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Quotient-4 Cordial Labeling of Some Ladder Graphs

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Abstract

Let G (V, E) be a simple graph with p vertices and q edges. Let φ: V (G) → Z_5 – {0} be a function and φ^*: E(G) → Z_4 by $\varphi^*(uv)=\left\lceil\frac{\varphi(u)}{\varphi(v)}\right\rceil$(modulo 4) where $\varphi(u) \geq \varphi(v)$. If $|v_\varphi(i) - v_\varphi(j)| \leq 1$, $1 \leq i, j \leq 4$, i≠j and $|e_\varphi(k) - e_\varphi(l)| \leq 1$, $0 \leq k, l \leq 3, k \neq l$ then φ is Quotient-4 cordial labeling of G, where $v_\varphi(x)$ and $e_\varphi(y)$ denote the number of vertices labeled with x and the number of edges labeled with y. Here some types of ladder graphs are proved to be quotient-4 cordial graphs.

Keywords: Mobius Ladder, Circular Ladder Pentagonal Ladder, Pentagonal circular Ladder and Hexagonal Ladder.

MSC Classification Code: 05C78

1. INTRODUCTION

In graph theory, graph labelling has made tremendous development and has a large number of applications. More information can be found in Gallian [4]. The cordial labelling concept was first presented by Cohit. [2]. Freeda Selvanayagam, Robinson, and ChellathuraiR.S pioneered H- and H2-cordial labelling. [3]. The Mean Cordial was developed by Albert William, Indira Rajasingh, and S Roy [1]. We developed quotient-4 cordial labeling and shown that several non-trivial, finite, undirected, and simple graphs can be labeled as quotient-4 cordial [5]. Open Ladder, Closed Ladder, Slanting Ladder, Mobius Ladder, Circular Ladder, Pentagonal Ladder, Pentagonal circular Ladder and Hexagonal Ladder are proved to be quotient-4 cordial graphs in this paper.

2. DEFINITIONS

Definition: 2.1 Let G (V, E) be a p vertices and q edges simple graph. Let φ: V(G) → Z_5 – {0} be a function. For each edge in E (G), the function φ^*: E (G) → Z_4 defined by $\varphi^*(uv) = \left\lceil\frac{\varphi(u)}{\varphi(v)}\right\rceil$ (modulo 4) where $\varphi(u) \geq \varphi(v)$. If $|v_\varphi(i) - v_\varphi(j)| \leq 1, 1 \leq i, j \leq 4$, i≠j and $|e_\varphi(k) - e_\varphi(l)| \leq 1$, $0 \leq k, l \leq 3$, $k \neq l$, then φ is Quotient-4 cordial labeling of G, where $v_\varphi(x)$ and $e_\varphi(y)$ signify the number of vertices and edges respectively labeled with x and y.

Definition: 2.2[6] An open Ladder graph OL (β), $\beta \geq 3$ is obtained from two copies of path $P_{\beta-1}$ with the vertex set V (OL (β)) = $\{u_\theta : 1 \leq \theta \leq \beta\} \cup \{v_\theta : 1 \leq \theta \leq \beta\}$ and E(OL (β)) = $\{u_\theta u_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{v_\theta v_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{u_\theta v_\theta : 1 < \theta < \beta\}$.

Definition: 2.3[6] A closed Ladder graph CL (β), $\beta \geq 3$ is obtained by combining two copies of path $P_{\beta-1}$ with the vertex set V (CL (β)) = $\{u_\theta : 1 \leq \theta \leq \beta\} \cup \{v_\theta : 1 \leq \theta \leq \beta\}$ and E (CL (β)) = $\{u_\theta u_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{v_\theta v_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{u_\theta v_\theta : 1 \leq \theta \leq \beta\}$.

JOURNAL OF ALGEBRAIC STATISTICS
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Definition: 2.4[6] A slanting Ladder graph SL (β), $\beta \geq 3$ is obtained by combining two copies of path $P_{\beta-1}$ with the vertex set V (SL (β) $= \{u_\theta : 1 \leq \theta \leq \beta\} \cup \{v_\theta : 1 \leq \theta \leq \beta\}$ and E (SL (β)) = $\{u_\theta u_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{v_\theta v_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{u_\theta v_{\theta+1} : 1 \leq \theta \leq \beta - 1\}$.

Definition: 2.5[6] A Mobius Ladder graph ML (β), $\beta \geq 3$ is obtained by combining two copies path $P_{\beta-1}$ with the vertex set V (ML (β)) $= \{u_\theta : 1 \leq \theta \leq \beta\} \cup \{v_\theta : 1 \leq \theta \leq \beta\}$ and E (ML (β)) = $\{u_\theta u_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{v_\theta v_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{u_\theta v_\theta : 1 \leq \theta \leq \beta\} \cup \{u_1 v_\beta\} \cup \{v_1 u_\beta\}$.

Definition: 2.6[6] A Circular Ladder graph CRL (β), $\beta \geq 3$ is obtained by combining two copies of path $P_{\beta-1}$ with the vertex set V (CRL (β)) $= \{u_\theta : 1 \leq \theta \leq \beta\} \cup \{v_\theta : 1 \leq \theta \leq \beta\}$ and E (CRL (β)) = $\{u_\theta u_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{v_\theta v_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{u_\theta v_\theta : 1 \leq \theta \leq \beta\} \cup \{u_1 u_\beta\} \cup \{v_1 v_\beta\}$.

Definition: 2.7 Consider the Closed Ladder graph CL (β), $\beta \geq 2$, introducing a vertex w_θ between the vertices v_θ and $v_{\theta+1}$, for $1 \leq \theta \leq \beta - 1$ resulting a new graph called pentagonal ladder graph denoted by PL(β).

Definition: 2.8 Consider the Pentagonal Ladder graph PL (β), $\beta \geq 3$, by connecting the vertices v_1 and v_β by a new vertex w_β and connecting u_1 and $u_\beta$by an edge resulting a new graph called pentagonal circular ladder graph denoted by PCL (β).

Definition: 2.9 Consider the Closed Ladder graph CL (β), $\beta \geq 2$, by adding a new vertices t_θ between the vertices u_θ and $u_{\theta+1}$, for $1 \leq \theta \leq \beta - 1$ and w_θ between the vertices v_θ and $v_{\theta+1}$, for $1 \leq \theta \leq \beta - 1$ resulting a new graph called hexagonal ladder graph denoted by HL (β).

3. MAIN RESULT

Theorem: 3.1. Any open ladder OL(β) is quotient-4 cordial if $\beta \geq 3$.

Proof: Consider G is an open ladder graph OL (β). Let V (G) = $\{u_\theta : 1 \leq \theta \leq \beta\} \cup \{v_\theta : 1 \leq \theta \leq \beta\}$ and E (G) = $\{u_\theta u_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{v_\theta v_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{u_\theta v_\theta : 1 < \theta < \beta\}$. Here $|V(G)| = 2\beta$, $|E(G)| = 3\beta - 4$.

Define φ: V (G) → {1, 2, 3, 4}.

The values of u_θ are labeled the following.

Case (i): When $\beta \equiv 0, 1, 6$ (modulo 8).

For $1 \leq \theta \leq \beta$.

φ (u_θ) = 1 if $\theta \equiv 2, 4$ (modulo 8).

φ (u_θ) = 2 if $\theta \equiv 6, 7$ (modulo 8).

φ (u_θ) = 4 if $\theta \equiv 0, 1, 3, 5$ (modulo 8).

Case (ii): When $\beta \equiv 2, 3, 5$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of u_θ values are same as case (i).

φ (u_β) = 2.

Case (iii): When $\beta \equiv 4, 7$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of u_θ values are same as case (i).

φ (u_β) = 3.

The values of v_θ are labeled the following.

JOURNAL OF ALGEBRAIC STATISTICS
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Case (i): When $\beta \equiv 0, 1, 2$ (modulo 8).

For $1 \leq \theta \leq \beta$.

φ (v_θ) = 1 if $\theta \equiv 1,3$ (modulo 8).

φ (v_θ) = 2 if $\theta \equiv 6, 7$ (modulo 8).

φ (v_θ) = 3 if $\theta \equiv 0, 2, 4, 5$ (modulo 8).

Case (ii): When $\beta \equiv 3$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of v_θ values are same as case (i).

φ (v_β) = 3.

Case (iii): When $\beta \equiv 4$ (modulo 8).

For $1 \leq \theta \leq \beta - 5$, the labeling of v_θ values are same as case (i).

φ (v_β) = φ ($v_{\beta-2}$) = 3, φ ($v_{\beta-1}$) = 1, φ ($v_{\beta-3}$) = φ ($v_{\beta-4}$) = 2.

Case (iv): When $\beta \equiv 5$ (modulo 8).

For $1 \leq \theta \leq \beta - 3$, the labeling of v_θ values are same as case (i).

φ (v_β) = 2, φ ($v_{\beta-1}$) = φ ($v_{\beta-2}$) = 3.

Case (v): When $\beta \equiv 6$(modulo 8).

For $1 \leq \theta \leq \beta - 4$, the labeling of v_θ values are same as case (i).

φ (v_β) = φ ($v_{\beta-1}$) = 2, φ ($v_{\beta-2}$) = φ ($v_{\beta-3}$) = 3.

Case (vi): When $\beta \equiv 7$(modulo 8).

For $1 \leq \theta \leq \beta - 5$, the labeling of v_θ values are same as case (i).

φ (v_β) = φ ($v_{\beta-1}$) = 2, φ ($v_{\beta-2}$)= 1, φ ($v_{\beta-3}$) = φ ($v_{\beta-4}$) = 3

Table 1

Nature of β	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$\beta \equiv 0, 2, 4, 6$ (modulo 8)	$\frac{\beta}{2}$	$\frac{\beta}{2}$	$\frac{\beta}{2}$	$\frac{\beta}{2}$
$\beta \equiv 1$(modulo 8)	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2} - 1$	$\frac{\beta+1}{2} - 1$	$\frac{\beta+1}{2}$
$\beta \equiv 3, 5, 7$ (modulo 8)	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2} - 1$	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2} - 1$

Table 2

Nature of β	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
https://publishoa.com
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$\beta \equiv 0,4$(modulo 8)	$\frac{3\beta-4}{4}$	$\frac{3\beta-4}{4}$	$\frac{3\beta-4}{4}$	$\frac{3\beta-4}{4}$
$\beta \equiv 1$(modulo 8)	$\frac{3\beta-3}{4}-1$	$\frac{3\beta-3}{4}$	$\frac{3\beta-3}{4}$	$\frac{3\beta-3}{4}$
$\beta \equiv 2$ (modulo 8)	$\frac{3\beta-2}{4}-1$	$\frac{3\beta-2}{4}-1$	$\frac{3\beta-2}{4}$	$\frac{3\beta-2}{4}$
$\beta \equiv 3$ (modulo 8)	$\frac{3\beta-1}{4}-1$	$\frac{3\beta-1}{4}-1$	$\frac{3\beta-1}{4}-1$	$\frac{3\beta-1}{4}$
$\beta \equiv 5$ (modulo 8)	$\frac{3\beta-3}{4}$	$\frac{3\beta-3}{4}-1$	$\frac{3\beta-3}{4}$	$\frac{3\beta-3}{4}$
$\beta \equiv 6$ (modulo 8)	$\frac{3\beta-2}{4}$	$\frac{3\beta-2}{4}-1$	$\frac{3\beta-2}{4}$	$\frac{3\beta-2}{4}-1$
$\beta \equiv 7$ (modulo 8)	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}-1$	$\frac{3\beta-1}{4}-1$	$\frac{3\beta-1}{4}-1$

The above tables 1 and 2 shows that $|v_\varphi(i) - v_\varphi(j)| \leq 1$ and $|e_\varphi(k) - e_\varphi(l)| \leq 1$. Hence the open ladder graph OL (β) is quotient-4 cordial labeling.

Theorem: 3.2. Any closed ladder CL (β) is quotient-4 cordial if $\beta \geq 3$.

Proof: Let G be a closed ladder graph CL (β). Let V(G) = $\{u_\theta : 1 \leq \theta \leq \beta\} \cup \{v_\theta : 1 \leq \theta \leq \beta\}$ and E(G) = $\{u_\theta u_{\theta+1}: 1 \leq \theta \leq \beta - 1\} \cup \{v_\theta v_{\theta+1}: 1 \leq \theta \leq \beta - 1\} \cup \{u_\theta v_\theta : 1 \leq \theta \leq \beta\}$. Here $|V(G)| = 2\beta$, $|E(G)| = 3\beta - 2$.

Define φ: V (G) → {1, 2, 3, 4}.

The values of u_θ are labeled the following.

Case (i): When $\beta \equiv 0, 1$ (modulo 8).

For $1 \leq \theta \leq \beta$.

φ (u_θ) = 1 if $\theta \equiv 1, 3$ (modulo 8).

φ (u_θ) = 2 if $\theta \equiv 6$ (modulo 8).

φ (u_θ) = 3 if $\theta \equiv 4, 5$ (modulo 8).

φ (u_θ) = 4 if $\theta \equiv 0, 2, 7$ (modulo 8).

Case (ii): When $\beta \equiv 2$ (modulo 8).

For $1 \leq \theta \leq \beta - 5$, the labeling of u_θ values are same as case (i).

φ (u_β) = φ ($u_{\beta-1}$) = φ ($u_{\beta-2}$) = φ ($u_{\beta-3}$) = 2, φ ($u_{\beta-4}$) = 4.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
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Case (iii): When $\beta \equiv 3, 5$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of u_θ values are same as case (i).

φ (u_β) = 2.

Case (iv): When $\beta \equiv 4$ (modulo 8).

For $1 \leq \theta \leq \beta - 2$, the labeling of u_θ values are same as case (i).

φ (u_β) = 2, φ ($u_{\beta-1}$) = 4.

Case (v): When $\beta \equiv 6$ (modulo 8).

For $1 \leq \theta \leq \beta - 4$, the labeling of u_θ values are same as case (i).

φ (u_β) = 2, φ ($u_{\beta-1}$) = φ ($u_{\beta-3}$) = 4, φ ($u_{\beta-2}$) = 1.

Case (vi): When $\beta \equiv 7$ (modulo 8).

For $1 \leq \theta \leq \beta - 2$, the labeling of u_θ values are same as case (i).

φ (u_β) = 4, φ ($u_{\beta-1}$) = 3.

The values of v_θ are labeled the following.

Case (i): When $\beta \equiv 0, 1$ (modulo 8).

For $1 \leq \theta \leq \beta$.

φ (v_θ) = 1 if $\theta \equiv 2, 4$ (modulo 8).

φ (v_θ) = 2 if $\theta \equiv 0, 6, 7$ (modulo 8).

φ (v_θ) = 3 if $\theta \equiv 3, 5$ (modulo 8).

φ (v_θ) = 4 if $\theta \equiv 1$ (modulo 8).

Case (ii): When $\beta \equiv 2$ (modulo 8).

For $1 \leq \theta \leq \beta - 5$, the labeling of v_θ values are same as case (i).

φ (v_β) = 2, φ ($v_{\beta-1}$) = 3, φ ($v_{\beta-2}$) = φ ($v_{\beta-3}$) = 4, φ ($v_{\beta-4}$) = 1.

Case (iii): When $\beta \equiv 3$ (modulo 8).

For $1 \leq \theta \leq \beta - 3$, the labeling of v_θ values are same as case (i).

φ (v_β) = 2, φ ($v_{\beta-1}$) = 1, φ ($v_{\beta-2}$) = 3.

Case (iv): When $\beta \equiv 4$ (modulo 8).

For $1 \leq \theta \leq \beta - 4$, the labeling of v_θ values are same as case (i).

φ (v_β) = 2, φ ($v_{\beta-1}$) = φ ($v_{\beta-3}$) = 3, φ ($v_{\beta-2}$) = 1.

Case (v): When $\beta \equiv 5$ (modulo 8).

For $1 \leq \theta \leq \beta - 4$, the labeling of v_θ values are same as case (i).

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
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φ (v_β) = 2, φ $(v_{\beta-1})$ = 1, φ $(v_{\beta-2})$ = 3, φ $(v_{\beta-3})$ = 4.

Case (vi): When $\beta \equiv 6$ (modulo 8).

For $1 \le \theta \le \beta - 6$, the labeling of v_θ values are same as case (i).

φ (v_β) = φ $(v_{\beta-1})$ = 2, φ $(v_{\beta-2})$ = φ $(v_{\beta-3})$ = φ $(v_{\beta-5})$ = 3, φ $(v_{\beta-4})$ = 1.

Case (vii): When $\beta \equiv 7$ (modulo 8).

For $1 \le \theta \le \beta - 3$, the labeling of v_θ values are same as case (i).

φ (v_β) = φ $(v_{\beta-1})$ = φ $(v_{\beta-2})$ = 2.

Table 3

Nature of β	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$\beta \equiv 0, 2, 4, 6$ (modulo 8)	$\frac{\beta}{2}$	$\frac{\beta}{2}$	$\frac{\beta}{2}$	$\frac{\beta}{2}$
$\beta \equiv 1,5$(modulo 8)	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2}-1$	$\frac{\beta+1}{2}-1$	$\frac{\beta+1}{2}$
$\beta \equiv 3$(modulo 8)	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2}-1$	$\frac{\beta+1}{2}-1$
$\beta \equiv 7$(modulo 8)	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2}-1$	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2}-1$

Table 4

Nature of β	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$\beta \equiv 0$(modulo 8)	$\frac{3\beta}{4}-1$	$\frac{3\beta}{4}$	$\frac{3\beta}{4}-1$	$\frac{3\beta}{4}$
$\beta \equiv 1$(modulo 8)	$\frac{3\beta+1}{4}$	$\frac{3\beta+1}{4}-1$	$\frac{3\beta+1}{4}-1$	$\frac{3\beta+1}{4}-1$
$\beta \equiv 2,6$(modulo 8)	$\frac{3\beta-2}{4}$	$\frac{3\beta-2}{4}$	$\frac{3\beta-2}{4}$	$\frac{3\beta-2}{4}$
$\beta \equiv 3,7$(modulo 8)	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}-1$	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}$
$\beta \equiv 4$ (modulo 8)	$\frac{3\beta}{4}-1$	$\frac{3\beta}{4}-1$	$\frac{3\beta}{4}$	$\frac{3\beta}{4}$
$\beta \equiv 5$(modulo 8)	$\frac{3\beta-3}{4}$	$\frac{3\beta-3}{4}$	$\frac{3\beta-3}{4}$	$\frac{3\beta-3}{4}+1$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
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The above tables 3 and 4 shows that $|v_\varphi(i) - v_\varphi(j)| \leq 1$ and $|e_\varphi(k) - e_\varphi(l)| \leq 1$. Hence the closed ladder graph CL (β) is quotient-4 cordial labeling.

Theorem: 3.3. Any slanting ladder graph SL (β) is quotient-4 cordial if $\beta \geq 3$.

Proof: Let G be a slanting ladder graph SL (β). Let V(G) = $\{u_\theta : 1 \leq \theta \leq \beta\} \cup \{v_\theta : 1 \leq \theta \leq \beta\}$ and E(G) = $\{u_\theta u_{\theta+1}: 1 \leq \theta \leq \beta - 1\} \cup \{v_\theta v_{\theta+1}: 1 \leq \theta \leq \beta - 1\} \cup \{u_\theta v_{\theta+1}: 1 \leq \theta \leq \beta - 1\}$.

Here $|V(G)| = 2\beta$, $|E(G)| = 3\beta - 3$.

Define φ: V (G) → {1, 2, 3, 4}.

The values of u_θ are labeled the following.

Case (i): When $\beta \equiv 0,1,6,7$ (modulo 8).

For $1 \leq \theta \leq \beta$.

$\varphi(u_\theta) = 1$ if $\theta \equiv 0, 2$ (modulo 8).

$\varphi(u_\theta) = 2$ if $\theta \equiv 4, 5, 6$ (modulo 8).

$\varphi(u_\theta) = 3$ if $\theta \equiv 7$ (modulo 8).

$\varphi(u_\theta) = 4$ if $\theta \equiv 1, 3$ (modulo 8).

Case (ii): When $\beta \equiv 2$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of u_θ values are same as case (i).

$\varphi(u_\beta) = 2$.

Case (iii): When $\beta \equiv 3$ (modulo 8).

For $1 \leq \theta \leq \beta - 2$, the labeling of u_θ values are same as case (i).

$\varphi(u_\beta) = 1$, $\varphi(u_{\beta-1}) = 4$.

Case (iv): When $\beta \equiv 4$ (modulo 8).

For $1 \leq \theta \leq \beta - 3$, the labeling of u_θ values are same as case (i).

$\varphi(u_\beta) = \varphi(u_{\beta-1}) = 2$, $\varphi(u_{\beta-2}) = 3$.

Case (v): When $\beta \equiv 5$ (modulo 8).

For $1 \leq \theta \leq \beta - 3$, the labeling of u_θ values are same as case (i).

$\varphi(u_\beta) = \varphi(u_{\beta-1}) = 2$, $\varphi(u_{\beta-2}) = 1$.

The values of v_θ are labeled the following.

Case (i): When $\beta \equiv 0, 1, 2, 7$(modulo 8).

For $1 \leq \theta \leq \beta$.

$\varphi(v_\theta) = 1$ if $\theta \equiv 2, 4$ (modulo 8).

$\varphi(v_\theta) = 2$ if $\theta \equiv 7$ (modulo 8).

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
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φ (v_θ) = 3 if $\theta \equiv 0, 1, 3$ (modulo 8).

φ (v_θ) = 4 if $\theta \equiv 5, 6$ (modulo 8).

Case (ii): When $\beta \equiv 3$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of v_θ values are same as case (i).

φ (v_β) = 2.

Case (iii): When $\beta \equiv 4$ (modulo 8).

For $1 \leq \theta \leq \beta - 2$, the labeling of v_θ values are same as case (i).

φ (v_β) = 4, φ $(v_{\beta-1})$ = 1.

Case (iv): When $\beta \equiv 5$ (modulo 8).

For $1 \leq \theta \leq \beta - 2$, the labeling of v_θ values are same as case (i).

φ (v_β) = φ $(v_{\beta-1})$ = 4.

Case (v): When $\beta \equiv 6$ (modulo 8).

For $1 \leq \theta \leq \beta - 3$, the labeling of v_θ values are same as case (i).

φ (v_β) = 3, φ $(v_{\beta-1})$ = 1, φ $(v_{\beta-2})$ = 4.

Table 5

Nature of β	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$\beta \equiv 0, 2, 4, 6$ (modulo 8)	$\frac{\beta}{2}$	$\frac{\beta}{2}$	$\frac{\beta}{2}$	$\frac{\beta}{2}$
$\beta \equiv 1$ (modulo 8)	$\frac{\beta+1}{2} - 1$	$\frac{\beta+1}{2} - 1$	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2}$
$\beta \equiv 3,5$ (modulo 8)	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2} - 1$	$\frac{\beta+1}{2} - 1$	$\frac{\beta+1}{2}$
$\beta \equiv 7$ (modulo 8)	$\frac{\beta+1}{2} - 1$	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2} - 1$	$\frac{\beta+1}{2}$

Table 6

Nature of β	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$\beta \equiv 0,4$ (modulo 8)	$\frac{3\beta}{4} - 1$	$\frac{3\beta}{4} - 1$	$\frac{3\beta}{4}$	$\frac{3\beta}{4} - 1$
$\beta \equiv 1,5$ (modulo 8)	$\frac{3\beta-3}{4}$	$\frac{3\beta-3}{4}$	$\frac{3\beta-3}{4}$	$\frac{3\beta-3}{4}$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
https://publishoa.com
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$\beta \equiv 2,6$ (modulo 8)	$\frac{3\beta-2}{4}$	$\frac{3\beta-2}{4}-1$	$\frac{3\beta-2}{4}$	$\frac{3\beta-2}{4}$
$\beta \equiv 3,7$ (modulo 8)	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}-1$	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}-1$

The above tables 5 and 6 shows that $|v_\varphi(i) - v_\varphi(j)| \leq 1$ and $|e_\varphi(k) - e_\varphi(l)| \leq 1$. Hence the slanting ladder graph SL (β) is quotient-4 cordial labeling.

Theorem: 3.4. Any Mobius ladder graph ML (β) is quotient-4 cordial if $\beta \geq 5$.

Proof: Let G be a Mobius ladder graph ML (β). Let V (G) = $\{u_\theta : 1 \leq \theta \leq \beta\} \cup \{v_\theta : 1 \leq \theta \leq \beta\}$ and E (G) = $\{u_\theta u_{\theta+1}: 1 \leq \theta \leq \beta - 1\} \cup \{v_\theta v_{\theta+1}: 1 \leq \theta \leq \beta - 1\} \cup \{u_\theta v_\theta: 1 \leq \theta \leq \beta\} \cup \{u_1 v_\beta\} \cup \{v_1 u_\beta\}$. Here $|V(G)| = 2\beta$, $|E(G)| = 3\beta$.

Define φ: V (G) → {1, 2, 3, 4}.

The values of u_θ are labeled the following.

Case (i): When $\beta \equiv 0, 3$ (modulo 8).

For $1 \leq \theta \leq \beta$.

φ (u_θ) = 1 if $\theta \equiv 1, 3$ (modulo 8).

φ (u_θ) = 2 if $\theta \equiv 6, 7$ (modulo 8).

φ (u_θ) = 3 if $\theta \equiv 5$ (modulo 8).

φ (u_θ) = 4 if $\theta \equiv 0, 2, 4$ (modulo 8).

Case (ii): When $\beta \equiv 1$(modulo 8).

For $1 \leq \theta \leq \beta - 2$, the labeling of u_θ values are same as case (i).

φ (u_β) = 3, φ ($u_{\beta-1}$) = 1.

Case (iii): When $\beta \equiv 2$(modulo 8).

For $1 \leq \theta \leq \beta - 3$, the labeling of u_θ values are same as case (i).

φ (u_β) = 1, φ ($u_{\beta-1}$) = 3, φ ($u_{\beta-2}$) = 2.

Case (iv): When $\beta \equiv 4$(modulo 8).

For $1 \leq \theta \leq \beta - 4$, the labeling of u_θ values are same as case (i).

φ (u_β) = φ ($u_{\beta-2}$) = 1,φ ($u_{\beta-1}$) = φ ($u_{\beta-3}$) = 4.

Case (v): When $\beta \equiv 5$ (modulo 8).

For $1 \leq \theta \leq \beta - 3$, the labeling of u_θ values are same as case (i).

φ (u_β) = 3, φ ($u_{\beta-1}$) = 1,φ ($u_{\beta-2}$) = 4.

Case (vi): When $\beta \equiv 6$ (modulo 8).

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
https://publishoa.com
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For $1 \leq \theta \leq \beta - 5$, the labeling of u_θ values are same as case (i).

φ (u_β) = φ ($u_{\beta-3}$) = 1, φ ($u_{\beta-1}$) = φ ($u_{\beta-2}$) = 4, φ ($u_{\beta-4}$) = 3.

Case (vii): When $\beta \equiv 7$ (modulo 8).

For $1 \leq \theta \leq \beta - 3$, the labeling of u_θ values are same as case (i).

φ (u_β) = 3, φ ($u_{\beta-1}$) = φ ($u_{\beta-2}$) = 4.

The values of v_θ are labeled the following.

Case (i): When $\beta \equiv 0$ (modulo 8).

For $1 \leq \theta \leq \beta$

φ (v_θ) = 1 if $\theta \equiv 2, 4$ (modulo 8).

φ (v_θ) = 2 if $\theta \equiv 6, 7$ (modulo 8).

φ (v_θ) = 3 if $\theta \equiv 1, 3, 5$ (modulo 8).

φ (v_θ) = 4 if $\theta \equiv 0$ (modulo 8).

Case (ii): When $\beta \equiv 1$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of v_θ values are same as case (i).

φ (v_β) = 4.

Case (iii): When $\beta \equiv 2$ (modulo 8).

For $1 \leq \theta \leq \beta - 2$, the labeling of v_θ values are same as case (i).

φ (v_β) = φ ($v_{\beta-1}$) = 4.

Case (iv): When $\beta \equiv 3$ (modulo 8).

For $1 \leq \theta \leq \beta - 4$, the labeling of $v_\theta$values are same as case (i).

φ (v_β) = 3, φ ($v_{\beta-1}$) = 4, φ ($v_{\beta-2}$) = φ ($v_{\beta-3}$) = 2.

Case (v): When $\beta \equiv 4$ (modulo 8).

For $1 \leq \theta \leq \beta - 5$, the labeling of v_θ values are same as case (i).

φ (v_β) = φ ($v_{\beta-1}$) = 3, φ ($v_{\beta-2}$) = 4, φ ($v_{\beta-3}$) = φ ($v_{\beta-4}$) = 2.

Case (vi): When $\beta \equiv 5$ (modulo 8).

For $1 \leq \theta \leq \beta - 3$, the labeling of v_θ values are same as case (i).

φ (v_β) = φ ($v_{\beta-1}$) = 2, φ ($v_{\beta-2}$) = 4.

Case (vii): When $\beta \equiv 6$ (modulo 8).

For $1 \leq \theta \leq \beta - 5$, the labeling of v_θ values are same as case (i).

φ (v_β) = 4, φ ($v_{\beta-1}$) = φ ($v_{\beta-2}$) = φ ($v_{\beta-3}$) = 2, φ ($v_{\beta-4}$) = 3.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
https://publishoa.com
ISSN: 1309-3452

Case (viii): When $\beta \equiv 7$ (modulo 8).

For $1 \leq \theta \leq \beta - 3$, the labeling of v_θ values are same as case (i).

φ (v_β) = φ ($v_{\beta-1}$) = φ ($v_{\beta-2}$) = 2.

Table 7

Nature of β	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$\beta \equiv 0,2,4,6$ (modulo 8)	$\frac{\beta}{2}$	$\frac{\beta}{2}$	$\frac{\beta}{2}$	$\frac{\beta}{2}$
$\beta \equiv 1$ (modulo 8)	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2} - 1$	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2} - 1$
$\beta \equiv 3$ (modulo 8)	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2} - 1$	$\frac{\beta+1}{2} - 1$
$\beta \equiv 5,7$ (modulo 8)	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2} - 1$	$\frac{\beta+1}{2} - 1$	$\frac{\beta+1}{2}$

Table 8

Nature of β	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$\beta \equiv 0,4$(modulo 8)	$\frac{3\beta}{4}$	$\frac{3\beta}{4}$	$\frac{3\beta}{4}$	$\frac{3\beta}{4}$
$\beta \equiv 1$ (modulo 8)	$\frac{3\beta+1}{4}$	$\frac{3\beta+1}{4}$	$\frac{3\beta+1}{4} - 1$	$\frac{3\beta+1}{4}$
$\beta \equiv 2$ (modulo 8)	$\frac{3\beta+2}{4} - 1$	$\frac{3\beta+2}{4}$	$\frac{3\beta+2}{4} - 1$	$\frac{3\beta+2}{4}$
$\beta \equiv 3$ (modulo 8)	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4} + 1$
$\beta \equiv 5$ (modulo 8)	$\frac{3\beta+1}{4}$	$\frac{3\beta+1}{4}$	$\frac{3\beta+1}{4}$	$\frac{3\beta+1}{4} - 1$
$\beta \equiv 6$ (modulo 8)	$\frac{3\beta+2}{4} - 1$	$\frac{3\beta+2}{4}$	$\frac{3\beta+2}{4}$	$\frac{3\beta+2}{4} - 1$
$\beta \equiv 7$ (modulo 8)	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4} + 1$	$\frac{3\beta-1}{4}$

The above tables 7 and 8 shows that $|v_\varphi(i) - v_\varphi(j)| \leq 1$ and $|e_\varphi(k) - e_\varphi(l)| \leq 1$. Hence the Mobius ladder graph ML (β) is quotient-4 cordial labeling.

Theorem: 3.5. Any Circular ladder CRL (β) is quotient-4 cordial if $\beta \geq 3$ and $\beta \neq 4$.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
https://publishoa.com
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Proof: Let G be a Circular ladder graph CRL (β). Let V(G) = $\{u_\theta : 1 \le \theta \le \beta\} \cup \{v_\theta : 1 \le \theta \le \beta\}$ and E(G) = $\{u_\theta u_{\theta+1} : 1 \le \theta \le \beta - 1\} \cup \{v_\theta v_{\theta+1} : 1 \le \theta \le \beta - 1\} \cup \{u_\theta v_\theta : 1 \le \theta \le \beta\} \cup \{u_1 u_\beta\} \cup \{v_1 v_\beta\}$. Here $|V(G)| = 2\beta$, $|E(G)| = 3\beta$.

Define φ: V (G) → {1, 2, 3, 4}.

The values of u_θ are labeled the following.

Case (i): When $\beta \equiv 0, 1, 2, 7$ (modulo 8).

For $1 \le \theta \le \beta$.

φ (u_θ) = 1 if $\theta \equiv 1, 3$ (modulo 8).

φ (u_θ) = 2 if $\theta \equiv 6, 7$ (modulo 8).

φ (u_θ) = 3 if $\theta \equiv 0, 2, 4, 5$ (modulo 8).

Case (ii): When $\beta \equiv 3$ (modulo 8).

For $1 \le \theta \le \beta - 2$, the labeling of u_θ values are same as case (i).

φ (u_β) = φ $(u_{\beta-1})$ = 3.

Case (iii): When $\beta \equiv 4$ (modulo 8).

For $1 \le \theta \le \beta - 1$, the labeling of u_θ values are same as case (i).

φ (u_β) = 4.

Case (iv): When $\beta \equiv 5$ (modulo 8).

For $1 \le \theta \le \beta - 3$, the labeling of u_θ values are same as case (i).

φ (u_β) = 3, φ $(u_{\beta-1})$ = 1, φ $(u_{\beta-2})$ = 4.

Case (v): When $\beta \equiv 6$ (modulo 8).

For $1 \le \theta \le \beta - 4$, the labeling of u_θ values are same as case (i).

φ (u_β) = φ $(u_{\beta-1})$ = φ $(u_{\beta-2})$ = 2, φ $(u_{\beta-3})$ = 4.

The values of v_θ are labeled the following.

Case (i): When $\beta \equiv 0, 3, 7$ (modulo 8).

For $1 \le \theta \le \beta$.

φ (v_θ) = 1 if $\theta \equiv 2, 4$ (modulo 8).

φ (v_θ) = 2 if $\theta \equiv 6, 7$ (modulo 8).

φ (v_θ) = 4 if $\theta \equiv 0, 1, 3, 5$ (modulo 8).

Case (ii): When $\beta \equiv 1$ (modulo 8).

For $1 \le \theta \le \beta - 2$, the labeling of v_θ values are same as case (i).

φ (v_β) = 4, φ $(v_{\beta-1})$ =1.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
https://publishoa.com
ISSN: 1309-3452

Case (iii): When $\beta \equiv 2$ (modulo 8).

For $1 \leq \theta \leq \beta - 2$, the labeling of v_θ values are same as case (i).

φ $(v_\beta) = 4$, φ $(v_{\beta-1}) = 2$.

Case (iv): When $\beta \equiv 4$ (modulo 8).

For $1 \leq \theta \leq \beta - 5$, the labeling of v_θ values are same as case (i).

φ (v_β) = φ $(v_{\beta-2}) = 4$, φ $(v_{\beta-1}) = 1$, φ $(v_{\beta-3})$ = φ $(v_{\beta-4}) = 2$.

Case (v): When $\beta \equiv 5$ (modulo 8).

For $1 \leq \theta \leq \beta - 3$, the labeling of v_θ values are same as case (i).

φ (v_β) = φ $(v_{\beta-1}) = 2$, φ $(v_{\beta-2}) = 4$.

Case (vi): When $\beta \equiv 6$ (modulo 8).

For $1 \leq \theta \leq \beta - 4$, the labeling of v_θ values are same as case (i).

φ (v_β) = φ $(v_{\beta-1}) = 3$, φ $(v_{\beta-2}) = 1$, φ $(v_{\beta-3}) = 4$.

Table 9

Nature of β	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$\beta \equiv 0, 2, 4, 6$ (modulo 8)	$\frac{\beta}{2}$	$\frac{\beta}{2}$	$\frac{\beta}{2}$	$\frac{\beta}{2}$
$\beta \equiv 1,3$ (modulo 8)	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2} - 1$	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2} - 1$
$\beta \equiv 5$ (modulo 8)	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2} - 1$	$\frac{\beta+1}{2} - 1$	$\frac{\beta+1}{2}$
$\beta \equiv 7$(modulo 8)	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2}$	$\frac{\beta+1}{2} - 1$	$\frac{\beta+1}{2} - 1$

Table 10

Nature of β	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$\beta \equiv 0,4$(modulo 8)	$\frac{3\beta}{4}$	$\frac{3\beta}{4}$	$\frac{3\beta}{4}$	$\frac{3\beta}{4}$
$\beta \equiv 1$ (modulo 8)	$\frac{3\beta+1}{4}$	$\frac{3\beta+1}{4}$	$\frac{3\beta+1}{4} - 1$	$\frac{3\beta+1}{4}$
$\beta \equiv 2$ (modulo 8)	$\frac{3\beta+2}{4} - 1$	$\frac{3\beta+2}{4}$	$\frac{3\beta+2}{4} - 1$	$\frac{3\beta+2}{4}$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
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$\beta \equiv 3$ (modulo 8)	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}+1$	$\frac{3\beta-1}{4}$
$\beta \equiv 5$ (modulo 8)	$\frac{3\beta+1}{4}$	$\frac{3\beta+1}{4}$	$\frac{3\beta+1}{4}$	$\frac{3\beta+1}{4}-1$
$\beta \equiv 6$ (modulo 8)	$\frac{3\beta+2}{4}-1$	$\frac{3\beta+2}{4}$	$\frac{3\beta+2}{4}$	$\frac{3\beta+2}{4}-1$
$\beta \equiv 7$ (modulo 8)	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}+1$

The above tables 9 and 10 shows that $|v_\varphi(i) - v_\varphi(j)| \leq 1$ and $|e_\varphi(k) - e_\varphi(l)| \leq 1$. Hence the Circular ladder graph CRL (β) is quotient-4 cordial labeling.

Theorem: 3.6. Any Pentagonal ladder PL (β) is quotient-4 cordial if $\beta \geq 2$.

Proof: Let G be a Pentagonal ladder graph PL (β). Let V (G) = $\{u_\theta: 1 \leq \theta \leq \beta\} \cup \{v_\theta: 1 \leq \theta \leq \beta\} \cup \{w_\theta: 1 \leq \theta \leq \beta - 1\}$ and E (G) = $\{u_\theta u_{\theta+1}: 1 \leq \theta \leq \beta - 1\} \cup \{u_\theta v_\theta: 1 \leq \theta \leq \beta\} \cup \{v_\theta w_\theta: 1 \leq \theta \leq \beta - 1\} \cup \{w_\theta v_{\theta+1}: 1 \leq \theta \leq \beta - 1\}$. Here $|V(G)| = 3\beta - 1$, $|E(G)| = 4\beta - 3$.

Define φ: V (G) → {1, 2, 3, 4}.

The values of u_θ are labeled the following.

Case (i): When $\beta \equiv 0, 1, 7$ (modulo 8).

For $1 \leq \theta \leq \beta$.

φ (u_θ) = 1 if $\theta \equiv 2, 4, 6$ (modulo 8).

φ (u_θ) = 2 if $\theta \equiv 7$ (modulo 8).

φ (u_θ) = 3 if $\theta \equiv 0, 1, 5$ (modulo 8).

φ (u_θ) = 4 if $\theta \equiv 3$ (modulo 8).

Case (ii): When $\beta \equiv 2, 3, 4, 5$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of u_θ values are same as case (i).

φ (u_β) = 2.

Case (iii): When $\beta \equiv 6$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of u_θ values are same as case (i).

φ (u_β) = 4.

The values of v_θ are labeled the following.

Case (i): When $\beta \equiv 0, 1, 2$ (modulo 8).

For $1 \leq \theta \leq \beta$.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
https://publishoa.com
ISSN: 1309-3452

φ (v_θ) = 1 if θ ≡ 1, 3, 5 (modulo 8).

φ (v_θ) = 2 if θ ≡ 0, 7 (modulo 8).

φ (v_θ) = 3 if θ ≡ 4 (modulo 8).

φ (v_θ) = 4 if θ ≡ 2, 6 (modulo 8).

Case (ii): When β ≡ 3, 4 (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of v_θ values are same as case (i).

φ (v_β) = 2.

Case (iii): When β ≡ 5, 6 (modulo 8).

For 1 ≤ θ ≤ β – 2, the labeling of v_θ values are same as case (i).

φ (v_β) = φ $(v_{\beta-1})$ = 2.

Case (iv): When β ≡ 7 (modulo 8).

For 1 ≤θ ≤ β – 3, the labeling of v_θ values are same as case (i).

φ (v_β) = φ $(v_{\beta-2})$ = 2, φ $(v_{\beta-1})$ = 4.

The values of w_θ are labeled the following.

Case (i): When β ≡ 0, 1, 2, 3, 5, 7 (modulo 8).

For 1 ≤ θ ≤ β.

φ (w_θ) = 2 if θ ≡ 4, 6, 7 (modulo 8).

φ (w_θ) = 3 if θ ≡ 2, 3 (modulo 8).

φ (w_θ) = 4 if θ ≡ 0, 1, 5 (modulo 8).

Case (ii): When β ≡ 4, 6 (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of w_θ values are same as case (i).

φ (w_β) = 2.

Table 11

Nature of β	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
β ≡ 0 (modulo 8)	$\frac{3\beta}{4}$	$\frac{3\beta}{4}$	$\frac{3\beta}{4}$	$\frac{3\beta}{4} - 1$
β ≡1 (modulo 8)	$\frac{3\beta+1}{4}$	$\frac{3\beta+1}{4} - 1$	$\frac{3\beta+1}{4}$	$\frac{3\beta+1}{4} - 1$
β ≡ 2 (modulo 8)	$\frac{3\beta-2}{4}$	$\frac{3\beta-2}{4}$	$\frac{3\beta-2}{4}$	$\frac{3\beta-2}{4} + 1$
β ≡ 3,7(modulo 8)	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
https://publishoa.com
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$\beta \equiv 4$ (modulo 8)	$\frac{3\beta}{4}$	$\frac{3\beta}{4}$	$\frac{3\beta}{4} - 1$	$\frac{3\beta}{4}$
$\beta \equiv 5$ (modulo 8)	$\frac{3\beta+1}{4}$	$\frac{3\beta+1}{4}$	$\frac{3\beta+1}{4} - 1$	$\frac{3\beta+1}{4} - 1$
$\beta \equiv 6$ (modulo 8)	$\frac{3\beta-2}{4}$	$\frac{3\beta-2}{4}$	$\frac{3\beta-2}{4} + 1$	$\frac{3\beta-2}{4}$

Table 12

Nature of β	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$\beta \equiv$ 0,1,6,7(modulo 8)	$\beta - 1$	$\beta - 1$	$\beta - 1$	β
$\beta \equiv$ 2,3 (modulo 8)	$\beta - 1$	$\beta - 1$	β	$\beta - 1$
$\beta \equiv$ 4,5(modulo 8)	β	$\beta - 1$	$\beta - 1$	$\beta - 1$

The above tables 11 and 12 shows that $|v_\varphi(i) - v_\varphi(j)| \leq 1$ and $|e_\varphi(k) - e_\varphi(l)| \leq 1$. Hence the Pentagonal ladder graph PL (β) is quotient-4 cordial labeling.

Theorem: 3.7. Any Pentagonal Circular ladder PCL (β) is quotient-4 cordial if $\beta \geq 3$.

Proof: Let G be a Pentagonal Circular ladder graph PCL (β). Let V(G) = $\{u_\theta : 1 \leq \theta \leq \beta\} \cup \{v_\theta : 1 \leq \theta \leq \beta\} \cup \{w_\theta : 1 \leq \theta \leq \beta\}$ and E(G) = $\{u_\theta u_{\theta+1}: 1 \leq \theta \leq \beta - 1\} \cup \{u_\theta v_\theta : 1 \leq \theta \leq \beta\} \cup \{v_\theta w_\theta: 1 \leq \theta \leq \beta\} \cup \{w_\theta v_{\theta+1}: 1 \leq \theta \leq \beta - 1\} \cup \{u_1 u_\beta\} \cup \{v_1 w_\beta\}$. Here $|V(G)| = 3\beta$, $|E(G)| = 4\beta$.

Define φ: V (G) → {1, 2, 3, 4}.

The values of u_θ are labeled the following.

Case (i): When $\beta \equiv 0, 5, 7$ (modulo 8).

For $1 \leq \theta \leq \beta$.

$\varphi(u_\theta) = 1$ if $\theta \equiv 2, 4, 7$ (modulo 8).

$\varphi(u_\theta) = 3$ if $\theta \equiv 0, 3$ (modulo 8).

$\varphi(u_\theta) = 4$ if $\theta \equiv 1, 5, 6$ (modulo 8).

Case (ii): When $\beta \equiv 1$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of u_θ values are same as case (i).

$\varphi(u_\beta) = 1$.

Case (iii): When $\beta \equiv 2, 3$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of u_θ values are same as case (i).

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
https://publishoa.com
ISSN: 1309-3452

φ (u_β) = 4.

Case (iv): When $\beta \equiv 4$ (modulo 8).

For $1 \leq \theta \leq \beta - 2$, the labeling of u_θ values are same as case (i).

φ (u_β) = φ ($u_{\beta-1}$) = 4.

Case (v): When $\beta \equiv 6$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of u_θ values are same as case (i).

φ (u_β) = 2.

The values of v_θ are labeled the following.

Case (i): When $\beta \equiv 0, 7$ (modulo 8).

For $1 \leq \theta \leq \beta$.

φ (v_θ) = 1 if $\theta \equiv 0, 1, 3$ (modulo 8).

φ (v_θ) = 2 if $\theta \equiv 5, 6, 7$ (modulo 8).

φ (v_θ) = 3 if $\theta \equiv 2, 4$ (modulo 8).

Case (ii): When $\beta \equiv 1$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of v_θ values are same as case (i).

φ (v_β) = 4.

Case (iii): When $\beta \equiv 2$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of v_θ values are same as case (i).

φ (v_β) = 1.

Case (iv): When $\beta \equiv 3$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of v_θ values are same as case (i).

φ (v_β) = 2.

Case (v): When $\beta \equiv 4$ (modulo 8).

For $1 \leq \theta \leq \beta - 3$, the labeling of v_θ values are same as case (i).

φ (v_β) = 1, φ ($v_{\beta-1}$) = φ ($v_{\beta-2}$) = 2.

Case (vi): When $\beta \equiv 5$ (modulo 8).

For $1 \leq \theta \leq \beta - 2$, the labeling of v_θ values are same as case (i).

φ (v_β) = φ ($v_{\beta-1}$) = 2.

Case (vii): When $\beta \equiv 6$ (modulo 8).

For $1 \leq \theta \leq \beta - 3$, the labeling of v_θ values are same as case (i).

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
https://publishoa.com
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φ (v_β) = φ $(v_{\beta-1})$ = 2, φ $(v_{\beta-2})$ = 1.

The values of w_θ are labeled the following.

Case (i): When $\beta \equiv 0, 6$ (modulo 8).

For $1 \leq \theta \leq \beta$.

φ (w_θ) = 2 if $\theta \equiv 5, 6, 7$ (modulo 8).

φ (w_θ) = 3 if $\theta \equiv 2, 4$ (modulo 8).

φ (w_θ) = 4 if $\theta \equiv 0, 1, 3$ (modulo 8).

Case (ii): When $\beta \equiv 1$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of w_θ values are same as case (i).

φ (w_β) = 2.

Case (iii): When $\beta \equiv 2$ (modulo 8).

For $1 \leq \theta \leq \beta - 2$, the labeling of w_θ values are same as case (i).

φ (w_β) = 2, φ $(w_{\beta-1})$ = 3.

Case (iv): When $\beta \equiv 3$ (modulo 8).

For $1 \leq \theta \leq \beta - 3$, the labeling of w_θ values are same as case (i).

φ (w_β) = 2, φ $(w_{\beta-1})$ = 1, φ $(w_{\beta-2})$ = 3.

Case (v): When $\beta \equiv 4$ (modulo 8).

For $1 \leq \theta \leq \beta - 4$, the labeling of w_θ values are same as case (i).

φ (w_β) = φ $(w_{\beta-1})$ = φ $(w_{\beta-3})$ = 3, φ $(w_{\beta-2})$ = 2.

Case (vi): When $\beta \equiv 5$ (modulo 8).

For $1 \leq \theta \leq \beta - 2$, the labeling of w_θ values are same as case (i).

φ (w_β) = φ $(w_{\beta-1})$ = 2.

Case (vii): When $\beta \equiv 7$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of w_θ values are same as case (i).

φ (w_β) = 3.

Table 13

Nature of β	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$\beta \equiv 0,4$(modulo 8)	$\frac{3\beta}{4}$	$\frac{3\beta}{4}$	$\frac{3\beta}{4}$	$\frac{3\beta}{4}$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
https://publishoa.com
ISSN: 1309-3452

$\beta\ \equiv$1,5(modulo 8)	$\frac{3\beta+1}{4}$	$\frac{3\beta+1}{4}$	$\frac{3\beta+1}{4}-1$	$\frac{3\beta+1}{4}$
$\beta\ \equiv$ 2 (modulo 8)	$\frac{3\beta+2}{4}$	$\frac{3\beta+2}{4}-1$	$\frac{3\beta+2}{4}-1$	$\frac{3\beta+2}{4}$
$\beta\ \equiv$ 3 (modulo 8)	$\frac{3\beta-1}{4}+1$	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}$
$\beta\ \equiv$ 6 (modulo 8)	$\frac{3\beta+2}{4}$	$\frac{3\beta+2}{4}$	$\frac{3\beta+2}{4}-1$	$\frac{3\beta+2}{4}-1$
$\beta\ \equiv$ 7 (modulo 8)	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}$	$\frac{3\beta-1}{4}+1$	$\frac{3\beta-1}{4}$

Table 14

Nature of β	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$\beta\equiv$ 0,1,2,3,4,5,6,7(modulo 8)	β	β	β	β

The above tables 13 and 14 shows that $|v_\varphi(i) - v_\varphi(j)| \leq 1$ and $|e_\varphi(k) - e_\varphi(l)| \leq 1$. Hence the Pentagonal Circular ladder graph PCL (β) is quotient-4 cordial labeling.

Theorem: 3.8. Any Hexagonal ladder HL (β) is quotient-4 cordial if $\beta \geq 2$.

Proof: Let G be a Hexagonal ladder graph HL (β). Let V (G) = $\{t_\theta : 1 \leq \theta \leq \beta - 1\} \cup \{u_\theta : 1 \leq \theta \leq \beta\} \cup \{v_\theta : 1 \leq \theta \leq \beta\} \cup \{w_\theta : 1 \leq \theta \leq \beta - 1\}$ and E (G) = $\{t_\theta u_\theta : 1 \leq \theta \leq \beta - 1\} \cup \{t_\theta u_{\theta+1} : 1 \leq \theta \leq \beta - 1\} \cup \{u_\theta v_\theta : 1 \leq \theta \leq \beta\} \cup \{v_\theta w_\theta : 1 \leq \theta \leq \beta - 1\} \cup \{v_{\theta+1} w_\theta : 1 \leq \theta \leq \beta - 1\}$. Here $|V(G)| = 4\beta - 2$, $|E(G)| = 5\beta - 4$.

Define φ: V (G) → {1, 2, 3, 4}.

The values of t_θ are labeled the following.

Case (i): When $\beta\ \equiv$ 0, 1, 2, 7 (modulo 8).

For $1 \leq \theta \leq \beta$.

φ (t_θ) = 2 if $\theta \equiv$ 0, 1, 2, 3, 4, 5 (modulo 8).

φ (t_θ) = 4 if $\theta \equiv$ 6, 7 (modulo 8).

Case (ii): When $\beta\ \equiv$ 3 (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of t_θ values are same as case (i).

φ (t_β) = 3.

Case (iii): When $\beta\ \equiv$ 4 (modulo 8).

For $1 \leq \theta \leq \beta - 2$, the labeling of t_θ values are same as case (i).

φ (t_β) = 2, φ ($t_{\beta-1}$) = 3.

Case (iv): When $\beta\ \equiv$ 5 (modulo 8).

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
https://publishoa.com
ISSN: 1309-3452

For $1 \leq \theta \leq \beta - 2$, the labeling of t_θ values are same as case (i).

φ (t_β) = 1, φ ($t_{\beta-1}$) = 2.

Case (v): When $\beta \equiv 6$ (modulo 8).

For $1 \leq \theta \leq \beta - 2$, the labeling of t_θ values are same as case (i).

φ (t_β) = 4, φ ($t_{\beta-1}$) = 3.

The values of u_θ are labeled the following.

Case (i): When $\beta \equiv 0, 1, 3, 4$ (modulo 8).

For $1 \leq \theta \leq \beta$.

φ (u_θ) = 1 if $\theta \equiv 1, 2, 4, 5$ (modulo 8).

φ (u_θ) = 2 if $\theta \equiv 0, 3, 7$ (modulo 8).

φ (u_θ) = 4 if $\theta \equiv 6$ (modulo 8).

Case (ii): When $\beta \equiv 2$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of u_θ values are same as case (i).

φ (u_β) = 2.

Case (iii): When $\beta \equiv 5$ (modulo 8).

For $1 \leq \theta \leq \beta - 2$, the labeling of u_θ values are same as case (i).

φ (u_β) = 3, φ ($u_{\beta-1}$) = 4.

Case (iv): When $\beta \equiv 6$ (modulo 8).

For $1 \leq \theta \leq \beta - 2$, the labeling of u_θ values are same as case (i).

φ (u_β) = 2, φ ($u_{\beta-1}$) = 3.

Case (v): When $\beta \equiv 7$ (modulo 8).

For $1 \leq \theta \leq \beta - 3$, the labeling of u_θ values are same as case (i).

φ (u_β) = 2, φ ($u_{\beta-1}$) = 1, φ ($u_{\beta-2}$) = 4.

The values of v_θ are labeled the following.

Case (i): When $\beta \equiv 0, 1, 3, 4, 5, 7$ (modulo 8).

For $1 \leq \theta \leq \beta$.

φ (v_θ) = 1 if $\theta \equiv 0$ (modulo 8).

φ (v_θ) = 2 if $\theta \equiv 3, 7$ (modulo 8).

φ (v_θ) = 3 if $\theta \equiv 1, 4, 5, 6$ (modulo 8).

φ (v_θ) = 4 if $\theta \equiv 2$ (modulo 8).

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
https://publishoa.com
ISSN: 1309-3452

Case (ii): When $\beta \equiv 2, 6$ (modulo 8).

For $1 \leq \theta \leq \beta - 1$, the labeling of v_θ values are same as case (i).

φ (v_β) = 2.

The values of w_θ are labeled the following.

Case (i): When $\beta \equiv 0, 1, 2, 5, 6, 7$ (modulo 8).

For $1 \leq \theta \leq \beta$.

φ (w_θ) = 1 if $\theta \equiv 1, 4, 5$ (modulo 8).

φ (w_θ) = 2 if $\theta \equiv 2$ (modulo 8).

φ (w_θ) = 3 if $\theta \equiv 0, 3, 6, 7$ (modulo 8).

Case (ii): When $\beta \equiv 3$ (modulo 8).

For $1 \leq \theta \leq \beta - 2$, the labeling of w_θ values are same as case (i).

φ (w_β) = 2, φ ($w_{\beta-1}$) = 3.

Case (iii): When $\beta \equiv 4$ (modulo 8).

For $1 \leq \theta \leq \beta - 2$, the labeling of w_θ values are same as case (i).

φ (w_β) = 2, φ ($w_{\beta-1}$) = 4.

Table 15

Nature of β	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$\beta \equiv 0,2,4$ (modulo 8)	β	β	$\beta - 1$	$\beta - 1$
$\beta \equiv 1,5,6$ (modulo 8)	β	$\beta - 1$	β	$\beta - 1$
$\beta \equiv 3$ (modulo 8)	$\beta - 1$	β	β	$\beta - 1$
$\beta \equiv 7$ (modulo 8)	β	$\beta - 1$	$\beta - 1$	β

Table 16

Nature of β	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$\beta \equiv 0,4$ (modulo 8)	$\frac{5\beta - 4}{4}$	$\frac{5\beta - 4}{4}$	$\frac{5\beta - 4}{4}$	$\frac{5\beta - 4}{4}$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3243-3264
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$\beta\ \equiv 1$ (modulo 8)	$\frac{5(\beta-1)}{4}$	$\frac{5(\beta-1)}{4}$	$\frac{5(\beta-1)}{4}$	$\frac{5(\beta-1)}{4}+1$
$\beta\ \equiv 2,6$ (modulo 8)	$\frac{5\beta-2}{4}-1$	$\frac{5\beta-2}{4}-1$	$\frac{5\beta-2}{4}$	$\frac{5\beta-2}{4}$
$\beta\ \equiv 3$ (modulo 8)	$\frac{5\beta-3}{4}$	$\frac{5\beta-3}{4}$	$\frac{5\beta-3}{4}$	$\frac{5\beta-3}{4}-1$
$\beta\ \equiv 5$ (modulo 8)	$\frac{5(\beta-1)}{4}+1$	$\frac{5(\beta-1)}{4}$	$\frac{5(\beta-1)}{4}$	$\frac{5(\beta-1)}{4}$
$\beta\ \equiv 7$(modulo 8)	$\frac{5\beta-3}{4}$	$\frac{5\beta-3}{4}$	$\frac{5\beta-3}{4}-1$	$\frac{5\beta-3}{4}$

The above tables 15 and 16 shows that $|v_\varphi(i) - v_\varphi(j)| \le 1$ and $|e_\varphi(k) - e_\varphi(l)| \le 1$. Hence the Pentagonal Circular ladder graph HL (β) is quotient-4 cordial labeling.

4. CONCLUSION

Some ladder graphs which are quotient-4 cordial are studied and works are presented in this paper. Our future work will be on finding the existence of this labeling on varies families of graphs.

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REFERENCES

1. Albert William, IndraRajasingh and S Roy, Mean Cordial Labeling of Certain graphs, J.Comp.& Math. Sci. Vol.4 (4),274-281 (2013).
2. I.Cahit and R. Yilmaz, E_3-Cordial graphs, ArsCombin., 54 (2000) 119-127.
3. Freeda Selvanayagam, Robinson and R. S. Chellathurai, H- and H_2 - Cordial Labeling of Some Graphs *Open J. Discrete Math., 2 (2012) 149-155.*
4. Joseph A. Gallian, A Dynamic survey of Graph Labeling, Twenty-first edition, December 21, 2018.
5. P.Sumathi, S.Kavitha, Quotient-4 cordial labeling for path related graphs, The International Journal of Analytical and Experimental Modal analysis, Volume XII, Issue I, January – 2020, pp. 2983-2991.
6. P.Sumathi, J.Suresh Kumar, Fuzzy Quotient-3 Cordial Labeling on Some types of Ladder Graphs, Science, Technology and Development, Volume X Issue V MAY 2021.