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Prime Labeling of m- Shadow Graph of Some Star Related Graphs

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Abstract. A graph $G = (V(G), E(G))$ is observed to admit prime labeling when the vertices of the graph are labeled with unique integral values from $[1,|V|]$ in a way that for every edge uv the labels designated to u and v share no common positive factors except 1. In this research manuscript we examine and manifest that the m-shadow graph of some star related graphs such as $K_{1,n}$, $B(n,n)$ and $Spl(K_{1,n})$ admits prime labeling.

AMS Subject Classification: 05C78

Keywords: Prime labeling, Shadow graph, Star graph, Bistar graph, Split graph

1.INTRODUCTION

In this research manuscript we deal with graphs that are finite, connected, not directed, with no loops and multiple edges. We indicate the set of vertices by $V(G)$ and the set of edges by $E(G)$. The cardinality of the vertex set is denoted by $|V(G)|$. Labeling of graphs has significant applications in various fields like the study of the physical universe, telecommunication network, circuit design, manage data in a database and much more. The concept of *Prime labeling* was established by R. Entringer. A graph $G = (V(G), E(G))$ is observed to admit prime labeling when the vertices of the graph are labeled with unique integral values from $[1, |V|]$ in a way that for every edge uv the labels designated to u and v share no common positive factors except 1[4]. A *prime graph* is the one that accepts a *prime labeling*. In our previous works we have investigated the prime labeling of 2-shadow and 3-shadow graphs. In this research article we generalize the result and we prove that the *m-Shadow graphs* of some *star* related graphs such as *star graph $K_{1,n}$*, *Bistar graph $B(n,n)$* and *Splitting star graph Spl $(K_{1,n})$* admits *Prime labeling.*

2.PRELIMINARIES

Definition 2.1.

Consider a bipartite graph with partitions $V_1 = \{u\}$ and $V_2 = \{u_1, u_2, u_3, \dots , u_n,\}$ such that the vertex u of V_1 is adjacent to every vertex of the partition V_2. Such a graph is known as the star graph $K_{1,n}$ [2].

Definition 2.2.

Consider a star graph $K_{1,n}$ with partitions $V_1 = \{u\}$ and $V_2 = \{u_1, u_2, u_3, \dots , u_n,\}$ and its facsimile copy with partitions $U_1 = \{v\}$ and $U_2 = \{v_1, v_2, v_3, \dots , v_n,\}$. A bistar graph is attained by joining the vertex u of V_1 with the vertex v of U_1 by an edge [3].

Definition 2.3.

Consider a graph G. The splitting graph $Spl(G)$ of the graph G is established by affixing a new vertex v_i corresponding to each vertex u_i of G in a way that the vertices adjacent to u_i are now adjacent to v_i in $Spl(G)$ [1].

Definition 2.4.

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The m-shadow graph $D_m(G)$ of a connected graph G is constructed by taking m facsimile copies of G, say G_1, G_2, $G_3, \ldots, G_m$ and then joining each vertex u in G_i to the neighbours of the corresponding vertex v in G_j, $1 \le i, j \le m$ [5].

3.RESULTS

3.1. $\boldsymbol{m}$-Shadow Graph $\boldsymbol{D_m(K_{1,n})}$ of Star Graph $\boldsymbol{K_{1,n}}$

Theorem 3.1.

m- Shadow graph $D_m(K_{1,n})$ of star graph $K_{1,n}$ admits prime labeling for $n \ge m$ and $m \ge 2$.

***Proof*:**

Let G denote a Star graph $K_{1,n}$.

Let G_1, G_2, G_3,…, G_m denote m- facsimile copies of G.

In general, $\{ G_k \, / \, 1 \le \ k \ \le \ m\}$ denote kth- facsimile copy of G.

Let $V(G_k) \ = \ \{ u_k, v_i / \, 1 \le \ k \ \le \ m, \ (k-1)n+1 \le \ i \ \le \ kn\}$ constitute the vertex set of G_k

Let $E(G_k) \ = \ \{ u_k v_i / \, 1 \le \ k \ \le \ m, \ (k-1)n+1 \le \ i \ \le \ kn\}$ constitute the edge set of G_k

Let $D_m(G)$ be the m- Shadow graph of G.

$V[D_m(G)] = \{ u_k, v_i / \, 1 \le \ k \ \le \ m, 1 \le \ i \ \le \ kn\}$ is the vertex set of $D_m(G)$.

$E[D_m(G)] = \{ u_k v_i / \, 1 \le \ k \ \le \ m, 1 \le \ i \ \le \ kn\}$ is the edge set of $D_m(G)$.

$|V[D_m(G)]| = \ m(n+1)$.

We define a bijective function,

$f: V[D_m(G)] \rightarrow \{ 1,2,3, \ldots, m(n+1)\}$ given by,

$f\ (u_1) = 1, f\ (u_k) = p_{k-1}$ for $2 \le \ k \ \le \ m$

wherein p_1 is the highest prime less than the number $m(n+1)$

p_2 is the second highest prime less than the number $m(n+1)$ and $p_2 < p_1$.

p_3 is the third highest prime less than the number $m(n+1)$ and $p_3 < p_2 < p_1$.

…

p_{k-1} is the (k-1) th highest prime less than the number $m(n+1)$ and

$p_{k-1} < p_{k-2} < p_{k-3} < \cdots < p_3 < p_2 < p_1$.

For all $1 \le i \le m(n+1)$, where $f\ (v_i) \ne p_j$ for $1 \le \ j \ \le \ m-1$, we define

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$$f(v_i) = \begin{cases} i+1, & 1 \le i \le p_{m-1} - 2 \\ i+2, & p_{m-1} - 1 \le i \le p_{m-2} - 3 \\ i+3, & p_{m-2} - 2 \le i \le p_{m-3} - 4 \\ i+4, & p_{m-3} - 3 \le i \le p_{m-4} - 5 \\ \cdot & \\ \cdot & \\ \cdot & \\ i+k, & p_{m-k+1} - (k-1) \le i \le p_{m-1} - (k+1) \\ \cdot & \\ \cdot & \\ \cdot & \\ i+m, & p_1 - (m-1) \le i \le m(n+1) \end{cases}$$

For the edges $u_k v_i$, $1 \le k \le m, 1 \le i \le kn$

$$gcd(f(u_k), f(v_i)) = \begin{cases} \gcd(1, f(v_i)) = 1 \\ \gcd(1, i+2) = 1 \ \ if\ q-1 \le i \le p-3 \\ \gcd(1, i+3) = 1\ if \quad p-2 \le i \le 3n \end{cases}$$

Therefore, the Shadow graph $D_m(K_{1,n})$ of star graph $K_{1,n}$ is a prime graph for $n \ge m$ and $m \ge 2$.

Illustration 3.1.1.

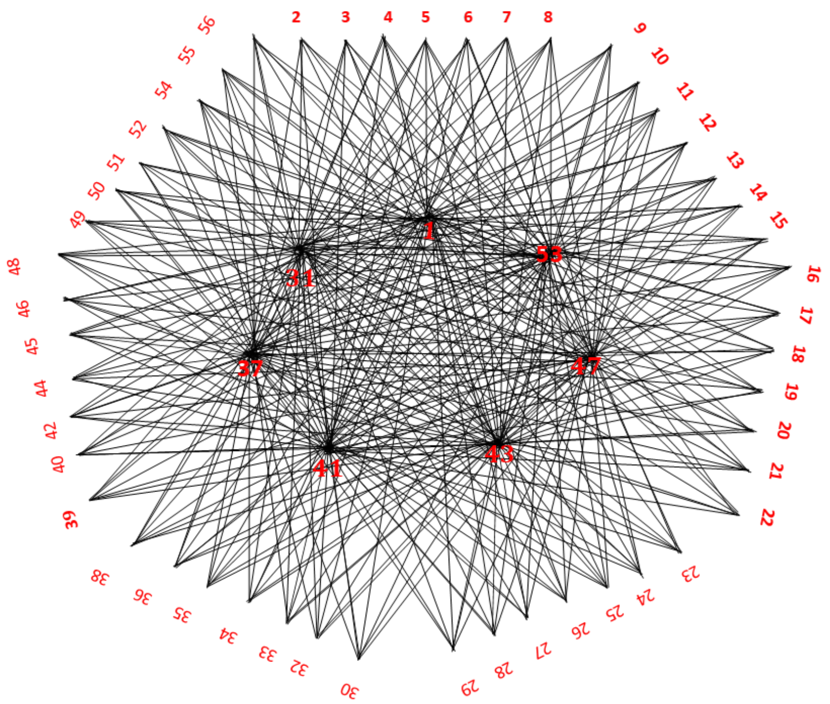

Figure 2. Prime Labeling of 7- Shadow graph $D_7(K_{1,7})$ of star graph $K_{1,7}$

3.2. m-Shadow Graph $D_m(B(n,n))$ of Bistar Graph $B(n,n)$

Theorem 3.2.

m-Shadow graph $D_m(B(n,n))$ of bistar graph $B(n,n)$ is a prime graph for $n \ge m$ and $m \ge 2$.

Proof:

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Let G denote Bistar graph $B(n,n), n \geq 2$.

Let G_1, G_2, G_3,…, G_m denote m- facsimile copies of G.

In general, $\{ G_k \ / \ 1 \leq k \leq m\}$ denote kth- facsimile copy of G.

Let $V(G_k) = \{ u'_k, v'_k, \ u_i, v_i / \ 1 \leq k \leq m, \ (k-1)n+1 \leq i \leq kn\}$ constitute the vertex set of G_k.

Let $E(G_k) = \{u'_k \, u_i / \ 1 \leq k \leq m, \ (k-1)n+1 \leq i \leq kn\} \cup$

$\{v'_k v_i / \ 1 \leq k \leq m, \ (k-1)n+1 \leq i \leq kn\} \cup \{u'_k v'_k / \ 1 \leq k \leq m, \ (k-1)n+1 \leq i \leq kn\}$ constitute the edge set of G_k.

Let $D_m(G)$ be the m- Shadow graph of G.

$V[D_m(G)] = \{ u'_k, v'_k, \ u_i, v_i \ / \ 1 \leq k \leq m, 1 \leq i \leq kn\}$ is the vertex set of $D_m(G)$.

$E[D_m(G)] = \{u'_k \, u_i / \ 1 \leq k \leq m, 1 \leq i \leq kn\} \cup \{v'_k v_i / \ 1 \leq k \leq m, 1 \leq i \leq kn\} \cup$

$\{u'_k \, v'_k / \ 1 \leq k \leq m, 1 \leq i \leq kn\}$ is the edge set of $D_m(G)$.

$|V[D_m(G)]| = 2m(n+1)$.

We define a bijective function,

$f: V[D_m(G)] \rightarrow \{ 1,2,3, \ldots, 2m(n+1)\}$ given by,

$f\,(u'_1) = 1, f\,(u'_k) = p_{k-1}$ for $2 \leq k \leq m$

Wherein, p_1 is the highest prime less than the number $m(n+1)$

p_2 is the second highest prime less than the number $m(n+1)$ and $p_2 < p_1$.

p_3 is the third highest prime less than the number $m(n+1)$ and $p_3 < p_2 < p_1$.

…

p_{k-1} is the (k-1) th highest prime less than the number $m(n+1)$ and

$p_{k-1} < p_{k-2} < p_{k-3} < \cdots < p_3 < p_2 < p_1$.

$$f\,(v'_i) = 2^i, 1 \leq i \leq m$$

$$f(u_i) = \begin{cases} 2i+2j, & 2^{j-1}-j+1 \leq i \leq 2^j - j - 1 \ \ where\ j = 2,3,4, \ldots m \\ 2i+2m, & 2^{j-1}-j+1 \leq i \leq mn \qquad for\ m \geq 2\ and\ n \geq m \end{cases}$$

$f(v_i) = 2j+1, where\ j = 1,2,3 \ldots and\ 2j+1 \neq p_{k-1} \forall\ 2 \leq k \leq m$

(i.e., $f(v_i)$ maps all odd numbers from 3 to 2m(n+1)-1 except for $p_{k-1} \forall\ 2 \leq k \leq m$)

We observe that,

For $1 \leq i \leq mn, 1 \leq k \leq m$

For the edges $u'_k \, u_i$, $\gcd\big(f(u'_k\,), f(u_i)\big)$=1

For the edges $v'_k v_i$, $\gcd\big(f(v'_k\,), f(v_i)\big)$=1

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For the edges $u'_k v'_k$, $\gcd\big(f(u'_k), f(v'_k)\big)$=1

Therefore, the Shadow graph $D_m(B(n,n))$ of bistar graph $B(n,n)$ is a prime graph for $n \geq m$ and $m \geq 2$

Illustration 3.2.1.

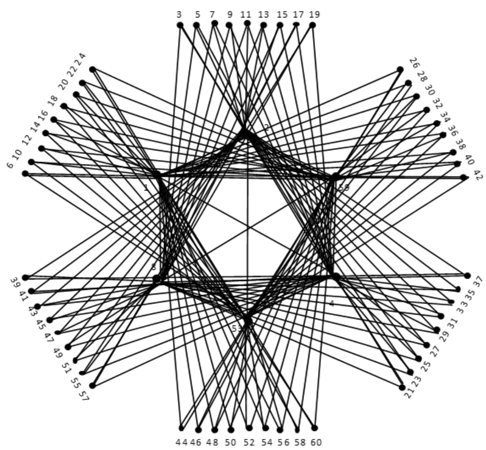

Figure 5. 3-Shadow graph $D_3(B(9,9))$ of bistar graph $B(9,9)$

3.3. m-Shadow graph $\mathbf{D_m(Spl(K_{1,n}))}$ of Splitting Star Graph $\mathbf{Spl(K_{1,n})}$

Theorem 3.3.

m-Shadow graph $D_m(Spl(K_{1,n}))$ of splitting star graph $Spl(K_{1,n})$ admits prime labeling for $n \geq m$ and $m \geq 2$.

Proof:

Let G denote a splitting star graph $Spl\big(K_{1,n}\big)$.

Let G_1, G_2, G_3,…, G_m denote m- facsimile copies of G**.**

In general, $\{ G_k \ / \ 1 \leq \ k \ \leq \ m\}$ denote kth- facsimile copy of G.

Let $V(G_k) \ = \ \{ u'_k, v'_k, \ u_i, v_i / \ 1 \leq \ k \ \leq \ m, \ (k-1)n+1 \leq \ i \ \leq \ kn\}$ constitute the vertex set of G_k

Let $E(G_k) \ = \ \{u'_k \, u_i / \ 1 \leq \ k \ \leq \ m, \ (k-1)n+1 \leq \ i \ \leq \ kn\} \cup$

$\{u'_k \, v_i / \ 1 \leq \ k \ \leq \ m, \ (k-1)n+1 \leq \ i \ \leq \ kn\} \cup$

$\{v'_k v_i / \ 1 \leq \ k \ \leq \ m, \ (k-1)n+1 \leq \ i \ \leq \ kn\}$ constitute the edge set of G_k.

Let $D_m(G)$ be the m- Shadow graph of G.

$V[D_m(G)] = \{ u'_k, v'_k, \ u_i, v_i \ / \ 1 \leq \ k \ \leq \ m, 1 \leq \ i \ \leq \ kn\}$ is the vertex set of $D_m(G)$.

$E[D_m(G)] = \{u'_k u_i / \ 1 \leq \ k \ \leq \ m, 1 \leq \ i \ \leq \ kn\} \cup$

$\{u'_k v_i / \ 1 \leq \ k \ \leq \ m, 1 \leq \ i \ \leq \ kn\} \cup$

$\{v'_k v_i / \ 1 \leq \ k \ \leq \ m, 1 \leq \ i \ \leq \ kn\}$ is the edge set of $D_m(G)$.

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$|V[D_m(G)]| = 2m(n+1)$.

We define a bijective function,

$f: V[D_m(G)] \rightarrow \{1,2,3, \ldots, 2m(n+1)\}$ given by,

$f\,(u_1') = 1, f\,(u_k') = p_{k-1}$ for $2 \leq\ k\ \leq\ m$

Wherein, p_1 is the highest prime less than the number $m(n+1)$

p_2 is the second highest prime less than the number $m(n+1)$ and $p_2 < p_1$.

p_3 is the third highest prime less than the number $m(n+1)$ and $p_3 < p_2 < p_1$.

…

p_{k-1} is the (k-1) th highest prime less than the number $m(n+1)$ and

$p_{k-1} < p_{k-2} < p_{k-3} < \cdots < p_3 < p_2 < p_1$.

$$f\,(v_i') = 2^i, 1 \leq\ i\ \leq\ m$$

$$f(u_i) = \begin{cases} 2i+2j, & 2^{j-1}-j+1 \leq i \leq 2^j - j - 1 \ \ where\ j = 2,3,4, \ldots m \\ 2i+2m, & 2^{j-1}-j+1 \leq i \leq mn \qquad for\ m \geq 2\ and\ n \geq m \end{cases}$$

$f(v_i) = 2j+1, where\ j = 1,2,3 \ldots and\ 2j+1 \neq p_{k-1} \forall\ 2 \leq k \leq m$

(i.e., $f(v_i)$ maps all odd numbers from 3 to $2m(n+1)-1$ except for $p_{k-1} \forall\ 2 \leq k \leq m$)

We observe that,

For $1 \leq i \leq mn$, $1 \leq k \leq m$

For the edges $u_k' u_i$, $\gcd\big(f(u_k'), f(u_i)\big)$=1

For the edges $u_k'\ v_i$, $\gcd\big(f(u_k'), f(v_i)\big)$=1

For the edges $v_k' v_i$, $\gcd\big(f(v_k'), f(v_i)\big)$=1

Therefore, the Shadow graph $D_m(Spl(K_{1,n}))$ of splitting star graph $Spl(K_{1,n})$ is a prime graph for $n \geq m$ and $m \geq 2$.

Illustration 3.3.1

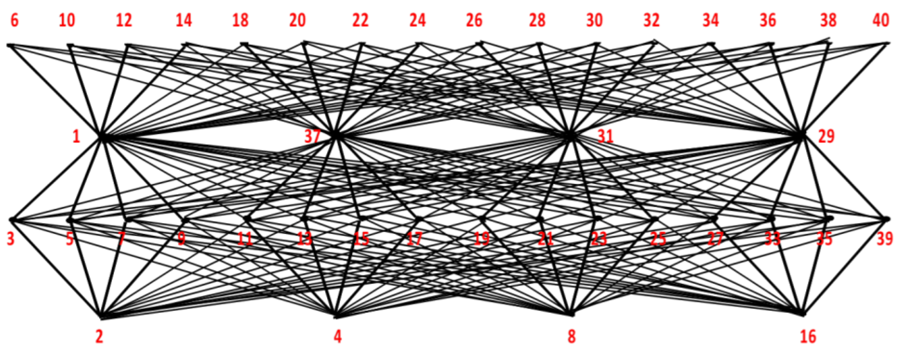

Figure 7. 4-Shadow graph $D_4(Spl(K_{1,4}))$ of splitting star graph $Spl(K_{1,4})$

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Conclusion

In this research article we conclude that the m-Shadow graph of some star related graphs such as Star graph $K_{1,n}$, Bistar graph $B(n,n)$ and Splitting star graph $Spl(K_{1,n})$ admit Prime Labeling for $n \geq m$ and m ≥ 2. For $n < m$, these graphs admit prime labeling for lesser values of m and as the value of m increases they do not admit prime labeling. For instance, $D_3(K_{1,2})$, $D_4(B(2,2))$, $D_4(Spl(K_{1,2}))$ are prime graphs but $D_{10}(K_{1,2})$, $D_5(B(2,2))$, $D_6(Spl(K_{1,3}))$ are not prime graphs. Hence in general we conclude that Star graph $K_{1,n}$, Bistar graph $B(n,n)$ and Splitting star graph $Spl(K_{1,n})$ admit Prime Labeling for $n \geq m$ and $m \geq 2$. The study of m-shadow graph of Star graph, Bistar graph and splitting star graph which can be used for future study and research work as it is very gripping for inspection and exploration in the field of graph theory.

References

[1] Broersma, H., Kretsch, D., Woeginger, G.J. 2009 Fully decomposable split graphs. In: Fiala, J., Kratochvil J., Miller, M (eds) Combinatorial Algorithms. IWOCA 2009. Lecture Notes in Computer Science, 5874. Springer, Berlin, Heidelberg

[2] Diefenderfer Nathan, Hastings Michael, Heath Levi N., Prawzinsky Hannah, Preston Briahna White Emily, Whittemore and Alyssa 2015 Prime vertex labelings of families of unicyclic graphs *Rose Hulman Undergraduate Mathematics Journal* **16** Issue 1 Article 14

[3] Hattingh, J.H., Joubert, E.J. 2014 Some bistar bipartite Ramsey numbers Graphs and Combinatorics **30** 1175-1181

[4] Joseph. A. Gallian 2019 A dynamic survey of graph labeling *The Electronic Journal of Combinatorics*

[5] Vaidya S K and Popat K M 2017 Energy of m-splitting and m-shadow graphs *Far East Journal of Mathematical Sciences* **102** Issue 8 pp 1571-78