathcal{A}$ be a subset of an ITS of $\mathcal{H}$. Then ***$I\alpha_g^\wedge$-closure*** of $\mathcal{A}$ is defined as the intersection of all $I\alpha_g^\wedge$-CS of $\mathcal{H}$ containing $\mathcal{A}$. It is denoted by $I\alpha_g^\wedge cl(\mathcal{A})$.

Theorem 5.2: If $\mathcal{A}$ is any subset of $\mathcal{H}$, $I\alpha_g^\wedge cl(\mathcal{A})$ is $I\alpha_g^\wedge$-C. In fact $I\alpha_g^\wedge cl(\mathcal{A})$ is the smallest $I\alpha_g^\wedge$-CS in $\mathcal{H}$ Containing $\mathcal{A}$

Proof: Follows from Definition 5.1 and Theorem 3.3 .

Theorem 5.3: A subset $\mathcal{A}$ of $\mathcal{H}$ is $I\alpha_g^\wedge$-C iff $I\alpha_g^\wedge cl(\mathcal{A})$= $\mathcal{A}$.

Proof: Suppose $\mathcal{A}$ is $I\alpha_g^\wedge$-C implies $I\alpha_g^\wedge cl(\mathcal{A})$= $\mathcal{A}$ is obvious. Conversely, suppose $I\alpha_g^\wedge cl(\mathcal{A})$= $\mathcal{A}$. By Theorem 5.2, $I\alpha_g^\wedge cl(\mathcal{A})$ is $I\alpha_g^\wedge$-C and hence $\mathcal{A}$ is $I\alpha_g^\wedge$-C.

Theorem 5.4: If $\mathcal{A}$ and $\mathfrak{B}$ be subsets of an ITS ($\mathcal{H}$,$I\tau$), then the following results hold:

(i) $I\alpha_g^\wedge cl(\varphi)$= φ.

(ii) $I\alpha_g^\wedge cl(\mathcal{H})$= $\mathcal{H}$.

(iii) $\mathcal{A} \subseteq I\alpha_g^\wedge cl(\mathcal{A})$.

(iv) $\mathcal{A} \subseteq \mathfrak{B} \Longrightarrow I\alpha_g^\wedge cl(\mathcal{A}) \subseteq I\alpha_g^\wedge cl(\mathfrak{B})$.

(v) $I\alpha_g^\wedge cl(I\alpha_g^\wedge cl(\mathcal{A}))$= $I\alpha_g^\wedge cl(\mathcal{A})$.

(vi) $I\alpha_g^\wedge cl(\mathcal{A}$ U$\mathfrak{B}) \supseteq I\alpha_g^\wedge cl(\mathcal{A})$ U $I\alpha_g^\wedge cl(\mathfrak{B})$.

(vii) $I\alpha_g^\wedge cl(\mathcal{A} \cap \mathfrak{B}) \subseteq I\alpha_g^\wedge cl(\mathcal{A}) \cap I\alpha_g^\wedge cl(\mathfrak{B})$.

Proof: (i), (ii), (iii) and (iv) follows from Definition 5.1. (v) Follows from Theorem 5.2 and Theorem 5.3 (vi) from (iv) $I\alpha_g^\wedge$cl ($\mathcal{A}$) ⊆ $I\alpha_g^\wedge$cl($\mathcal{A}$ U$\mathfrak{B}$) and $I\alpha_g^\wedge$cl ($\mathfrak{B}$) ⊆ $I\alpha_g^\wedge$cl ($\mathcal{A}$ U$\mathfrak{B}$) which implies $I\alpha_g^\wedge$cl ($\mathcal{A}$ U$\mathfrak{B}$) ⊇ $I\alpha_g^\wedge$cl ($\mathcal{A}$)U $I\alpha_g^\wedge$cl ($\mathfrak{B}$). (vii) Again, from (iv) $I\alpha_g^\wedge$cl ($\mathcal{A}$) ⊇ $I\alpha_g^\wedge$cl ($\mathcal{A} \cap \mathfrak{B}$) and $I\alpha_g^\wedge$cl ($\mathfrak{B}$) ⊇ $I\alpha_g^\wedge$cl ($\mathcal{A} \cap \mathfrak{B}$) ⇒ $I\alpha_g^\wedge$cl ($\mathcal{A} \cap \mathfrak{B}$) ⊆ $I\alpha_g^\wedge$cl ($\mathcal{A}$) ∩ $I\alpha_g^\wedge$cl ($\mathfrak{B}$).

Definition 5.5: Let $\mathcal{A}$ be a subset of an ITS of $\mathcal{H}$. Then ***$I\alpha_g^\wedge$-interior*** of $\mathcal{A}$ is defined as the union of all $I\alpha_g^\wedge$-OS of $\mathcal{H}$ contained in $\mathcal{A}$. It is denoted by $I\alpha_g^\wedge int(\mathcal{A})$.

Theorem 5.6: If $\mathcal{A}$ is any subset of $\mathcal{H}$, $I\alpha_g^\wedge$int($\mathcal{A}$) is $I\alpha_g^\wedge$-O. In fact $I\alpha_g^\wedge$ $int(\mathcal{A})$ is the largest $I\alpha_g^\wedge$-OS contained in $\mathcal{A}$.

Proof: Follows from Definition 5.5 and Theorem 4.2.

Theorem 5.7: Let $\mathcal{A}$ be subset $\mathcal{A}$ of $\mathcal{H}$. Then A is $I\alpha_g^\wedge$-O if and only if $I\alpha_g^\wedge$int($\mathcal{A}$) = $\mathcal{A}$.

Proof: $\mathcal{A}$ is $I\alpha_g^\wedge$-O implies $I\alpha_g^\wedge int(\mathcal{A})$= $\mathcal{A}$ is obvious. Conversely, let $I\alpha_g^\wedge int(\mathcal{A})$= $\mathcal{A}$. By Theorem 5.6, $I\alpha_g^\wedge int(\mathcal{A})$ is $I\alpha_g^\wedge$-O and hence $\mathcal{A}$ is $I\alpha_g^\wedge$-O.

Theorem 5.8: If $\mathcal{A}$ and $\mathfrak{B}$ are subsets of an ITS ($\mathcal{H}$, $I\tau$), then the following results hold:

(i) $I\alpha_g^\wedge$int(φ)= φ.

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(ii) $I\alpha_g^{\wedge}\mathrm{int}(\mathcal{H})=\mathcal{H}$.

(iii) $I\alpha_g^{\wedge}int(\mathcal{A})\subseteq \mathcal{A}$.

(iv) If $\mathcal{A} \subseteq \mathfrak{B}$, then $I\alpha_g^{\wedge}int\ (\mathcal{A}) \subseteq I\alpha_g^{\wedge}int(\mathfrak{B})$.

(v) $I\alpha_g^{\wedge}int(\ I\alpha_g^{\wedge}int(\mathcal{A}))=\ I\alpha_g^{\wedge}int(\mathcal{A})$.

(vi) $I\alpha_g^{\wedge}int(\mathcal{A}\ \mathrm{U}\ \mathfrak{B}) \supseteq\ I\alpha_g^{\wedge}int(\mathcal{A})\mathrm{U}\ I\alpha_g^{\wedge}int(\mathfrak{B})$.

(vii) $I\alpha_g^{\wedge}int(\mathcal{A} \cap \mathfrak{B}) \subseteq\ I\alpha_g^{\wedge}int(\mathcal{A}) \cap\ I\alpha_g^{\wedge}int(\mathfrak{B})$.

Proof: (i), (ii), (iii) and (iv) follows from Definition 5.5 (v) follows from Theorem 5.6 and Theorem 5.7 (vi) from (iv) $I\alpha_g^{\wedge}int\ (\mathcal{A}) \subseteq\ I\alpha_g^{\wedge}int(\mathcal{A}\ \mathrm{U}\ \mathfrak{B})$ and $I\alpha_g^{\wedge}int(\mathfrak{B}) \subseteq\ I\alpha_g^{\wedge}int(\mathcal{A}\ \mathrm{U}\ \mathfrak{B})$ which implies $I\alpha_g^{\wedge}int(\mathcal{A}\ \mathrm{U}\ \mathfrak{B}) \supseteq$ $I\alpha_g^{\wedge}int(\mathcal{A})\mathrm{U}\ I\alpha_g^{\wedge}int(\mathfrak{B})$. Again, from (iv) $I\alpha_g^{\wedge}int\ (\mathcal{A}) \supseteq\ I_{ntu}\alpha_g^{\wedge}int(\mathcal{A} \cap \mathfrak{B})$ and $I\alpha_g^{\wedge}int(\mathfrak{B}) \supseteq\ I\alpha_g^{\wedge}int(\mathcal{A} \cap \mathfrak{B}) \Rightarrow$ $I\alpha_g^{\wedge}int(\mathcal{A} \cap \mathfrak{B}) \subseteq\ I\alpha_g^{\wedge}int(\mathcal{A}) \cap\ I\alpha_g^{\wedge}int(\mathfrak{B})$.

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