JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3197-3217
https://publishoa.com
ISSN: 1309-3452

Fuzzy Quotient-3 Cordial Labeling on some Unicyclic Graphs with Pendant Edges

Dr. P. Sumathi[1], J.Suresh Kumar[2]

[1]Department of Mathematics,

C. Kandaswami Naidu College for Men, Chennai, Tamil Nadu, India.

[2]Department of Mathematics,

St. Thomas College of Arts and Science, Chennai, Tamil Nadu, India.

[1] Sumathipaul@gmail.com

[2]jskumar.robo@gmail.com (Corresponding author)

Received 2022 March 25; **Revised** 2022 April 28; **Accepted** 2022 May 15.

Abstract. Consider a non-trivial, simple, undirected graph G having a vertex set $V(G)$ with p vertices and edge sets $E(G)$ with q edges. Let the function

$\sigma : V(G) \rightarrow [0,1]$ defined by $\sigma(\alpha) = \frac{r}{10}, r \in Z_4 - \{0\}$ and for each $\alpha\beta \in E(G)$, the induced function $\mu : E(G) \rightarrow [0,1]$ assigns the label for $\mu(\alpha\beta) = \frac{1}{10}\left\lceil \frac{3\sigma(\alpha)}{\sigma(\beta)} \right\rceil$ where $\sigma(\alpha) \leq \sigma(\beta)$. Then σ is called fuzzy quotient 3 cordial labeling if $|v_\sigma(h) - v_\sigma(\kappa)| \leq 1$ and $|\varepsilon_\mu(h) - \varepsilon_\mu(\kappa)| \leq 1$. For $h \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$, $v_\sigma(h)$ and $\varepsilon_\mu(h)$ represent the number of vertices and edges assigned the labels h respectively, If a graph admit this labeling, then it is fuzzy quotient 3 cordial. The existence of above labeling on $C_\eta[m], C_\eta[m,l], C_{2\eta}[m]A$, $C_\eta \odot K_{1,m}, C_\eta[a,d]$ and $C_\eta[a,r]$ are examined and the results are provided in this paper.

AMS Subject Classification: 05C78.

Keywords: Cycle, Pendant edges, Fuzzy quotient 3cordial graph.

1. Introduction

Labeling is a process of assigning values to vertices, edges, or both of a graph based on certain conditions [1-2]. Rosa and Graham and Sloane were the first to use this technique [3]. The researchers are highly motivated and enthusiastic about labeling the graph. Joseph A. Gallian summarises a comprehensive discussion of graph labelling. As a result of these labelings, we introduced fuzzy quotient-3 cordial labeling in [4-2] and analysed some graph families as fuzzy quotient 3 cordial [14]. This paper investigates fuzzy quotient-3 cordial labeling on several subdivision graphs and demonstrates that the graphs are naturally fuzzy quotient 3 cordial.

2. Definitions

Definition 2.1. A graph denoted by $C_\eta[m]$, is produced by linking a vertex of the cycle C_η with m leaves.

Definition 2.2. The graph is $C_\eta[m,l]$, obtained by attaching m pendant edges to the first vertex and l pendant edges to the $\left(\frac{\eta}{2}+1\right)^{th}$ vertex of a cycle , if η is even or by attaching m pendant edges to the first and l pendant edges to the $\left(\frac{\eta+1}{2}\right)^{th}$ vertex of a cycle C_η, if η is odd.

Definition 2.3. A graph results by connecting the m leaves to the non-adjacent vertices of a cycle $C_{2\eta}$ is denoted by $C_{2\eta}[m]A$, $\eta \geq 2$.

Definition 2.4. The graph $C_\eta \odot K_{1,m}$ is obtained by attaching m leaves toeach vertex of a cycle Cη.

Definition 2.5. Attaching $a + (i-1)d$, $a, d \geq 1$ leaves to the i^{th} vertex of a cycle

$C\eta$ yields the new graph and it is denoted by $C\eta\,[a, d]$.

Definition 2.6. Attaching $\frac{a(r^i-1)}{r-1}$, $a, r \geq 1$ leaves to the i^{th} vertex of a cycle C_η yields the new graph and it is denoted by $C\eta\,[a, r]$.

Definition 2.7. Consider a non-trivial, simple, undirected graph G having a vertex set $V(G)$ with p vertices and edge sets $E(G)$ with q edges. Let the function

$\sigma : V(G) \to [0,1]$ defined by $\sigma(\alpha) = \frac{r}{10}, r \in Z_4 - \{0\}$ and for each $\alpha\beta \in E(G)$, the induced function $\mu : E(G) \to [0,1]$ assigns the label for $\mu(\alpha\beta) = \frac{1}{10}\left\lceil\frac{3\sigma(\alpha)}{\sigma(\beta)}\right\rceil$ where $\sigma(\alpha) \leq \sigma(\beta)$. Then σ is called fuzzy quotient 3 cordial labeling if $|v_\sigma(h) - v_\sigma(\kappa)| \leq 1$ and $|\varepsilon_\mu(h) - \varepsilon_\mu(\kappa)| \leq 1$. For $h \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$, $v_\sigma(h)$ and $\varepsilon_\mu(h)$ represent the number of vertices and edges assigned the labels h respectively, If a graph admit this labeling, then it is fuzzy quotient 3 cordial.

3. Main Results

Theorem 3.1: The graph $C_\eta(m)$, $\eta \geq 3, m \geq 1$ is fuzzy quotient 3 cordial.

Proof: Let $V\left(C_\eta(m)\right) = \{y\} \cup \{x_i : 2 \leq i \leq \eta\} \cup \{y_\kappa : 1 \leq \kappa \leq m\}$ and

$E\left(C_\eta(m)\right) = \{y\,x_2\} \cup \{x_i x_{i+1} : 2 \leq i \leq \eta - 1\} \cup \{x_\eta y\} \cup \{y\,y_\kappa : 1 \leq \kappa \leq m\}$.

For $C_\eta(m)$, $p = \eta + m = q$. Assigning labels to this graph involves,

Case 1. $\boldsymbol{\eta \equiv 0 (mod\ 6)}$

$\sigma(y) = 0.1$

$$\sigma(x_i) = \begin{cases} 0.1 & if\ i = 6S+1\ or\ 6s+2 \\ 0.2 & if\ i = 6S+4\ or\ 6S+5 \\ 0.3 & if\ i = 6S\ or\ 6S+3 \end{cases}, \text{ for all } i \in \{2,3,4,\ldots\eta\} \text{ and } S \geq 0.$$

$$\sigma(y_\kappa) = \begin{cases} 0.1 & if\ i = 3S+1 \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S \end{cases}, \text{ for all } \kappa \in \{1,2,3,4,\ldots m\} \text{ and } S \geq 0.$$

Case 2. $\boldsymbol{\eta \equiv 1 (mod\ 6)}$

$\sigma(y) = 0.1$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3197-3217
https://publishoa.com
ISSN: 1309-3452

For all $i \in \{2,3,4,\dots\eta\}$ $\sigma(x_i)$ is same as in Case 1.

$$\sigma(y_\kappa) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S+1 \end{cases}, \text{ for all } \kappa \in \{1,2,3,4,\dots m\} \text{ and } S \geq 0.$$

Case 3. $\boldsymbol{\eta \equiv 2(mod\ 6)}$

$\sigma(y) = 0.1$

For all $i \in \{2,3,4,\dots\eta-3\}$ $\sigma(x_i)$ is same as in Case 1.

$\sigma(x_{\eta-2}) = 0.1, \sigma(x_{\eta-2}) = \sigma(x_\eta) = 0.3$

$$\sigma(y_\kappa) = \begin{cases} 0.1 & if\ i = 3S+2 \\ 0.2 & if\ i = 3S+1 \\ 0.3 & if\ i = 3S \end{cases}, \text{ for all } \kappa \in \{1,2,3,4,\dots m\} \text{ and } S \geq 0.$$

Case 4. $\boldsymbol{\eta \equiv 3(mod\ 6)}$

$\sigma(y) = 0.3$

$$\sigma(x_i) = \begin{cases} 0.1 & if\ i = 6S+2\ or\ 6s+3 \\ 0.2 & if\ i = 6S\ or\ 6S+5 \\ 0.3 & if\ i = 6S+1\ or\ 6S+4 \end{cases}, \text{ for all } i \in \{2,3,4,\dots\eta\} \text{ and } S \geq 0.$$

$\sigma(y_1) = 0.2$

$$\sigma(y_\kappa) = \begin{cases} 0.1 & if\ i = 3S+1 \\ 0.2 & if\ i = 3S \\ 0.3 & if\ i = 3S+2 \end{cases}, \text{ for all } i \in \{2,3,4,\dots m\} \text{ and } S \geq 0.$$

Case 5. $\boldsymbol{\eta \equiv 4(mod\ 6)}$

$\sigma(y) = 0.3$

For all $i \in \{2,3,4,\dots\eta\}$ $\sigma(x_i)$ is same as in Case 4.

$\sigma(y_1) = \sigma(y_2) = 0.2$ and for all $\kappa \in \{3,4,\dots m\}$ $\sigma(y_\kappa)$ is same as in case 4.

Case 6. $\boldsymbol{\eta \equiv 5(mod\ 6)}$

$\sigma(y) = 0.1$

For all $i \in \{2,3,4,\dots\eta\}$ $\sigma(x_i)$ is same as in Case 1.

For all $\kappa \in \{1,2,3,4,\dots m\}$ $\sigma(y_\kappa)$ is same as in Case 2.

By the result of above assignment we could see that the elements of $E\left(C_\eta(m)\right)$ receives the label $\iota \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$ and also for $\iota \neq h \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$, $|v_\sigma(\iota) - v_\sigma(h)| \leq 1$ and $\left|\varepsilon_\mu(\iota) - \varepsilon_\mu(h)\right| \leq 1$. Then by definition 2.7, $C_\eta(m)$ is fuzzy quotient 3 cordial.

Theorem 3.2

The graph $C_\eta(m,l)$ is fuzzy quotient 3 cordial for all odd $\eta \geq 3$ and $m, l \geq 1$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3197-3217
https://publishoa.com
ISSN: 1309-3452

Proof: Let $V\left(C_\eta(m,l)\right) = \{x_i : 1 \le i \le \eta\} \cup \{y_j : 1 \le j \le m\} \cup \{z_\kappa : 1 \le \kappa \le l\}$ and $E\left(C_\eta(m,l)\right) = \{x_i x_{i+1} : 1 \le i \le \eta - 1\} \cup \{x_\eta x_1\} \cup \{x_1 y_j : 1 \le j \le m\} \cup \left\{x_{\frac{\eta+1}{2}} z_\kappa : 1 \le \kappa \le l\right\}$.

For $C_\eta(m,l)$, $p = \eta + m + l = q$. Assigning labels to this graph involves,

Case 1: $\eta \equiv 1(mod\ 6)$

$$\sigma(x_i) = \begin{cases} 0.1 & if\ i = 6S\ or\ 6s+5 \\ 0.2 & if\ i = 6S+2\ or\ 6S+3 \\ 0.3 & if\ i = 6S+1\ or\ 6S+4 \end{cases}, \text{ for all } i \in \{1,2,3,4,\dots\eta\} \text{ and } S \ge 0.$$

$$\sigma(y_j) = \begin{cases} 0.1 & if\ i = 3S+1 \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S \end{cases}, \text{ for all } j \in \{1,2,3,4,\dots m\} \text{ and } S \ge 0.$$

Subcase 1.1: $m \equiv 0(mod\ 3)$

$$\sigma(z_\kappa) = \begin{cases} 0.1 & if\ i = 3S+1 \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S \end{cases}, \text{ for all } \kappa \in \{1,2,3,4,\dots l\} \text{ and } S \ge 0.$$

Subcase 1.2: $m \equiv 1(mod\ 3)$

$$\sigma(z_\kappa) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S+1 \\ 0.3 & if\ i = 3S+2 \end{cases}, \text{ for all } \kappa \in \{1,2,3,4,\dots l\} \text{ and } S \ge 0.$$

Subcase 1.3: $m \equiv 2(mod\ 3)$

$\sigma(z_\kappa)$ is same as in subcase 1.1.

Case 2: $\eta \equiv 3(mod\ 6)$

i. **If $\frac{\eta+1}{2}$ is even**

$\sigma(x_1) = \sigma(x_{\eta-2}) = 0.1, \sigma(x_{\eta-1}) = \sigma(x_\eta) = 0.3$ and for all $i \in \{2,3,4,\dots\eta-3\}$ $\sigma(x_i)$ is same as in case 1.

ii. **If $\frac{\eta+1}{2}$ is odd**

$\sigma(x_1) = 0.1$, $\sigma(x_{\eta-2}) = \sigma(x_{\eta-1}) = 0.3, \sigma(x_\eta) = 0.1$ and for all $i \in \{2,3,4,\dots\eta-3\}$ $\sigma(x_i)$ is same as in case 1.

Subcase 2.1: $m \equiv 0(mod\ 3)$

$$\sigma(y_j) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S+1 \\ 0.3 & if\ i = 3S+2 \end{cases}, \text{ for all } j \in \{1,2,3,4,\dots m-1\} \text{ and } S \ge 0.$$

$\sigma(y_m) = 0.2$

$\sigma(z_\kappa)$ is same as in subcase 1.2.

Subcase 2.2: $m \equiv 1,2(mod\ 3)$

$\sigma(y_j)$ is same as in Subcase 2.1 for all $j \in \{1,2,3,4,\dots m-1\}$ and $\sigma(y_m) = 0.3$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3197-3217
https://publishoa.com
ISSN: 1309-3452

$\sigma(z_{\kappa})$ is same as in subcase 1.2.

Case 3: $\eta \equiv 5(mod\ 6)$

$$\sigma(x_i) = \begin{cases} 0.1 & if\ i = 6S+1\ or\ 6s+2 \\ 0.2 & if\ i = 6S+4\ or\ 6S+5 \\ 0.3 & if\ i = 6S\ or\ 6S+3 \end{cases}, \text{ for all } i \in \{1,2,3,4,\dots\eta\} \text{ and } S \geq 0.$$

Subcase 3.1: $m \equiv 0(mod\ 3)$

$$\sigma(y_j) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S+1 \end{cases}, \text{ for all } j \in \{1,2,3,4,\dots m-3\} \text{ and } S \geq 0.$$

$\sigma(y_{m-2}) = \sigma(y_{m-1}) = 0.3$, $\sigma(y_m) = 0.1$

$$\sigma(z_{\kappa}) = \begin{cases} 0.1 & if\ i = 3S+2 \\ 0.2 & if\ i = 3S+1 \\ 0.3 & if\ i = 3S \end{cases}, \text{ for all } \kappa \in \{1,2,3,4,\dots l\} \text{ and } S \geq 0.$$

Subcase 3.2: $m \equiv 1(mod\ 3)$

$\sigma(y_j)$ is same as in Subcase 3.1 for all $j \in \{1,2,3,4,\dots m-3\}$ and
$\sigma(y_{m-2}) = 0.2, \sigma(y_{m-1}) = 0.1, \sigma(y_m) = 0.3$

$\sigma(z_{\kappa})$ is same as in subcase 3.1.

Subcase 3.3: $m \equiv 2(mod\ 3)$

$\sigma(y_j)$ is same as in Subcase 3.2 for all $j \in \{1,2,3,4,\dots m\}$.

$$\sigma(z_{\kappa}) = \begin{cases} 0.1 & if\ i = 3S+1 \\ 0.2 & if\ i = 3S \\ 0.3 & if\ i = 3S+2 \end{cases}, \text{ for all } \kappa \in \{1,2,3,4,\dots l\} \text{ and } S \geq 0.$$

By the result of above assignment we could see that the elements of $E(C_\eta(m,l))$ receives the label $\iota \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$ and also for $\iota \neq h \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$, $|v_\sigma(\iota) - v_\sigma(h)| \leq 1$ and $|\varepsilon_\mu(\iota) - \varepsilon_\mu(h)| \leq 1$. Then by definition 2.7, $C_\eta(m,l)$ is fuzzy quotient 3 cordial for all odd $\eta \geq 3$ and $m, l \geq 1$.

Theorem 3.3

The graph $C_\eta(m,l)$ is fuzzy quotient 3 cordial for all even $\eta \geq 4$ and $m, l \geq 1$

Proof: Let $V(C_\eta(m,l)) = \{x_i : 1 \leq i \leq \eta\} \cup \{y_j : 1 \leq j \leq m\} \cup \{z_\kappa : 1 \leq \kappa \leq l\}$ and $E(C_\eta(m,l)) = \{x_i x_{i+1} : 1 \leq i \leq \eta - 1\} \cup \{x_\eta x_1\} \cup \{x_1 y_j : 1 \leq j \leq m\} \cup \left\{x_{\frac{\eta}{2}+1} z_\kappa : 1 \leq \kappa \leq l\right\}$.

For $C_\eta(m,l)$, $p = \eta + m + l = q$. Assigning labels to this graph involves,

Case 1: $\eta \equiv 0(mod\ 6)$

$$\sigma(x_i) = \begin{cases} 0.1 & if\ i = 6S+2\ or\ 6s+3 \\ 0.2 & if\ i = 6S\ or\ 6S+5 \\ 0.3 & if\ i = 6S+1\ or\ 6S+4 \end{cases}, \text{ for all } i \in \{1,2,3,4,\dots\eta\} \text{ and } S \geq 0.$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3197-3217
https://publishoa.com
ISSN: 1309-3452

$$\sigma(y_j) = \begin{cases} 0.1 & if\ i = 3S+1 \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S \end{cases}, \text{ for all } j \in \{1,2,3,4,\ldots m\} \text{ and } S \geq 0.$$

Subcase 1.1: $m \equiv 0(mod\ 3)$

$$\sigma(z_\kappa) = \begin{cases} 0.1 & if\ i = 3S+1 \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S \end{cases}, \text{ for all } \kappa \in \{1,2,3,4,\ldots l\} \text{ and } S \geq 0.$$

Subcase 1.2: $m \equiv 1(mod\ 3)$

$$\sigma(z_\kappa) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S+1 \\ 0.3 & if\ i = 3S+2 \end{cases}, \text{ for all } \kappa \in \{1,2,3,4,\ldots l\} \text{ and } S \geq 0.$$

Subcase 1.3: $m \equiv 2(mod\ 3)$

$$\sigma(z_\kappa) = \begin{cases} 0.1 & if\ i = 3S+2 \\ 0.2 & if\ i = 3S \\ 0.3 & if\ i = 3S+1 \end{cases}, \text{ for all } \kappa \in \{1,2,3,4,\ldots l\} \text{ and } S \geq 0.$$

Case 2: $\eta \equiv 2(mod\ 6)$ and $\frac{\eta}{2}$ is even

$\sigma(x_1) = 0.1, \sigma(x_2) = 0.3$

$$\sigma(x_i) = \begin{cases} 0.1 & if\ i = 6S+2\ or\ 6s+5 \\ 0.2 & if\ i = 6S+3\ or\ 6S+4 \\ 0.3 & if\ i = 6S+1\ or\ 6S \end{cases}, \text{ for all } i \in \{3,4,5,\ldots \eta\} \text{ and } S \geq 0.$$

$$\sigma(y_j) = \begin{cases} 0.1 & if\ i = 3S+2 \\ 0.2 & if\ i = 3S+1 \\ 0.3 & if\ i = 3S \end{cases}, \text{ for all } j \in \{1,2,3,4,\ldots m\} \text{ and } S \geq 0,$$

Subcase 2.1: $m \equiv 0(mod\ 3)$

$$\sigma(z_\kappa) = \begin{cases} 0.1 & if\ i = 3S+2 \\ 0.2 & if\ i = 3S+1 \\ 0.3 & if\ i = 3S \end{cases}, \text{ for all } \kappa \in \{1,2,3,4,\ldots l\} \text{ and } S \geq 0.$$

Subcase 2.2: $m \equiv 1(mod\ 3)$

$\sigma(z_\kappa)$ is same as in subcase 1.1.

Subcase 2.3: $m \equiv 2(mod\ 3)$

$$\sigma(z_\kappa) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S+1 \end{cases}, \text{ for all } \kappa \in \{1,2,3,4,\ldots l\} \text{ and } S \geq 0.$$

Case 3: $\eta \equiv 2(mod\ 6)$ and $\frac{\eta}{2}$ is odd

$\sigma(x_1) = 0.3, \sigma(x_2) = 0.1$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3197-3217
https://publishoa.com
ISSN: 1309-3452

$$\sigma(x_i) = \begin{cases} 0.1 & if\ i = 6S\ or\ 6s+1 \\ 0.2 & if\ i = 6S+3\ or\ 6S+4 \\ 0.3 & if\ i = 6S+2\ or\ 6S+5 \end{cases}\Bigg\}, \text{ for all } i \in \{3,4,5,\ldots\eta\} \text{ and } S \geq 0.$$

$\sigma(y_j)$ is same as in Case 2.

Subcase 3.1: $m \equiv 0(mod\ 3)$

$\sigma(z_\kappa)$ is same as in subcase 2.1.

Subcase 3.2: $m \equiv 1(mod\ 3)$

$\sigma(z_\kappa)$ is same as in subcase 1.1.

Subcase 3.3: $m \equiv 2(mod\ 3)$

$\sigma(z_\kappa)$ is same as in subcase 2.3.

Case 4: $\eta \equiv 4(mod\ 6)$ and $\eta = 4$

$\sigma(x_1) = \sigma(x_2) = 0.1, \sigma(x_3) = \sigma(x_4) = 0.3$

Subcase 4.1: $m \equiv 0(mod\ 3)$

If $m = 3, \sigma(y_1) = \sigma(y_2) = \sigma(y_3) = 0.2$

$$\sigma(z_\kappa) = \begin{cases} 0.1 & if\ i = 3S+1 \\ 0.2 & if\ i = 3S \\ 0.3 & if\ i = 3S+2 \end{cases}\Bigg\}, \text{ for all } \kappa \in \{1,2,3,4,\ldots l\} \text{ and } S \geq 0.$$

If $m \geq 6$

$\sigma(y_j)$ is same as in Case 1.

$\sigma(z_\kappa)$ is same as in subcase 2.3 for all $\kappa \in \{1,2,3,4,\ldots l-1\}$ and $\sigma(z_l) = 0.2$

Subcase 4.2: $m \equiv 1(mod\ 3)$

If $m = 1, \sigma(y_1) = 0.2$

$\sigma(z_\kappa)$ is same as in subcase 2.1 for all $\kappa \in \{1,2,3,4,\ldots l\}$

If $m \geq 4$

$$\sigma(y_j) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S+1 \\ 0.3 & if\ i = 3S+2 \end{cases}\Bigg\}, \text{ for all } j \in \{1,2,3,4,\ldots m\} \text{ and } S \geq 0.$$

$\sigma(z_\kappa)$ is same as in subcase 2.1 for all $\kappa \in \{1,2,3,4,\ldots l\}$

Subcase 4.3: $m \equiv 2(mod\ 3)$

If $m = 2, \sigma(y_1) = \sigma(y_2) = 0.2$

$\sigma(z_\kappa)$ is same as in subcase 1.1 for all $\kappa \in \{1,2,3,4,\ldots l\}$

If $m \geq 5$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3197-3217
https://publishoa.com
ISSN: 1309-3452

$$\sigma(y_j) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S+1 \\ 0.3 & if\ i = 3S+2 \end{cases}, \text{ for all } j \in \{1,2,3,4,\dots m-1\} \text{ and } S \geq 0,$$

$\sigma(y_m) = 0.2$

$\sigma(z_\kappa)$ is same as in subcase 1.1 for all $\kappa \in \{1,2,3,4,\dots l\}$

Case 5: $\equiv 4(mod\ 6)$ **,** $\frac{\eta}{2}$ **is even and** $\eta > 4$

$\sigma(x_1) = 0.3, \sigma(x_2) = \sigma(x_3) = 0.1, \sigma(x_4) = \sigma(x_5) = 0.2, \sigma(x_6) = 0.3$

$$\sigma(x_i) = \begin{cases} 0.1 & if\ i = 6S+4\ or\ 6s+5 \\ 0.2 & if\ i = 6S+1\ or\ 6S+2 \\ 0.3 & if\ i = 6S\ or\ 6S+3 \end{cases}, \text{ for all } i \in \{7,8,9,\dots \eta\} \text{ and } S \geq 0.$$

$\sigma(y_j)$ is same as in Case 2.

Subcase 5.1: $m \equiv 0(mod\ 3)$

$\sigma(z_\kappa)$ is same as in subcase 4.1 for all $\kappa \in \{1,2,3,4,\dots l\}$

Subcase 5.2: $m \equiv 1(mod\ 3)$

$\sigma(z_\kappa)$ is same as in subcase 2.3 for all $\kappa \in \{1,2,3,4,\dots l\}$

Subcase 5.3: $m \equiv 2(mod\ 3)$

$\sigma(z_\kappa)$ is same as in subcase 2.1 for all $\kappa \in \{1,2,3,4,\dots l\}$

Case 6: $\equiv 4(mod\ 6)$ **,** $\frac{\eta}{2}$ **is odd**

$$\sigma(x_i) = \begin{cases} 0.1 & if\ i = 6S\ or\ 6s+1 \\ 0.2 & if\ i = 6S+3\ or\ 6S+4 \\ 0.3 & if\ i = 6S+2\ or\ 6S+5 \end{cases}, \text{ for all } i \in \{1,2,3,\dots \eta\} \text{ and } S \geq 0.$$

$\sigma(y_j)$ is same as in Case 2.

Subcase 6.1: $m \equiv 0(mod\ 3)$

$\sigma(z_\kappa)$ is same as in subcase 1.3 for all $\kappa \in \{1,2,3,4,\dots l\}$

Subcase 6.2: $m \equiv 1(mod\ 3)$

$\sigma(z_\kappa)$ is same as in subcase 2.3 for all $\kappa \in \{1,2,3,4,\dots l\}$

Subcase 6.3: $m \equiv 2(mod\ 3)$

$\sigma(z_\kappa)$ is same as in subcase 1.2 for all $\kappa \in \{1,2,3,4,\dots l\}$

By the result of above assignment we could see that the elements of $E(C_\eta(m,l))$ receives the label $\iota \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$ and also for $\iota \neq h \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$, $|v_\sigma(\iota) - v_\sigma(h)| \leq 1$ and $|\varepsilon_\mu(\iota) - \varepsilon_\mu(h)| \leq 1$. Then by definition 2.7, $C_\eta(m,l)$ is fuzzy quotient 3 cordial for all even $\eta \geq 4$ and $m, l \geq 1$.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3197-3217
https://publishoa.com
ISSN: 1309-3452

Theorem 3.4

The graph $C_{2\eta}[m]_A$ is fuzzy quotient 3 cordial for all even $\eta \geq 2$ and $m \geq 1$

Proof: Let $V(C_{2\eta}[m]_A) = \{x_i : 1 \leq i \leq \eta\} \cup \left\{y_j : 1 \leq j \leq \frac{\eta m}{2}\right\}$ and

$E(C_{2\eta}[m]_A) = \{x_i x_{i+1} : 1 \leq i \leq \eta - 1\} \cup \{x_\eta x_1\} \cup$

$\left\{x_{2i} y_j : 1 \leq i \leq \frac{\eta}{2}, 1 + (i-1)m \leq j \leq im\right\}$.

For $C_{2\eta}[m]_A$, $p = \eta + \frac{\eta m}{2} = q$. Assigning labels to this graph involves,

Case 1: $\boldsymbol{\eta \equiv 0 (mod\ 6)}$

Subcase: 1.1: $\boldsymbol{m = 1}$

Subcase: 1.1.1

If $\frac{\eta}{2}$ is even

$\sigma(x_i) = \begin{cases} 0.1 & if\ i = 4S\ or\ 4s+1 \\ 0.3 & if\ i = 4S+2\ or\ 4s+3 \end{cases}$, for all $i \in \{1, 2, 3, \dots \eta\}$ and $S \geq 0$.

$\sigma(y_j) = 0.2$, for all $j \in \{1, 2, 3, 4, \dots \frac{\eta}{2}\}$

Subcase: 1.1.2

If $\frac{\eta}{2}$ is odd

$\sigma(x_i) = \begin{cases} 0.1 & if\ i = 4S\ or\ 4s+1 \\ 0.3 & if\ i = 4S+2\ or\ 4s+3 \end{cases}$, for all $i \in \{1, 2, 3, \dots \eta - 1\}$ and $S \geq 0$.

$\sigma(x_\eta) = 0.2$

For all $\in \left\{1, 2, 3, 4, \dots \frac{\eta}{2} - 2\right\} \sigma(y_j) = 0.2$, $\sigma\left(y_{\frac{\eta}{2}-1}\right) = 0.3$ and $\sigma\left(y_{\frac{\eta}{2}}\right) = 0.2$

Subcase: 1.2: $\boldsymbol{m = 2}$

Subcase: 1.2.1

If $\frac{\eta}{2}$ is even

$\sigma(x_i)$ is same as in Subcase 1.1, for all $i \in \{1, 2, 3, \dots \eta\}$

$\sigma(y_{2j-1}) = 0.2$, for all $\in \{1, 2, 3, 4, \dots \frac{\eta}{2}\}$ and

$\sigma(y_j) = \begin{cases} 0.1 & if\ i = 3S+1 \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S \end{cases}$, for all $j \in \{1, 2, 3, 4, \dots \frac{\eta}{2}\}$ and $S \geq 0$.

Subcase: 1.2.2

If $\frac{\eta}{2}$ is odd

$\sigma(x_i)$ is same as in Subcase 1.1, for all $i \in \{1, 2, 3, \dots \eta\}$

$\sigma(y_{2j-1}) = 0.2$, for all $j \in \{1, 2, 3, 4, \ldots \frac{\eta}{2}\}$ and

$$\sigma(y_j) = \begin{cases} 0.1 & if\ i = 3S+1 \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S \end{cases}$$, for all $j \in \{1, 2, 3, 4, \ldots \frac{\eta}{2} - 3\}$ and $S \geq 0$.

$\sigma\left(y_{\frac{\eta}{2}-2}\right) = 0.2$, $\sigma\left(y_{\frac{\eta}{2}-1}\right) = 0.1$, $\sigma\left(y_{\frac{\eta}{2}}\right) = 0.3$

Subcase: 1.3: $m = 3$

Subcase: 1.3.1

If $\frac{\eta}{2}$ is even

$\sigma(x_i)$ is same as in Subcase 1.1, for all $i \in \{1, 2, 3, \ldots \eta\}$

$\sigma(y_{3j-2}) = 0.2$, for all $j \in \{1, 2, 3, 4, \ldots \frac{\eta}{2}\}$ and

$$\sigma(y_{3j-1}) = \begin{cases} 0.1 & if\ i = 3S+1 \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S \end{cases}$$, for all $j \in \{1, 2, 3, 4, \ldots \frac{\eta}{2}\}$ and $S \geq 0$.

$$\sigma(y_{3j}) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S+1 \end{cases}$$, for all $j \in \{1, 2, 3, 4, \ldots \frac{\eta}{2}\}$ and $S \geq 0$.

Subcase: 1.3.2

If $\frac{\eta}{2}$ is odd

$\sigma(y_{3j-2}) = 0.2$, for all $j \in \{1, 2, 3, 4, \ldots \frac{\eta}{2}\}$ and

$$\sigma(y_{3j-1}) = \begin{cases} 0.1 & if\ i = 3S+1 \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S \end{cases}$$, for all $j \in \{1, 2, 3, 4, \ldots \frac{\eta}{2}\}$ and $S \geq 0$.

$$\sigma(y_{3j}) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S+1 \end{cases}$$, for all $j \in \{1, 2, 3, 4, \ldots \frac{\eta}{2} - 3\}$ and $S \geq 0$.

$\sigma\left(y_{3(\frac{\eta}{2}-2)}\right) = 0.2$, $\sigma\left(y_{3(\frac{\eta}{2}-1)}\right) = 0.1$, $\sigma\left(y_{3(\frac{\eta}{2})}\right) = 0.3$.

Subcase: 1.4: $m = 4$

$\sigma(x_i) = 0.3$, for all $i \in \{1, 2, 3, \ldots \eta\}$

$$\sigma(y_j) = \begin{cases} 0.1 & if\ j = 2S \\ 0.2 & if\ j = 2S+1 \end{cases}$$, for all $j \in \{1, 2, 3, \ldots \frac{\eta m}{2}\}$ and $S \geq 0$.

Subcase: 1.5: $\eta \equiv 0 (mod\ 6)$ and $m \equiv 0, 1, 2 (mod\ 3)$, $m \geq 5$

$\sigma(x_i) = 0.3$, for all $i \in \{1, 2, 3, \ldots \eta\}$

$\sigma(y_j) = 0.3$, for all $i \in \{1, 2, 3, \ldots \frac{\eta m - 4\eta}{6}\}$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3197-3217
https://publishoa.com
ISSN: 1309-3452

$\sigma(y_j) = \begin{cases} 0.1 & if\ j = 2S \\ 0.2 & if\ j = 2S+1 \end{cases}$, for all $j \in \{\frac{\eta m-4\eta}{6}+1, \frac{\eta m-4\eta}{6}+2, \ldots \frac{\eta m}{2}\}$ and $S \geq 0$.

Case 2: $\eta \equiv 2(mod\ 6)$

Subcase: 2.1 $m = 1$

Subcase: 2.1.1

If $\frac{\eta}{2}$ is even

$\sigma(x_i)$ is same as in Subcase 1.1, for all $i \in \{1, 2, 3, \ldots \eta\}$

$\sigma(y_j) = 0.2$, for all $j \in \{1, 2, 3, 4, \ldots \frac{\eta}{2}\}$

Subcase: 2.1.2

If $\frac{\eta}{2}$ is odd

$\sigma(x_i)$ is same as in Subcase 1.1, for all $i \in \{1, 2, 3, \ldots \eta - 1\}$ and $\sigma(x_\eta) = 0.2$

$\sigma(y_j) = 0.2$, for all $j \in \{1, 2, 3, 4, \ldots \frac{\eta}{2} - 2\}$

$\sigma\left(y_{\frac{\eta}{2}-1}\right) = 0.3$, $\sigma\left(y_{\frac{\eta}{2}}\right) = 0.2$

Subcase: 2.2 $m = 2$

Subcase: 2.2.1

If $\frac{\eta}{2}$ is even

$\sigma(x_i)$ is same as in Subcase 1.1, for all $i \in \{1, 2, 3, \ldots \eta\}$

$\sigma(y_{2j-1}) = 0.2$, for all $j \in \{1, 2, 3, 4, \ldots \frac{\eta}{2}\}$ and

$\sigma(y_j) = \begin{cases} 0.1 & if\ i = 3S+1 \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S \end{cases}$, for all $j \in \{1, 2, 3, 4, \ldots \frac{\eta}{2}\}$ and $S \geq 0$.

Subcase: 2.2.2

If $\frac{\eta}{2}$ is odd

$\sigma(x_i)$ is same as in Subcase 1.1, for all $i \in \{1, 2, 3, \ldots \eta\}$

$\sigma(y_{2j-1}) = 0.2$, for all $j \in \{1, 2, 3, 4, \ldots \frac{\eta}{2}\}$ and

$\sigma(y_j) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S+1 \end{cases}$, for all $j \in \{1, 2, 3, 4, \ldots \frac{\eta}{2}\}$ and $S \geq 0$.

Subcase: 2.3 $m = 3$

Subcase: 2.3.1

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3197-3217
https://publishoa.com
ISSN: 1309-3452

If $\frac{\eta}{2}$ is even

$\sigma(x_i)$ is same as in Subcase 1.1, for all $i \in \{1,2,3,\dots\eta\}$

$\sigma(y_{3j-2}) = 0.2$, for all $j \in \{1,2,3,4,\dots\frac{\eta}{2}\}$ and

$$\sigma(y_{3j-1}) = \begin{cases} 0.1 & if\ i = 3S+1 \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S \end{cases}, \text{ for all } j \in \{1,2,3,4,\dots \tfrac{\eta}{2}\} \text{ and } S \geq 0.$$

$$\sigma(y_{3j}) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S+1 \end{cases}, \text{ for all } j \in \{1,2,3,4,\dots \tfrac{\eta}{2}\} \text{ and } S \geq 0.$$

Subcase: 2.3.2

If $\frac{\eta}{2}$ is odd

$\sigma(x_i)$ is same as in Subcase 1.1.2, for all $i \in \{1,2,3,\dots\eta-1\}$

$\sigma(x_\eta) = 0.1$

$\sigma(y_{3j-2}) = 0.2$, for all $j \in \{1,2,3,4,\dots\frac{\eta}{2}\}$ and

$$\sigma(y_{3j-1}) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S+1 \end{cases}, \text{ for all } j \in \{1,2,3,4,\dots \tfrac{\eta}{2}\} \text{ and } S \geq 0.$$

$$\sigma(y_{3j}) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S+1 \end{cases}, \text{ for all } j \in \{1,2,3,4,\dots \tfrac{\eta}{2}\} \text{ and } S \geq 0.$$

Subcase: 2.4 $m = 4$

$\sigma(x_i) = 0.3$, for all $i \in \{1,2,3,\dots\eta\}$

$$\sigma(y_{3j}) = \begin{cases} 0.1 & if\ j = 2S \\ 0.2 & if\ j = 2S+1 \end{cases}, \text{ for all } j \in \{1,2, \dots\tfrac{\eta m}{2}\} \text{ and } S \geq 0.$$

Subcase: 2.5 $\eta \equiv 2(mod\ 6)$ and $m \equiv 0(mod\ 3), m \geq 5$

$\sigma(x_i) = 0.3$, for all $i \in \{1,2,3,\dots\eta\}$

$\sigma(y_j) = 0.3$, for all $j \in \{1,2,3,4,\dots\frac{\eta m-4\eta-4}{6}\}$

$$\sigma(y_j) = \begin{cases} 0.1 & if\ j = 2S \\ 0.2 & if\ j = 2S+1 \end{cases}, \text{ for all } j \in \{\tfrac{\eta m-4\eta-4}{6}+1, \tfrac{\eta m-4\eta-4}{6}+2, \dots\tfrac{\eta m}{2}\} \text{ and } S \geq 0.$$

Subcase: 2.6 $\eta \equiv 2(mod\ 6)$ and $m \equiv 1(mod\ 3), m \geq 5$

$\sigma(x_i)$ =0.3, for all $i \in \{1,2,3,\dots\eta\}$

$\sigma(y_j) = 0.3$, for all $j \in \{1,2,3,4,\dots\frac{\eta m-4\eta}{6}\}$

$$\sigma(y_j) = \begin{cases} 0.1 & if\ j = 2S \\ 0.2 & if\ j = 2S+1 \end{cases}, \text{ for all } j \in \{\tfrac{\eta m-4\eta}{6}+1, \tfrac{\eta m-4\eta}{6}+2, \dots\tfrac{\eta m}{2}\} \text{ and } S \geq 0.$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3197-3217
https://publishoa.com
ISSN: 1309-3452

Subcase: 2.7 $\eta \equiv 2(mod\ 6)$ and $m \equiv 2(mod\ 3), m \geq 5$

$\sigma(x_i) = 0.3$, for all $i \in \{1,2,3,\dots\eta\}$

$\sigma(y_j) = 0.3$, for all $j \in \{1,2,3,4,\dots\frac{\eta m-4\eta-2}{6}\}$

$\sigma(y_j) = \begin{cases} 0.1 & if\ j = 2S \\ 0.2 & if\ j = 2S+1 \end{cases}$, for all $j \in \{\frac{\eta m-4\eta-2}{6}+1, \frac{\eta m-4\eta-2}{6}+\ 2,\ \dots\frac{\eta m}{2}\}$ and $S \geq 0$.

Case 3: $\eta \equiv 4(mod\ 6)$

Subcase: 3.1 $m = 1$

Subcase: 3.1.1

If $\frac{\eta}{2}$ is even

$\sigma(x_i)$ is same as in Subcase 1.1 , for all $i \in \{1,2,3,\dots\eta\}$

$\sigma(y_j) = 0.2$, for all $j \in \{1,2,3,4,\dots\frac{\eta}{2}\}$

Subcase: 3.1.2

If $\frac{\eta}{2}$ is odd

$\sigma(x_i)$ is same as in Subcase 1.1.2, for all $i \in \{1,2,3,\dots\eta-1\}$ and $\sigma(x_\eta)$ =0.2

$\sigma(y_j) = 0.2$, for all $j \in \{1,2,3,4,\dots\frac{\eta}{2}-2\}$ and $\sigma\left(y_{\frac{\eta}{2}-1}\right) = 0.3$, $\sigma\left(y_{\frac{\eta}{2}}\right) = 0.2$

Subcase: 3.2 $m = 2$

Subcase: 3.2.1

If $\frac{\eta}{2}$ is even

$\sigma(x_i)$ is same as in Subcase 1.1 , for all $i \in \{1,2,3,\dots\eta\}$

$\sigma(y_{2j-1}) = 0.2$, for all $j \in \{1,2,3,4,\dots\frac{\eta}{2}\}$

$\sigma(y_{2j}) = \begin{cases} 0.1 & if\ i = 3S+1 \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S \end{cases}$, for all $j \in \{1,2,3,4,\dots\ \frac{\eta}{2}\}$ and $S \geq 0$.

Subcase: 3.2.2

If $\frac{\eta}{2}$ is odd

$\sigma(x_i)$ is same as in Subcase 1.1 , for all $i \in \{1,2,3,\dots\eta\}$

$\sigma(y_{2j-1}) = 0.2$, for all $j \in \{1,2,3,4,\dots\frac{\eta}{2}\}$

$\sigma(y_{2j}) = \begin{cases} 0.1 & if\ i = 3S+1 \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S \end{cases}$, for all $j \in \{1,2,3,4,\dots\ \frac{\eta}{2}\}$ and $S \geq 0$.

Subcase: 3.3 $m = 3$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3197-3217
https://publishoa.com
ISSN: 1309-3452

Subcase: 3.3.1

If $\frac{\eta}{2}$ is even

$\sigma(x_i)$ is same as in Subcase 1.1, for all $i \in \{1, 2, 3, \dots \eta\}$

$\sigma(y_{3j-2}) = 0.2$, for all $j \in \{1, 2, 3, 4, \dots \frac{\eta}{2}\}$ and

$$\sigma(y_{3j-1}) = \begin{cases} 0.1 & if\ i = 3S + 1 \\ 0.2 & if\ i = 3S + 2 \\ 0.3 & if\ i = 3S \end{cases}, \text{ for all } j \in \{1, 2, 3, 4, \dots \tfrac{\eta}{2}\} \text{ and } S \geq 0.$$

$$\sigma(y_{3j}) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S + 2 \\ 0.3 & if\ i = 3S + 1 \end{cases}, \text{ for all } j \in \{1, 2, 3, 4, \dots \tfrac{\eta}{2}\} \text{ and } S \geq 0.$$

Subcase: 3.3.2

If $\frac{\eta}{2}$ is odd

$\sigma(x_i)$ is same as in Subcase 1.1.2, for all $i \in \{1, 2, 3, \dots \eta - 1\}$ and $\sigma(x_\eta) = 0.1$

$\sigma(y_{3j-2}) = 0.2$, for all $j \in \{1, 2, 3, 4, \dots \frac{\eta}{2}\}$ and

$$\sigma(y_{3j-1}) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S + 2 \\ 0.3 & if\ i = 3S + 1 \end{cases}, \text{ for all } j \in \{1, 2, 3, 4, \dots \tfrac{\eta}{2}\} \text{ and } S \geq 0.$$

$$\sigma(y_{3j}) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S + 2 \\ 0.3 & if\ i = 3S + 1 \end{cases}, \text{ for all } j \in \{1, 2, 3, 4, \dots \tfrac{\eta}{2}\} \text{ and } S \geq 0.$$

Subcase: 3.3 $m = 4$

Subcase: 3.3.1

If $\frac{\eta}{2}$ is even

$\sigma(x_i)$ is same as in Subcase 1.1, for all $i \in \{1, 2, 3, \dots \eta\}$

$\sigma(y_{3j-2}) = 0.2$, for all $j \in \{1, 2, 3, 4, \dots \frac{\eta}{2}\}$ and

$$\sigma(y_{3j-1}) = \begin{cases} 0.1 & if\ i = 3S + 1 \\ 0.2 & if\ i = 3S + 2 \\ 0.3 & if\ i = 3S \end{cases}, \text{ for all } j \in \{1, 2, 3, 4, \dots \tfrac{\eta}{2}\} \text{ and } S \geq 0.$$

$$\sigma(y_{3j}) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S + 2 \\ 0.3 & if\ i = 3S + 1 \end{cases}, \text{ for all } j \in \{1, 2, 3, 4, \dots \tfrac{\eta}{2}\} \text{ and } S \geq 0.$$

Subcase: 3.3.2

If $\frac{\eta}{2}$ is odd

$\sigma(x_i)$ is same as in Subcase 1.1.2, for all $i \in \{1, 2, 3, \dots \eta - 1\}$ and $\sigma(x_\eta) = 0.1$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3197-3217
https://publishoa.com
ISSN: 1309-3452

$\sigma\left(y_{3j-2}\right) = 0.2$, for all $j \in \{1, 2, 3, 4, \ldots \frac{\eta}{2}\}$ and

$$\sigma\left(y_{3j-1}\right) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S+1 \end{cases}, \text{ for all } j \in \{1, 2, 3, 4, \ldots \tfrac{\eta}{2}\} \text{ and } S \geq 0.$$

$$\sigma\left(y_{3j}\right) = \begin{cases} 0.1 & if\ i = 3S \\ 0.2 & if\ i = 3S+2 \\ 0.3 & if\ i = 3S\ +1 \end{cases}, \text{ for all } j \in \{1, 2, 3, 4, \ldots \tfrac{\eta}{2}\} \text{ and } S \geq 0.$$

Subcase: 3.4 $m = 4$

$\sigma(x_i)$ =0.3, for all $i \in \{1, 2, 3, \ldots \eta\}$

$$\sigma\left(y_j\right) = \begin{cases} 0.1 & if\ j = 2S \\ 0.2 & if\ j = 2S+1 \end{cases}, \text{ for all } j \in \{1, 2, \ \ldots \tfrac{\eta m}{2}\} \text{ and } S \geq 0.$$

Subcase: 3.5 $\eta \equiv 4(mod\ 6)$ and $m \equiv 0(mod\ 3), m \geq 5$

$\sigma(x_i)$ = 0.3, for all $i \in \{1, 2, 3, \ldots \eta\}$

$\sigma\left(y_j\right) = 0.3$, for all $j \in \{1, 2, \ \ldots \frac{\eta m - 4\eta - 2}{6}\}$ and $S \geq 0$.

$$\sigma\left(y_j\right) = \begin{cases} 0.1 & if\ j = 2S \\ 0.2 & if\ j = 2S+1 \end{cases}, \text{ for all } j \in \{\tfrac{\eta m-4\eta-2}{6} + 1, \tfrac{\eta m-4\eta-2}{6} + \ 2, \ \ldots \tfrac{\eta m}{2}\} \text{ and } S \geq 0.$$

Subcase: 3.6 $\eta \equiv 4(mod\ 6)$ and $m \equiv 1(mod\ 3), m \geq 5$

$\sigma(x_i)$ = 0.3, for all $i \in \{1, 2, 3, \ldots \eta\}$

$\sigma\left(y_j\right) = 0.3$, for all $j \in \{1, 2, \ \ldots \frac{\eta m - 4\eta}{6}\}$ and $S \geq 0$.

$$\sigma\left(y_j\right) = \begin{cases} 0.1 & if\ j = 2S \\ 0.2 & if\ j = 2S+1 \end{cases}, \text{ for all } j \in \{\tfrac{\eta m-4\eta}{6} + 1, \tfrac{\eta m-4\eta}{6} + \ 2, \ \ldots \tfrac{\eta m}{2}\} \text{ and } S \geq 0.$$

Subcase: 3.7 $\eta \equiv 4(mod\ 6)$ and $m \equiv 2(mod\ 3), m \geq 5$

$\sigma(x_i)$ = 0.3, for all $i \in \{1, 2, 3, \ldots \eta\}$

$\sigma\left(y_j\right) = 0.3$, for all $j \in \{1, 2, \ \ldots \frac{\eta m - 4\eta - 4}{6}\}$ and $S \geq 0$.

$$\sigma\left(y_{m\,j}\right) = \begin{cases} 0.1 & if\ j = 2S \\ 0.2 & if\ j = 2S+1 \end{cases}, \text{ for all } j \in \{\tfrac{\eta m-4\eta-4}{6} + 1, \tfrac{\eta m-4\eta-4}{6} + 2, \ \ldots \tfrac{\eta m}{2}\} \text{ and } S \geq 0.$$

By the result of above assignment we could see that the elements of $E\left(C_{2\eta}[m]_A\right)$ receives the label $\iota \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$ and also for $\iota \neq h \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$, $|v_\sigma(\iota) - v_\sigma(h)| \leq 1$ and $\left|\varepsilon_\mu(\iota) - \varepsilon_\mu(h)\right| \leq 1$. Then by definition 2.7, $C_{2\eta}[m]_A$ is fuzzy quotient 3 cordial for all even $\eta \geq 2$ and $m \geq 1$

Theorem 3.5

The graph $C_\eta \odot K_{1,m}$ is fuzzy quotient 3 cordial for all $m \geq 2$

Proof: Let $V\left(C_\eta \odot K_{1,m}\right) = \{x_i : 1 \leq i \leq \eta\} \cup \left\{y_j : 1 \leq j \leq m\right\}$ and

$E\left(C_\eta \odot K_{1,m}\right) = \{x_i x_{i+1} : 1 \leq i \leq \eta - 1\} \cup \left\{x_\eta x_1\right\} \cup$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3197-3217
https://publishoa.com
ISSN: 1309-3452

$\{x_i y_j : 1 \leq i \leq \eta, 1+(i-1)m \leq j \leq im\}$.

For $C_\eta \odot K_{1,m}$, $p = \eta + \eta m = q$. Assigning labels to this graph involves,

For $m = 2$

$\sigma(x_i)$ = 0.3, for all $i \in \{1, 2, 3, \dots \eta\}$

$\sigma(y_j)$ = 0.1, for all $j \in \{1, 2, 3, \dots \frac{\eta m}{3}\}$

$\sigma\left(y_{\frac{\eta m}{3}+j}\right)$ = 0.2, for all $j \in \{1, 2, 3, \dots \frac{\eta m}{3}\}$

For $m \geq 2$

Case 1: $\eta \equiv 0 (mod\ 3)$

Subcase 1.1: $m \equiv 0 (mod\ 3)$

$\sigma(x_i)$ = 0.3, for all $i \in \{1, 2, 3, \dots \eta\}$

$\sigma(y_j)$ = 0.3, for all $j \in \{1, 2, 3, \dots \frac{\eta m-2\eta}{3}\}$

$\sigma\left(y_{\frac{\eta m-2\eta}{3}+j}\right)$ = 0.2, for all $j \in \{1, 2, 3, \dots \frac{\eta m+\eta}{3}\}$

$\sigma\left(y_{\frac{2\eta m-\eta}{3}+j}\right)$ = 0.1, for all $j \in \{1, 2, 3, \dots \frac{\eta m+\eta}{3}\}$

Subcase 1.2: $m \equiv 1, 2 (mod\ 3)$

$\sigma(x_i)$ and $\sigma(y_j)$ are same as in Subacase 1.1

Case 2: $\eta \equiv 1 (mod\ 3)$

Subcase 2.1: $m \equiv 0 (mod\ 3)$

$\sigma(x_i)$ = 0.3, for all $i \in \{1, 2, 3, \dots \eta\}$

$\sigma(y_j)$ = 0.3, for all $j \in \{1, 2, 3, \dots \frac{\eta m-2\eta-1}{3}\}$

$\sigma\left(y_{\frac{\eta m-2\eta-1}{3}+j}\right)$ = 0.2, for all $j \in \{1, 2, 3, \dots \frac{\eta m+\eta-1}{3}\}$

$\sigma\left(y_{\frac{2\eta m-\eta-2}{3}+j}\right)$ = 0.1, for all $j \in \{1, 2, 3, \dots \frac{\eta m+\eta+2}{3}\}$

Subcase 2.2: $m \equiv 1 (mod\ 3)$

$\sigma(x_i)$ = 0.3, for all $i \in \{1, 2, 3, \dots \eta\}$

$\sigma(y_j)$ = 0.3, for all $j \in \{1, 2, 3, \dots \frac{\eta m-2\eta-2}{3}\}$

$\sigma\left(y_{\frac{\eta m-2\eta-2}{3}+j}\right)$ = 0.2, for all $j \in \{1, 2, 3, \dots \frac{\eta m+\eta+1}{3}\}$

$\sigma\left(y_{\frac{2\eta m-\eta-1}{3}+j}\right)$ = 0.1, for all $j \in \{1, 2, 3, \dots \frac{\eta m+\eta+1}{3}\}$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3197-3217
https://publishoa.com
ISSN: 1309-3452

Subcase 2.3: $m \equiv 2(mod\ 3)$

$\sigma(x_i)$ and $\sigma(y_j)$ are same as in Subcase 1.1

Case 3: $\eta \equiv 2(mod\ 3)$

Subcase 3.1: $m \equiv 0(mod\ 3)$

$\sigma(x_i)$ and $\sigma(y_j)$ are same as in Subcase 2.2

Subcase 3.2: $m \equiv 1(mod\ 3)$

$\sigma(x_i)$ and $\sigma(y_j)$ are same as in Subcase 2.1

Subcase 3.3: $m \equiv 2(mod\ 3)$

$\sigma(x_i)$ and $\sigma(y_j)$ are same as in Subacase 1.1

By the result of above assignment we could see that the elements of $E(C_\eta \odot K_{1,m})$ receives the label $\iota \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$ and also for $\iota \neq h \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$, $|v_\sigma(\iota) - v_\sigma(h)| \leq 1$ and $|\varepsilon_\mu(\iota) - \varepsilon_\mu(h)| \leq 1$. Then by definition 2.7, $C_\eta \odot K_{1,m}$ is fuzzy quotient 3 cordial for all $m \geq 2$.

Theorem 3.6

The graph $C_\eta\ [a, d]$ is fuzzy quotient 3 cordial for all $a, d \geq 1$.

Proof: Let $V(C_\eta\ [a, d]) = \{x_i : 1 \leq i \leq \eta\} \cup \left\{y_j : 1 \leq j \leq \frac{\eta}{2}[2a + (\eta - 1)d]\right\}$ and

$E(C_\eta\ [a, d]) = \{x_i x_{i+1} : 1 \leq i \leq \eta - 1\} \cup \{x_\eta x_1\} \cup$

$\left\{x_i y_j : 1 \leq i \leq \eta, 1 + (i-1)a + \frac{(i-1)(i-2)d}{2} \leq j \leq ia + \frac{i(i-1)d}{2}\right\}$.

For $C_\eta\ [a, d]$, $p = \frac{\eta}{2}[2a + (\eta - 1)d + 2] = q$. Assigning labels to this graph involves,

Case 1: $\eta \equiv 0(mod\ 3)$

Subcase 1.1: $\eta = 3, a, d = 1$

$\sigma(x_i) = 0.3$, for all $i \in \{1, 2, 3, \ldots \eta\}$

$$\sigma(y_j) = \begin{cases} 0.1 & j \in \left\{1, 2, \ldots \frac{p}{3}\right\} \\ 0.2 & j \in \left\{\frac{p}{3} + 1, \frac{p}{3} + 2, \ldots \frac{2p}{3}\right\} \end{cases}$$

Subcase 1.2: $a \equiv 0, 1, 2(mod\ 3)$ and $d \equiv 0, 1, 2(mod\ 3)$

$\sigma(x_i) = 0.3$, for all $i \in \{1, 2, 3, \ldots \eta\}$

$$\sigma(y_j) = \begin{cases} 0.1 & j \in \left\{1, 2, \ldots \frac{p}{3}\right\} \\ 0.2 & j \in \left\{\frac{p}{3} + 1, \frac{p}{3} + 2, \ldots \frac{2p}{3}\right\} \\ 0.3 & j \in \left\{\frac{2p}{3} + 1, \frac{2p}{3} + 2, \ldots (p - \eta)\right\} \end{cases}$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3197-3217
https://publishoa.com
ISSN: 1309-3452

Case 2: $\eta \equiv 1(mod\ 3)$

Subcase 2.1: $\eta = 4, a, d = 1$

$\sigma(x_i) = 0.3$, for all $i \in \{1, 2, 3, \dots \eta\}$

$$\sigma(y_j) = \begin{cases} 0.1 & j \in \left\{1,2, \quad \dots \frac{p+1}{3}\right\} \\ 0.2 & j \in \left\{\frac{p+1}{3} + 1, \frac{p+1}{3} + 2, \dots \frac{2(p+1)}{3}\right\} \end{cases}$$

Subcase 2.2: $a \equiv 0(mod\ 3)$ and $d \equiv 0, 1, 2(mod\ 3)$

$\sigma(x_i) = 0.3$, for all $i \in \{1, 2, 3, \dots \eta\}$

$$\sigma(y_j) = \begin{cases} 0.1 & j \in \left\{1,2, \quad \dots \frac{p-1}{3}\right\} \\ 0.2 & j \in \left\{\frac{p-1}{3} + 1, \frac{p-1}{3} + 2, \quad \dots \frac{2(p-1)}{3}\right\} \\ 0.3 & j \in \left\{\frac{2(p-1)}{3} + 1, \frac{2(p-1)}{3} + 2, \quad \dots (p-\eta)\right\} \end{cases}$$

Subcase 2.3: $a \equiv 1(mod\ 3)$ and $d \equiv 0, 1, 2(mod\ 3)$

$\sigma(x_i) = 0.3$, for all $i \in \{1, 2, 3, \dots \eta\}$

$$\sigma(y_j) = \begin{cases} 0.1 & j \in \left\{1,2, \quad \dots \frac{p+1}{3}\right\} \\ 0.2 & j \in \left\{\frac{p+1}{3} + 1, \frac{p+1}{3} + 2, \quad \dots \frac{2(p+1)}{3}\right\} \\ 0.3 & j \in \left\{\frac{2(p+1)}{3} + 1, \frac{2(p+1)}{3} + 2, \quad \dots (p-\eta)\right\} \end{cases}$$

Subcase 2.4: $a \equiv 2(mod\ 3)$ and $d \equiv 0, 1, 2(mod\ 3)$

$\sigma(x_i) = 0.3$, for all $i \in \{1, 2, 3, \dots \eta\}$

$\sigma(x_i)$ and $\sigma(y_j)$ are same as in Subcase 1.2

Case 3: $\eta \equiv 2(mod\ 3)$

Subcase 3.1: $a \equiv 0(mod\ 3)$ and $d \equiv 1(mod\ 3)$

Or

$a \equiv 1(mod\ 3)$ and $d \equiv 2(mod\ 3)$

Or

$a \equiv 2(mod\ 3)$ and $d \equiv 0(mod\ 3)$

$\sigma(x_i)$ and $\sigma(y_j)$ are same as in Subcase 1.2

Subcase 3.2: $a \equiv 0(mod\ 3)$ and $d \equiv 2(mod\ 3)$

Or

$a \equiv 1(mod\ 3)$ and $d \equiv 0(mod\ 3)$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3197-3217
https://publishoa.com
ISSN: 1309-3452

Or

$\boldsymbol{a \equiv 2(mod\ 3)}$ **and** $\boldsymbol{d \equiv 1(mod\ 3)}$

$\sigma(x_i)$ and $\sigma(y_j)$ are same as in Subcase 2.2

Subcase 3.3: $\boldsymbol{a \equiv 0(mod\ 3)}$ **and** $\boldsymbol{d \equiv 0(mod\ 3)}$

Or

$\boldsymbol{a \equiv 1(mod\ 3)}$ **and** $\boldsymbol{d \equiv 1(mod\ 3)}$

Or

$\boldsymbol{a \equiv 2(mod\ 3)}$ **and** $\boldsymbol{d \equiv 2(mod\ 3)}$

$\sigma(x_i)$ and $\sigma(y_j)$ are same as in Subcase 2.3

By the result of above assignment we could see that the elements of $E(C_\eta\ [a,d]\ )$ receives the label $\iota \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$ and also for $\iota \neq h\ \in\ \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$, $|v_\sigma(\iota) - v_\sigma(h)| \leq 1$ and $|\varepsilon_\mu(\iota) - \varepsilon_\mu(h)| \leq 1$. Then by definition 2.7, $C_\eta\ [a,d]$ is fuzzy quotient 3 cordial for all $a,d\ \geq 1$.

Theorem 3.7

The graph $C_\eta\ [a,r]$ is fuzzy quotient 3 cordial for all $a,r\ \geq 1$.

Proof: Let $V(C_\eta\ [a,r]\ ) = \{x_i : 1 \leq i\ \leq\ \eta\} \cup \left\{y_j : 1 \leq j\ \leq \frac{a(r^\eta - 1)}{r-1}\right\}$ and

$E(C_\eta\ [a,r]) = \{x_i x_{i+1} : 1 \leq i\ \leq\ \eta - 1\} \cup \{x_\eta x_1\} \cup$

$\left\{x_i y_j : 1 \leq i\ \leq\ \eta, 1 + \frac{a(r^{i-1}-1)}{r-1} \leq j\ \leq \frac{a(r^i - 1)}{r-1}\right\}$.

For $C_\eta\ [a,r]$, $p =\ \eta + \frac{a(r^\eta - 1)}{r-1} = q$. Assigning labels to this graph involves,

Case 1: $\boldsymbol{\eta \equiv 0(mod\ 3)}$

Subcase 1.1: $\boldsymbol{a \equiv 0(mod\ 3)}$ **and** $\boldsymbol{r \equiv 0,1,2(mod\ 3)}$

$\sigma(x_i)$ = 0.3, for all $i \in \{1,2,3,\dots\eta\}$

$$\sigma(y_j) = \begin{cases} 0.1 & j \in \left\{1,2, \quad \dots\frac{p}{3}\right\} \\ 0.2 & j \in \left\{\frac{p}{3}+1, \frac{p}{3}+\ 2, \quad \dots\frac{2p}{3}\right\} \\ 0.3 & j \in \left\{\frac{2p}{3}+1, \frac{2p}{3}+\ 2, \quad \dots(p-\eta)\right\} \end{cases}$$

Subcase 1.2: $\boldsymbol{a \equiv 1,2(mod\ 3)}$ **and** $\boldsymbol{r \equiv 1(mod\ 3)}$

$\sigma(x_i)$ and $\sigma(y_j)$ are same as in Subcase 1.1

Subcase 1.3: $\boldsymbol{a \equiv 1(mod\ 3)}$ **and** $\boldsymbol{r \equiv 0,2(mod\ 3)}$

$\sigma(x_i)$ = 0.3, for all $i \in \{1,2,3,\dots\eta\}$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3197-3217
https://publishoa.com
ISSN: 1309-3452

$$\sigma(y_j) = \begin{cases} 0.1 & j \in \left\{1,2, \quad \dots \frac{p-1}{3}\right\} \\ 0.2 & j \in \left\{\frac{p-1}{3}+1, \frac{p-1}{3}+2, \quad \dots \frac{2(p-1)}{3}\right\} \\ 0.3 & j \in \left\{\frac{2(p+1)}{3}+1, \frac{2(p+1)}{3}+2, \quad \dots (p-\eta)\right\} \end{cases}$$

Case 2: $\eta \equiv 1(mod\ 3)$

Subcase 2.1: $a \equiv 2(mod\ 3)$ and $r \equiv 0,1(mod\ 3)$

$\sigma(x_i)$ and $\sigma(y_j)$ are same as in Subcase 1.1

Subcase 2.2: $a \equiv 0(mod\ 3)$ and $r \equiv 0,1,2(mod\ 3)$

$\sigma(x_i)$ and $\sigma(y_j)$ are same as in Subcase 1.3

Subcase 2.3: $a \equiv 1(mod\ 3)$ and $r \equiv 0,1(mod\ 3)$

$\sigma(x_i)$ and $\sigma(y_j)$ are same as in Subcase 1.4

Case 3: $\eta \equiv 2(mod\ 3)$

Subcase 3.1: $a \equiv 1(mod\ 3)$ and $r \equiv 0,2(mod\ 3)$

Or

$a \equiv 2(mod\ 3)$ and $r \equiv 1(mod\ 3)$

$\sigma(x_i)$ and $\sigma(y_j)$ are same as in Subcase 1.1

Subcase 3.2: $a \equiv 1(mod\ 3)$ and $r \equiv 1(mod\ 3)$

Or

$a \equiv 2(mod\ 3)$ and $r \equiv 0,2(mod\ 3)$

$\sigma(x_i)$ and $\sigma(y_j)$ are same as in Subcase 1.3

Subcase 3.3: $a \equiv 2(mod\ 3)$ and $r \equiv 0,1,2(mod\ 3)$

$\sigma(x_i)$ and $\sigma(y_j)$ are same as in Subcase 1.4

By the result of above assignment we could see that the elements of $E(C_\eta\ [a,r]\ )$ receives the label $\iota \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$ and also for $\iota \neq h \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$, $|v_\sigma(\iota) - v_\sigma(h)| \leq 1$ and $|\varepsilon_\mu(\iota) - \varepsilon_\mu(h)| \leq 1$. Then by definition 2.7, $C_\eta\ [a,r]$ is fuzzy quotient 3 cordial for all $a, r\ \geq 1$.

4. CONCLUSION

The presence of fuzzy quotient 3 labelling on some subdivision graphs is discussed and established in this study. Our next step will be to investigate this concept in various graph families and identify applications for it.

References

1. Gallian, J. A. (2014). A dynamic survey of graph labeling. *The Electronic journal of combinatorics,17*, 60-62.

2. Graham, R. L., and Sloane, N. J. A. (1980). On additive bases and harmonious graphs. SIAM Journal on Algebraic Discrete Methods, 1(4), 382-404.
3. Rosa, A. (1966, July). On certain valuations of the vertices of a graph. In Theory of Graphs (Internat. Symposium, Rome (pp. 349-355).
4. Sumathi, P., and J. Suresh Kumar. (2019). Fuzzy quotient-3 cordial labeling of star related graphs-Paper I. *Malaya Journal of Matematik*, 79-82.
5. Sumathi, P., & Suresh Kumar, J. (2021). Fuzzy Quotient-3 Cordial Labeling of Some Trees of Diameter 5. In Proceedings of First International Conference on Mathematical Modeling and Computational Science (pp. 275-289). Springer, Singapore.