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Labelings on a Special Digraph

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Abstract

In 1967, Rosa [7] was introduced by the concept of graph labeling. A graph labeling is an assignment of integers to the vertices (or) edges (or) both to certain conditions. If the domain of the mapping is the set of vertices (or edges), then the labeling is called a vertex labeling (or an edge labeling). In this paper we investigate the existence of Z_3 magic labeling, odd mean labeling, and signed product cordial labeling for the Cayley Digraph of 2-generated 2-groups.

Keywords: Cayley digraph, Graph labeling

1. Introduction

Cayley Digraph for the 2-generated 2-group:

In 2011, Bala et.al., introduced the concept of Cayley Digraph for the 2-generated 2-group and proved the existence of some graph labeling**.**

The Cayley digraph for the 2-generated 2-group Cay(G,(α,β)) has n vertices and 2n arcs. Let us denote the vertex set of Cay(G,(α,β)) as $V=\{v_1, v_2, v_3,...,v_n)$. Define the arc set of Cay(G,(α,β)) as $E(E_\alpha,E_\beta)$ where $E_\alpha=\{(v,\alpha v)|v\epsilon V\}$ and $E_\beta=\{(v,\beta v|v\epsilon V\}$. Denote the arcs in E_α as $\{g_\alpha(v_i)|v_i\epsilon V\}$ and E_β as $\{g_\beta(v_i)|v_i\epsilon V\}$. Clearly each vertex in Cay(G,(α,β)) has exactly two out going arcs out of which one arc is from the set E_α and another is from the set E_β.A group G is said to be a p-group if $o(G)=p^m$, $m\geq 1$. It is said to be 2-generated if the minimal generating set of G has exactly two elements. It is said to be a 2-group if p=2. We consider the groups of order $2^m=n$.

Cordial Labeling:

The concept of Cordial labeling was introduced by Cahit in 1967 [5].

A function $\phi: V\rightarrow \{0, 1\}$ is said to be a cordial labeling if each edge uv has the label $|\phi(u) - \phi(v)|$ such that the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most one and the number of edges labeled 0 and the number of edges labeled 1 differ by at most one.

Z_3-magic labeling:

The concept of Z_3 -magic labeling was introduced by Baskar Babujee et.al [3].

A graph G(V,E) is said to admit Z_3-magic labeling if there exists a function f from E onto the set { 1,2} such that the induced map f* on V defined by $f^*(v_i) = \sum f(e) \pmod 3 = k$, a constant, where $e = (v_iv_j) \in E$. A graph which admits Z_3-magic labeling is called Z_3-magic graph.

Odd mean labeling:

K. Manickam and M. Marudai [6] introduced by the concept of odd mean labeling

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A function $\emptyset$ is called an Odd mean labeling of a graph G with p vertices and q edges. If f is an injection from the vertices of G to the set{1,3,5,.., .2q-1} such that when each edge uv is assigned the label $[(\emptyset(u)+\emptyset(v))/2]$, then the resulting edge labels are distinct.

Signed Product Cordial Labeling:

The concept of signed product cordial labeling was introduced by Baskar Babujee etal., [4]

A vertex labeling of graph G, $f:V(G)\rightarrow\{-1,1\}$ with induced edge labeling $f^*: E(G)\rightarrow\{-1,1\}$ defined by $f^*(uv) = f(u) \times f(v)$ is called a signed product cordial labeling if $|v_f(-1) - v_f(1)|\leq 1$and $|e_f(-1) - e_f(1)|\leq 1$, where $v_f(-1)$ is the number of vertices labeled with `-1', $v_f(1)$ is the number of vertices labeled with 1, $e_f(-1)$ is the number of edges labeled with '-1' and $e_f(1)$ is the number of edges labeled with1.

Main Result

2. Z_3 -magic labeling:

In this section, we prove the existence of Z_3-magic labeling for Cayley digraph of 2-generated 2-groups by presenting algorithm.

Algorithm 2.1

Input: Cayley digraph for the 2-generated 2-groups

Procedure Z_3-magic labeling for Cayley digraph of 2-generated 2-groups

$V \leftarrow \{v_1, v_2, v_3, \ldots, v_n\}$

$E\ (E_{\alpha}, E_{\beta}) \leftarrow \{e_1, e_2, e_3, \ldots, e_{2n}\}$

$E_{\alpha}$is the set of all outgoing edges generated by α through each vertex of Cay(G,(α,β))

E_{β} is the set of all outgoing edges generated by β through each vertex of Cay(G,(α,β))

for i=1 to n do

$g_{\alpha}(v_i) \leftarrow 1$

$g_{\beta}(v_i) \leftarrow 2$

end for

end procedure

Theorem 2.1 The Cayley digraph associated with 2-generated 2-groups admits Z_3-magic labeling.

Proof

We know that, Cayley digraph for the 2-generated 2-groups has n vertices and 2n edges. Denote the edges set $E=\{e_1, e_2, e_3,\ldots,e_{2n}\}$ and vertices set $V=\{v_1,v_2,\ldots,v_n\}$. The edges are labeled by defining a function $\emptyset:E\rightarrow\{1, 2\}$ as given algorithm 2.1.

Clearly the number of edges labeled with '1' and '2' are as follows:

The number of edges labeled '1' is n and the number of edges labeled '2' is n.

Thus the number of edges labeled '1' and the number of edges labeled '2' different at most one.

In order to obtain the labels for the vertices, define the induced map $\emptyset^*: V \rightarrow \{0,1,2\}$ defined on $\emptyset^*(v_i)=\{\emptyset(g_\alpha(v_i))+\emptyset(g_\beta(v_i))\} \pmod 3 = 0$ for all i, where $\emptyset(g_\alpha(v_i))$ and $\emptyset(g_\beta(v_i))$ are incoming edges.

For $1 \leq i \leq n$

$\emptyset(v_i)= k= 0$, a constant

Hence, the Cayley digraph associated with 2-generated 2-groups admits Z_3-magic labeling.

Illustration: 2.1 Cayley digraph for the 2-generated 2-groups with its Z_3-magic labeling is given in figure.

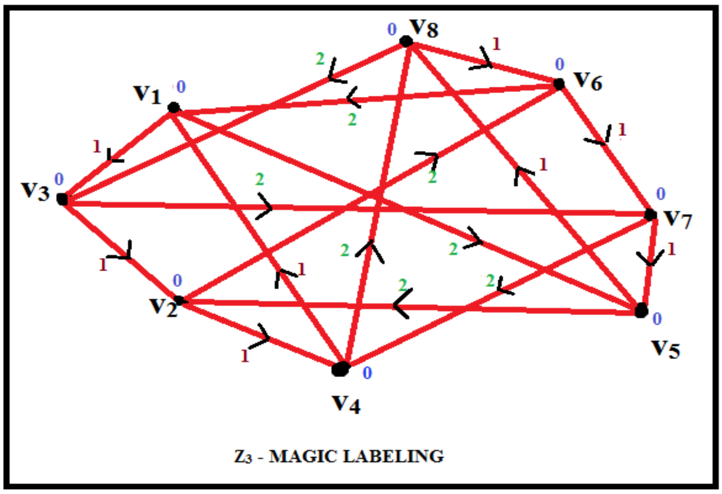

Fig 2.1: Z_3 -magic labeling

3. Odd mean labeling

In this section, we prove the existence of odd mean labeling for Cayley digraph of 2-generated 2-groups by presenting algorithm.

Algorithm 3.1

Input: Cayley digraph for the 2-generated 2-groups

Procedure Odd mean labeling for Cayley digraph of 2-generated 2-groups

$V \leftarrow \{v_1, v_2, v_3, \ldots, v_n\}$

$E\ (E_\alpha, E_\beta) \leftarrow \{e_1, e_2, e_3, \ldots, e_{2n}\}$

E_α is the set of all outgoing edges generated by α through each vertex of $Cay(G,(\alpha,\beta))$

E_β is the set of all outgoing edges generated by β through each vertex of $Cay(G,(\alpha,\beta))$

for $i=1$ to $\frac{n}{4}$ do

$g_\alpha(v_i) \leftarrow 2(1+i)$

end for

for $i=\frac{n}{4}+1$ to $\frac{n}{2}$ do

$g_\alpha(v_i) \leftarrow n-3-i$

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end for

for i=$\frac{n}{2}+1$to $\frac{n}{2}+2$ do

$g_\alpha(v_i)$←3n+1-2i

$g_\beta(v_i)$←2n-3i+2

end for

for i=$\frac{n}{2}+3$ to n do

$g_\alpha(v_i)$←2i-n+3

$g_\beta(v_i)$←3n-2i-3

end for

for i=1 to n/2 do

$g_\beta(v_i)$←n+2i-2

end for

end procedure

Theorem 3.1 The Cayley digraph associated with 2-generated 2-groups admits odd mean labeling.

Proof We know that, Cayley digraph for the 2-generated 2-groups has n vertices and 2n edges. Denote the edges set E= $\{e_1, e_2, e_3,\ldots,e_{2n}\}$ and vertices set $V=\{v_1,v_2,\ldots,v_n\}$. The edges are labeled by defining a function $\emptyset$:E→{0, 1, 2, …,2n-1} as given algorithm 3.1.

In order to obtain the labels for the vertices, define the induced map $\emptyset^*$:V→{1, 3, …,2n-1}such that for each vertices labeled by

$\emptyset(v_i)=\frac{\emptyset^*(g_\alpha(v_i))+\emptyset^*(g_\beta(v_i))}{2}$if $\emptyset^*(g_\alpha(v_i))+\emptyset^*(g_\beta(v_i))$ is even

$\emptyset(v_i)=\frac{\emptyset^*(g_\alpha(v_i))+\emptyset^*(g_\beta(v_i))+1}{2}$ if $\emptyset^*(g_\alpha(v_i))+\emptyset^*(g_\beta(v_i))$ is odd.

where $\emptyset(g_\alpha(v_i))$ and $\emptyset(g_\beta(v_i))$ are incoming edges.

For $1\leq i\leq n$

$\emptyset$ (v_i)= 2i-1 for all n.

Therefore the vertices are distinct.

Hence, the Cayley digraph associated with 2-generated 2-groups admits odd mean labeling.

Illustration 3.1 Cayley digraph for the 2-generated 2-groups with its odd mean labeling is given in figure.

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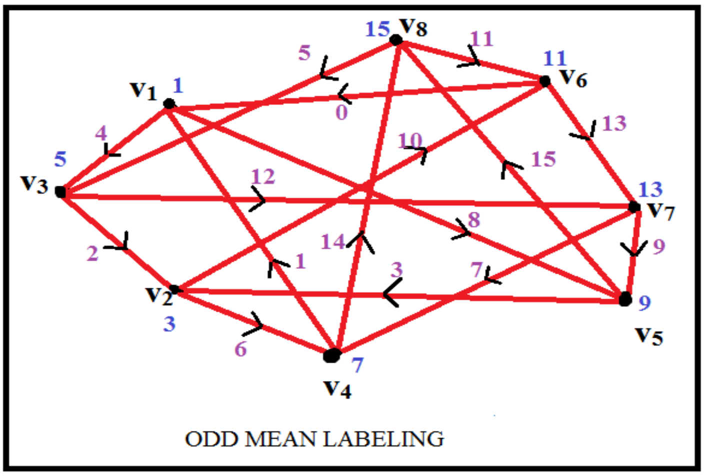

Fig 3.1 Odd mean labeling

4. Signed product cordial labeling

In this section, we prove the existence of signed product cordial labeling for Cayley digraph of 2-generated 2-groups by presenting algorithm.

Algorithm 4.1

Input: Cayley digraph for the 2-generated 2-groups

Procedure Signed product cordial labeling for Cayley digraph of 2-generated 2-groups

$V \leftarrow \{v_1, v_2, v_3, \ldots, v_n\}$

$E\ (E_{\alpha}, E_{\beta}) \leftarrow \{ e_1, e_2, e_3, \ldots, e_{2n}\}$

E_{α} is the set of all outgoing edges generated by α through each vertex of Cay(G,(α,β))

E_{β} is the set of all outgoing edges generated by β through each vertex of Cay(G,(α,β))

for $i=1$ to $\frac{n}{4}$ do

$$g_{\alpha}(v_i) \leftarrow \begin{cases} -1, & i \equiv 1 \pmod 2 \\ 1, & i \equiv 0 \pmod 2 \end{cases}$$

$$g_{\beta}(v_i) \leftarrow \begin{cases} 1, & i \equiv 1 \pmod 2 \\ -1, & i \equiv 0 \pmod 2 \end{cases}$$

end for

for $i=\frac{n}{4}+1$ to $\frac{n}{2}$ do

$$g_{\alpha}(v_i) \leftarrow 1$$

$$g_{\beta}(v_i) \leftarrow \begin{cases} -1, & i \equiv 1 \pmod 2 \\ 1, & i \equiv 0 \pmod 2 \end{cases}$$

end for

for $i=\frac{n}{2}+1$ to $\frac{n}{2}+2$ do

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$g_\alpha(v_i) \leftarrow -1$

$g_\beta(v_i) \leftarrow \begin{cases} -1, & i \equiv 1 \pmod 2 \\ 1, & i \equiv 0 \pmod 2 \end{cases}$

end for

for $i = \frac{n}{2} + 3$ to n do

$g_\alpha(v_i) \leftarrow 1$

$g_\beta(v_i) \leftarrow -1$

end for

end procedure

Theorem 4.1 The Cayley digraph associated with 2-generated 2-groups admits signed product cordial labeling.

Proof

We know that, Cayley digraph for the 2-generated 2-groups has n vertices and 2n edges. Denote the edges set E= $\{e_1, e_2, e_3, \ldots, e_{2n}\}$ and vertices set $V=\{v_1, v_2, \ldots, v_n\}$. The edges are labeled by defining a function $\emptyset: E \rightarrow \{-1, 1\}$ as given algorithm 4.1.

Clearly the number of edges labeled with '-1' and '1' are as follows:

The number of edges labeled '-1' is n and the number of edges labeled '1' is n.

Thus the number of edges labeled '-1' and the number of edges labeled '1' different at most one.

In order to obtain the labels for the vertices, define the induced map$\emptyset^*: V \rightarrow \{-1,1\}$ defined on $\emptyset(v_i) = \prod \emptyset^*(g_\alpha(v_i))$ is bijective. where $\emptyset(g_\alpha(v_i))$ and $\emptyset(g_\beta(v_i))$ are incoming edges.

For $1 \leq i \leq n$

$\emptyset(v_i) = \begin{cases} 1, & i \equiv 1 \pmod 2 \\ -1, & i \equiv 0 \pmod 2 \end{cases}$

Clearly the number of vertices labeled with '-1' and '1' are as follows:

Vertex label	-1	1
For all 'n'	$\frac{n}{2}$	$\frac{n}{2}$

Thus the number of vertices labeled '-1'and the number of vertices labeled '1' different at most one.

Hence, the Cayley digraph associated with 2-generated 2-groups admits signed product cordial labeling.

Illustration 4.1: Cayley digraph for the 2-generated 2-groups with its signed product cordial labeling is given in figure.

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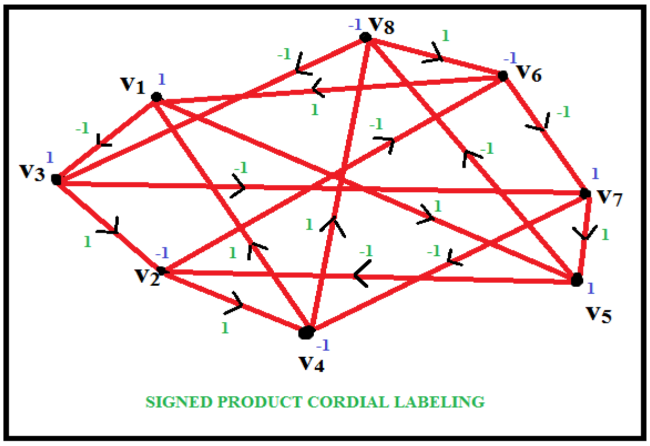

Fig 4.1 Signed product cordial labeling

Conclusion

In this paper, we proved existence of Z_3 magic labeling, odd mean labeling, and signed product cordial labeling for the Cayley Digraph of 2-generated 2-groups.

References

1. S.B. Akers, and B. Krishnamurthy, A group theoretic model for symmetric Interconnection networks, IEEE Trans. Comput.**38** (1989), 555-566.
2. F. Annexstein, M. Baumslag and A.L Rosenberg, Group action graphs & Parallel architectures. SIAM J. Comput.**19** (1990), 544-569.
3. E. Bala and K. Thirusangu, E-Cordial and Z_3-Magic Labelings in Extended Triplicate Graph of a Path, Gen. Math. Notes, 5 (2) (2011), 8-23.
4. J. Baskar Babujee, and L. Shobana, On Signed Product Cordial Labeling, Applied Mathematics, **12**(2011) 1525-1530.
5. J.A. Gallian.,A dynamic survey of graph labeling,The Electronic Journal of Combinatorics**18** (2018) #DS6.
6. K.Manickam and M.Marudai, Odd Mean Labeling of Graphs, Bulletin of Pure and Applied Sciences, 25 E(1) (2006), 149-153.
7. A. Rosa, On certain valuations of the vertices of a graph, Theory of graphs (Internat.Symposium, Rome, July 1996), Gordon and Breach, N.Y. and Dunod Paris (1967) 349-355.
8. K. Thirusangu., K. Atulyanagar, R. Rajeswari, Labelings in Cayley's digraphs, European Journal of Combinatorics, **32**(2011) 133-139.
9. K. Thirusangu, E. Bala, Magic and antimagic labeling in Cayley digraphs of 2-generated p-groups, International Journal of Mathematics Research**3**(**3**) (2011) 221-229.
10. R. Vasuki and A. Nagarajan, Odd mean labeling of the graphs $P_{a,b}$, P^b_a and P^b_{2a}, Kragujevac Journal of Mathematics**36** 1(2012), 141-150.