

Performance Efficiency of Tamil Nadu State Universities using Fuzzy DEA

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Abstract:

This study aims to evaluate the efficiency of the Tamil Nadu state government universities using D.E.A and Fuzzy - D.E.A approach. The study includes 16 Tamil Nadu state universities considering each university as decision making units. Each D.M.U includes 2 inputs namely number of students enrolled and the number of faculty and 3 outputs namely number of students graduated, number of students placed and enrolled for higher studies. The required data was collected from the universities official websites. The efficiency scores of each university have been calculated by using both classical DEA and fuzzy DEA. Further Comparison between these two methods has been performed.

AMS Subject Classification: 90C08; 93C42

Key words and Phrases: DEA, Fuzzy DEA, CCR, Efficiency, Universities.

1. Introduction:

Data Envelopment Analysis is widely used to evaluate the efficiency of the decision making units based on the given multiple crisp inputs and multiple crisp outputs. But one cannot get the crisp data in real life problem as are vagueness and fuzziness in data. In that situation Fuzzy DEA can be used to find the efficiency of the D.M.U'S where the inputs and outputs are fuzzy in nature. Sengupta (2022)^[14] was the first researcher who has applied the fuzzy set concept in D.E.A. The fuzzy D.E.A. methods can be classified under 6 categories and they are as follows:

- i. Tolerance Approach
- ii. Alpha – level based approach
- iii. Fuzzy Ranking Approach
- iv. Possibility Approach
- v. Fuzzy Arithmetic Approach
- vi. The Fuzzy Random/Type 2 Fuzzy Set.

There are many research papers done by applying the above concepts in literature. Since the review indicates that the alpha – level based approach is most familiar and applied by many researchers, the researcher used the alpha level based approach (which was proposed by Saati et.al. (2002)^[13]) in this paper to find the performance efficiency of the Tamil Nadu state government universities where the inputs and outputs are fuzzy in nature.

1.1. Objective of the Study:

The objective of this research study is

- i. To find the performance efficiency of Tamilnadu state government universities using data envelopment analysis and fuzzy data envelopment analysis.
- ii. To compare the efficiency scores of both the D.E.A and F.D.E.A.

2. Review of Literature:

Ana Lucia Miranda Lopes and Edgar Augusto Lanzer (2002)^[2] have made a study on the topic “ Data Envelopment Analysis – DEA and Fuzzy Sets to Assess the performance of Academic departments: A case study at federal university of santacatarina - UFSC”. In this study, the researchers analyze the performance efficiency of 58 academic departments of Federal university, Santa Catarina. The empirical results concluded that 15 departments are falls under low performance category. And also the researchers found that there is no relationship between the teaching level of departments, Research activity and Service quality. But there is a weak correlation between the research productivity index and quality index.

PredragMimovic and Ana Krstic (2016)^[12] have constructed a novel method called hybrid DEAHP to assess the performance efficiency of DMU's. In this paper, the researchers assessed the efficiency of 12 Higher education institutions of Republic of Serbia by using D.E.A.H.P. method (Data Envelopment Analytical Hierarchy Process). The empirical results concluded that, only 1 DMUs is efficient under AHP model, 4 DMUs are performed well under the classical CRS DEA model and 7 DMUs performed efficiently under the proposed model (DEAHP).

UmiMahmudah and MuhamadSafia Lola (2016)^[15] studied the efficiency of 25 Indonesian universities by using the conventional and Fuzzy D.E.A. The researchers found that out of 25 universities,9 (36%)_universities are efficient under fuzzy CRS model whereas 13(52%) Indonesian universities are performed efficiently under Fuzzy VRS model.

M.Maragatham et al (2021)^[10] have made a study to assess the efficiency of 27 affiliated colleges under Bharathidasan University using fuzzy data envelopment analysis. The authors used the fuzzy inference system to assess the performance efficiency. The researchers found that , only 3 colleges performed efficiently under CRS model whereas under VRS model 9 colleges are performed efficiently.

3. Research Design and Methodology:

3.1. Data:

. The required data has been collected from the universities official websites over the period of time 2016- 2020. The Number of students admitted, number of full time faculty of 16 universities were considered as inputs in this research paper. The number of students graduated, number of students placed and number of students opt higher studies are considered as output for this research study.

3.2. Methodology:

The Data Envelopment analysis (DEA) and the fuzzy DEA has been used to find and compare the performance efficiency of 16 Tamil Nadu State universities over the period of time 2016-17 – 2019-2020.

3.2.1. Data Envelopment Analysis:

D.E.A is a non-parameter approach which is used to find the efficiency of decision-making units (DMU). In this paper, the author uses the CCR and BCC multiplier DEA model (Output Maximization) as the output (No. of students graduated, placed and opt for higher studies) has to be maximized by keeping the inputs at the same level which was developed by Charnes et.al^[4] and presented below.

The CCR- Multiplier DEA model is,

$$\begin{aligned}
 \text{Max}\theta_k &= \sum_{r=1}^p u_r \cdot y_{rk} \\
 \text{subject to} \\
 \sum_{i=1}^m v_i \cdot x_{ij} &= 1 \\
 \sum_{r=1}^p u_r \cdot y_{rk} - \sum_{i=1}^m v_i \cdot x_{ij} &\leq 0; j = 1 \dots n \\
 u_r, v_i &\geq \varepsilon, r = 1 \dots p, i = 1 \dots m
 \end{aligned} \tag{1}$$

Where θ_k is the efficiency measure.

$Y = [Y_1, Y_2, \dots, Y_N]$ is the vector of all outputs.

$X = [X_1, X_2, \dots, X_N]$ is the vector of all inputs.

The BCC – Multiplier DEA^[3] model is,

$$\begin{aligned}
 \text{Max}\theta_k &= \sum_{r=1}^p u_r \cdot y_{rk} + u_0 \\
 \text{subject to} \\
 \sum_{i=1}^m v_i \cdot x_{ij} &= 1 \\
 \sum_{r=1}^p u_r \cdot y_{rk} - \sum_{i=1}^m v_i \cdot x_{ij} + u_0 &\leq 0; j = 1 \dots n \\
 u_r, v_i &\geq \varepsilon, r = 1 \dots p, i = 1 \dots m
 \end{aligned} \tag{2}$$

By solving the above CCR and BCC model, one can get the efficiency score (θ_k value) for each DMU. A DMU is said to be efficient if $\theta_k = 1$ and there is no slack. A D.M.U is said to be inefficient if $\theta_k < 1$.

3.2.2. FUZZY DATA ENVELOPMENT ANALYSIS:

Fuzzy D.E.A has been used to find the efficiency of decision making units when the inputs and outputs are fuzzy in nature. That is the fuzzy DEA will be used when there is fuzziness or ambiguity occurs in data.

In this paper, the author used the alpha level approach to find the efficiency of decision making units having fuzzy inputs (triangular) and fuzzy outputs (triangular). The Mathematical model for Fuzzy DEA which is proposed by Saati.et.al.^[13] is presented below.

The model (1) and (2) can be written as

$$\begin{aligned} \text{Max} \theta_k &= \sum_{r=1}^p u_r \cdot \tilde{y}_{rk} & \text{Max} \theta_k &= \sum_{r=1}^p u_r \cdot \tilde{y}_{rk} + u_0 \\ \text{subject to} & & \text{subject to} & \\ \sum_{i=1}^m v_i \cdot \tilde{x}_{ij} &= 1 & \sum_{i=1}^m v_i \cdot \tilde{x}_{ij} &= 1 \\ \sum_{r=1}^p u_r \cdot \tilde{y}_{rk} - \sum_{i=1}^m v_i \cdot \tilde{x}_{ij} &\leq 0; j = 1 \dots n & \sum_{r=1}^p u_r \cdot \tilde{y}_{rk} - \sum_{i=1}^m v_i \cdot \tilde{x}_{ij} + u_0 &\leq 0; j = 1 \dots n \\ u_r, v_i &\geq \varepsilon, r = 1..p, i = 1 \dots m & u_r, v_i &\geq \varepsilon, r = 1..p, i = 1 \dots m \end{aligned}$$

Where $\tilde{y}_{rk} = (y_{rk}^L, y_{rk}^M, y_{rk}^U)$ and $\tilde{x}_{ij} = (x_{ij}^L, x_{ij}^M, x_{ij}^U)$ are the triangular fuzzy numbers and the lower bound and upper bound of alpha cuts can be found by using the method which is proposed by Saati. et.al.^[13] and is

$$\begin{aligned} [\tilde{x}_{ij}]_{\alpha} &= (\alpha \cdot x_{ij}^M + (1-\alpha) \cdot x_{ij}^L, \alpha \cdot x_{ij}^M + (1-\alpha) \cdot x_{ij}^U) \\ [\tilde{y}_{rk}]_{\alpha} &= (\alpha \cdot y_{rk}^M + (1-\alpha) \cdot y_{rk}^L, \alpha \cdot y_{rk}^M + (1-\alpha) \cdot y_{rk}^U) \end{aligned}$$

And the fuzzy efficiency model under alpha cuts is

$$\begin{aligned} (\theta_k^L)_{\alpha} &= \text{Max.} \sum_{r=1}^p u_r \cdot y_{rk}^L & (\theta_k^U)_{\alpha} &= \text{Max.} \sum_{r=1}^p u_r \cdot y_{rk}^U \\ \text{subject to} & & \text{subject to} & \\ \sum_{i=1}^m v_i \cdot x_{ij}^U &= 1 & \sum_{i=1}^m v_i \cdot x_{ij}^L &= 1 \\ \sum_{r=1}^p u_r \cdot y_{rk}^L - \sum_{i=1}^m v_i \cdot x_{ij}^U &\leq 0; j = 1 \dots n & \sum_{r=1}^p u_r \cdot y_{rk}^U - \sum_{i=1}^m v_i \cdot x_{ij}^L &\leq 0; j = 1 \dots n \\ u_r, v_i &\geq \varepsilon, r = 1..p, i = 1 \dots m & u_r, v_i &\geq \varepsilon, r = 1..p, i = 1 \dots m \end{aligned}$$

By solving the above two models, we can get the interval efficiency score for different level of alpha cuts.

4. Analysis and Interpretation:

Descriptive measures and D.E.A analysis has been performed to assess the characteristic and efficiency of the 16 Tamil Nadu state universities and presented in this section.

4.1. Descriptive Analysis:

Descriptive measures help us to summarize the data's characteristics. The following table presents the descriptive measures of the collected inputs and outputs of the universities.

Table 4.1. Descriptive Statistics

Measures	No. of Students Graduated _Output 1	No. of Students Placed _ Output 2	Higher Studies_ Output 3	No. of Students Admitted_ Input 1	No.of Teaching Faculty
Min.	94	18	0	8	40
Max.	4628	1717	1140	4896	1437
Mean	1493.25	518.125	233.25	1886	451.3125
S.D.	1131.148	422.1251	1436.326	1393.976	442.6535

It is observed from the above table that on an average, 79% of the university students graduated every year with S.D. of 1131 and 27% of the students got placement with S.D. of 422 and only 12% of the university students opted higher studies with S.D. of 1436.

4.2. Conventional DEA efficiency Score:

The following table shows the Conventional - CCR and VRS Efficiency Score of T.N state government universities.

Table 4.2.**CCR and VRS Efficiency Score**

DMU	CCR (θ_k)	VRS (θ_k)
Alagappa University	.988	1.000
Anna University,	0.953	1.000
Annamalai University,	0.136	0.2
Bharathiar University	1.000	1.000
Bharathidasan University	0.993	1.000
Madurai Kamaraj University	0.598	0.683
Mother Teresa Women's University	0.981	1.000
Periyar University	0.929	0.949
Tamilnadu Agricultural University	0.803	.906
Tamilnadu Dr. Ambedkar Law University	0.864	.865
Tamilnadu Dr. M.G.R. Medical University	0.777	.954
Tamilnadu Fisheries University	0.720	1.000
Tamilnadu Veterinary & Animal Sciences University	1.000	1.000
Thiruvalluvar University	1.000	1.000
University of Madras	0.953	.954
Manonmaniam Sundarnar University	1.000	1.000
Average	.856	.907

The above table reveals that out of 16 D.M.U's, 4 D.M.U's were efficient and the remaining 12 D.M.U's are inefficient under Conventional C.C.R Model. It is also observed that 9 universities were efficient under conventional V.R.S model. It is concluded that 4 universities are performed well under both conventional C.C.R and Conventional V.R.S Model which included Bharathiar University, Tamil Nadu Veterinary & Animal Sciences University, Thiruvalluvar University and Manonmaniam Sundarnar University.

4.3. Fuzzy DEA Score:

The following table (4.3.) presents the alpha cut (.25 and .75) lower and upper bound efficiency score of 16 D.M.U's under fuzzy CCR model which is calculated based on fuzzy inputs (triangular) and fuzzy outputs (triangular) for different level of alpha.

Table 4.3.

CCR - Fuzzy Efficiency Score

DMU	$(\theta_k)_{.25}$		$(\theta_k)_{.75}$	
	$(\theta_k^L)_{.25}$	$(\theta_k^U)_{.25}$	$(\theta_k^L)_{.75}$	$(\theta_k^U)_{.75}$
Alagappa University	0.933	0.912	0.987	0.964
Anna University,	0.994	0.764	0.969	0.901
Annamalai University,	0.139	0.106	0.148	0.137
Bharathiar University	1	1	1	1
Bharathidasan University	0.991	0.859	0.996	0.945
Madurai Kamraj University	0.757	0.554	0.59	0.507
Mother Teresa Women's University	0.94	0.964	1	1
Periyar University	0.843	1	0.931	0.983
Tamilnadu Agricultural University	0.938	0.664	0.805	0.742
Tamilnadu Dr. Ambedkar Law University	0.623	1	0.816	0.884
Tamilnadu Dr. M.G.R. Medical University	0.318	1	0.659	0.932
Tamilnadu Fisheries University	0.481	0.688	0.633	0.698
Tamilnadu Veterinary & Animal Sciences University	1	1	1	1
Thiruvalluvar University	0.889	1	1	1
University of Madras	0.837	1	0.902	0.979
ManonmaniamSundarnar University	1	1	1	1

The above table reveals that the fuzzy efficiency scores of the 16 D.M.U's and it shows that out of 16, 3 universities were efficient at lower bound level at $\alpha=.25$ and 8 universities are efficient at the upper bound level at $\alpha=.25$. At $\alpha = .75$, 5 universities were efficient at both the upper and lower bound level.

It is observed that 3 universities performed efficiently at both the different level of α and they are Bharathiar University, Tamilnadu Veterinary & Animal Sciences University and Manonmaniam Sundaranar University and which are also efficient under conventional DEA model.

5. Summary and Conclusion:

In this study, 16 TN state universities were considered to find the performance efficiency using D.E.A and Fuzzy D.E.A. The period considered for this study was 2016-2020. The Conventional CCR and VRS – DEA model and Fuzzy CCR DEA model was used to assess the performance efficiency of 16 universities. The empirical investigation results were found to be that out of 16 universities, 4 states were efficient under conventional – CCR-Model and 9 were efficient under Conventional VRS model. Under Fuzzy – CCR Model, 3 universities performed efficiently at both level of α (.25 and .75) which includes Bharathiar University, Tamilnadu Veterinary & Animal Sciences University and Manonmaniam Sundaranar University and they were efficient even under conventional DEA model. Henceforth, it is concluded that the Fuzzy D.E.A. model helps the decision maker to know the performance efficiency of the D.M.U's at different levels of α .

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