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Construction of fuzzy relational map based on hesitant fuzzy sets with an application to model the causal influence of self-beliefs related to emotional intelligence on leadership

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Abstract

Fuzzy Relational Map is a relational structure between two disjoint fuzzy sets. The relation between any two fuzzy sets is represented with a fuzzy set which denotes the edge strength of the relation between the nodes of the sets. This paper proposes a new approach to construct a fuzzy relational map with hesitant fuzzy set, an extension of regular fuzzy set. The hesitant fuzzy relational map is way more effective in capturing the uncertainty of the problem and the hesitancy of the expert than the regular fuzzy relational map. Effective leadership is the guiding force of a civilised society and high-performance institutions. Emotional intelligence capabilities are most closely linked to effective leadership. In this paper, the influence of beliefs and values related to emotional intelligence on effective leadership is studied with Hesitant Fuzzy Relational Map.

Key words

Fuzzy relational map, hesitant fuzzy set, beliefs, emotional intelligence, leadership

1. Introduction

Fuzzy Relational Map (FRM) is a soft computing technique to model the human knowledge concerning the complex and dynamic real-world problems [12]. Like FCM, FRM is also a combination of fuzzy logic and neural network that can model complex systems effectively [4]. The dynamics of FRM is similar to that of FCM except that the causal relations are between two disjoint fuzzy sets involved in FRM [12]. Since the onset of Fuzzy set theory, several extensions of fuzzy sets have come to existence depending on the need and necessity of the problem. Hesitant fuzzy set is one of the extensions of the fuzzy set that takes into account the imprecise and hesitant information of experts [13]. In this paper, new approach is proposed by combining the hesitant fuzzy set (HFS) theory with FRM to deal with a special kind of uncertainty, that is, hesitancy. The hesitant fuzzy relational map (HFRM), that is FRM based on HFS, is effective in dealing with the issues where the experts have varying opinions. The hesitant fuzzy influence of the relationships among the factors can be quantified by the proposed method.

2. Hesitant Fuzzy Sets

The concept of hesitant fuzzy set (HFS) was introduced by Torra and Narukawa, as one of the extensions of Zadeh (1965)'s fuzzy set, in 2009 [9,10,13]. The hesitant fuzzy set allows the membership degree of an element belonging to a set is denoted by several possible values. Compared to other extensions of fuzzy sets, HFS can express the hesitant information more comprehensively besides the uncertainty. Xu and Xia (2011) defined the concept of hesitant fuzzy element (HFE) which can be considered as the basic unit of an HFS, a simple tool used to express the experts' hesitant opinion [13]. The fuzzy set theory relies on linguistic variables that describes the information qualitatively [15]. The fuzzy linguistic variables that are described with single term, is inadequate to evaluate language variants involving hesitation. In order to resolve this kind of issue Hesitant Fuzzy Linguistic Term Sets

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(HFLTSs) have been proposed as a solution by Rodriguez et al. in 2012 [6].

Let X be a fixed set, a hesitant fuzzy set (HFS) on X is defined in terms of a function that returns a subset of [0,1], that is $h_A: X \rightarrow \{[0,1]\}$ [9]. A hesitant fuzzy set is denoted by $A = \{x,\ h_A(x) | x \in X\}$ where $h_A(x)$, the hesitant fuzzy element, is a set of some values from [0,1], denoting the possible membership degrees of the element $x \in X$ to the set A. A hesitant fuzzy linguistic term set (HFLTS), denoted by H_s, is an ordered finite subset of consecutive linguistic terms of the linguistic term set S. The envelope of an HFLTS $env(H_S)$, is a linguistic interval with its upper bound and lower bound as limits. Liu and Rodríguez (2014) proposed a new method to obtain a fuzzy envelope for HFLTS. This is a trapezoidal fuzzy membership function (TFMF) obtained by aggregating the fuzzy membership functions of the linguistic terms of the HFLTS according to their relevance [5],[6].

2.1. Hesitant Fuzzy Relational Map Model

Like FRM, HFRM is also a soft computing technique to model the human knowledge in the decision-making process. The HFRM represent the unstructured knowledge through causalities expressed in Hesitant Fuzzy Elements (HFEs). The experts provide information represented by HFEs to describe the influence of one variable on another. An illustrative example of a HFRM is shown in Figure1.

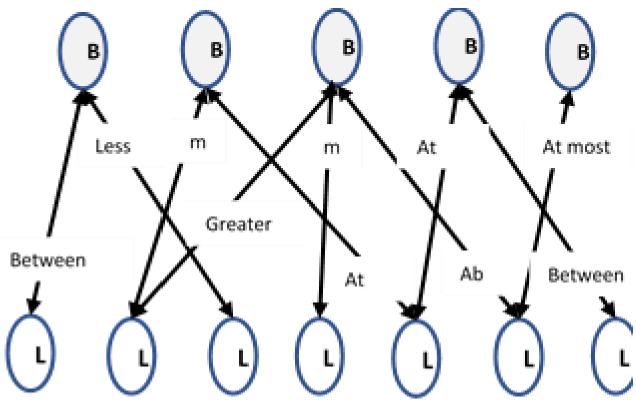

Figure 1: A Hesitant Fuzzy Relational Map Model

3. Description of the Problem

The greatest challenge of our generation of leadership is to overcome the misconceptions of control and result-oriented approach and move to new understanding to include values and spiritual qualities. The era where people believed that capacity, intelligence and power alone is required to lead an organisation is gone. The recent research on successful and effective leadership focus on emotional and spiritual intelligence and the beliefs that influence such intelligences. Recent researches have studied the idea that emotional intelligence is the central theme of successful performance in any sphere of life. In particular, leadership is a powerful role that demands a high-level intelligence in handling one's own emotions and that of others. The emotional impact of a leader is the least discussed fact but the most influential in every sphere of performance of the organisation. Emotional leadership is the spark that ignites a company's performance, creating a bonfire of success or a landscape of ashes.

Every individual's actions, feelings, behaviours and abilities are results of one's core beliefs [1]. The beliefs make up the cognitive component of human emotions. The dormant beliefs that are stored in the rational brain contribute to emotions and the actions related to it. Emotions are functional tools that are helpful in identifying one's beliefs that operate them. Studying the relationship between emotional intelligence and principles of leadership gives the criteria of the successful leadership. In this paper, an attempt is made to study the intrinsic relationship between the beliefs that underlie emotional intelligence and its influence on leadership.

David Ryback, Ph.D. in his book, "Putting Emotional Intelligence to Work: Successful Leadership Is More Than IQ" describes the Seven Core Qualities of Leadership [7]. 1. Strategic Planning: The art of creating specific professional strategies, employing them, and assessing the results in regard to long-term goals. 2. Communication and Alignment: An array of characteristics necessary to successfully achieve a mission, involving a larger number of people working together. 3. Team Building: Supporting individual empowerment, a sense of egalitarian inclusion and a team approach. 4. Continuous Learning: A commitment to continuing education. 5. Dynamic Accountability: Trust, maturity and professional standards to consider the means as equal as ends. 6. Systemic Results: Achievement from all facets and levels of the organizational operation. 7. Actualized Integrity: An approach of openness and integrity of character.

The domains of beliefs pertaining to emotional intelligence are taken from Chapter 6 of Lynn's The EQ

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Difference [1]: 1. Self-Awareness and Self-Control: Understanding oneself and managing emotions productively. 2. Empathy: Understanding the views of others. 3. Social Expertness: Building genuine relationships with care and concern and expressing conflict in healthy ways. 4. Personal Influence: Skills to positively lead and inspire others. 5. Mastery of Purpose and Vision: Authenticity and a life based on deeply felt purposes and values.

Notation	Factors of Domain Space	Notation	Factors of Range Space
B_1	Self-awareness and Self-control	L_1	Strategic Planning
B_2	Empathy	L_2	Communication and alignment
B_3	Social Expertness	L_3	Team building
B_4	Personal Influence	L_4	Continuous Learning
B_5	Mastery of Purpose and Vision	L_5	Dynamic accountability
		L_6	Systemic Results
		L_7	Actualised Integrity

Table 1: Factors of HFRM

4. Construction and Analysis of the Problem using HFRM

Step 1: The variables of the problem understudy are chosen. These variables are taken to be the nodes of the HFRM that describes the complex structure that is to be modelled.

Step 2: The relationships between nodes are identified based on the experts' domain knowledge. Using the linguistic evaluations that define the causal relations among concepts graph-based HFRM model is constructed.

Step 3: The linguistic estimations are transformed to HFLTS according to the linguistic term set such as at least medium, between very low and medium, and lower than very much.

Step 4: The membership functions of HFLTS are aggregated using OWA method and a trapezoidal fuzzy membership function (TFMF) $\hat{A} = (a, b, c, d)$, is obtained.

Step 5: The TFMFs are converted into crisp values using defuzzification method.

Step 6: The values obtained in the above step is taken as the causal relation between the variables. These causal values of the concepts constitute the adjacency matrix of the HFRM.

Step 7: Initial state vector of the concepts is passed through the relational matrix in an iterative manner until the fixed point is reached.

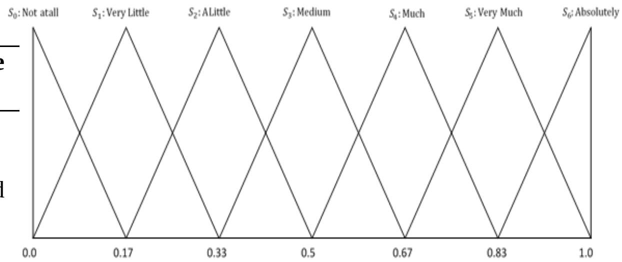

Figure 2: Membership functions of Hesitant Linguistic Term Sets

The following linguistic term set, $S = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6\}$ is adopted to describe the causal relations between the concepts. An illustrative example of hesitant linguistic terms and their corresponding membership values are given in figure 2. In regular fuzzy set approach, the experts are usually provided with single linguistic terms to express their opinions. There may arise situations where the experts might hesitate among different linguistic terms to express their preferences. In circumstances, where high degree of uncertainty is involved the experts would like to have a greater flexibility to propose their views in linguistic expressions. This context needs a space for the experts express their hesitancy with proper linguistic expressions. Hesitant Fuzzy Linguistic Term Sets (HFLTSs) [5,6] provide rules to generate linguistic expressions that are nearly close to humans' cognitive thinking. The causal relations among the factors are defined with the linguistic evaluations of the experts with the help of HFLTSs. An example of an HFLTS for the linguistic variable $v = Greater\ than\ medium$ is given as follows. $H_S(v) = \{Greater\ than\ medium\} = \{Much, Very\ Much, Absolutely\} = \{S_4, S_5, S_6\}$. The HFRM model, constructed with the HFLTS provided by the experts, is given in table 2.

	L_1	L_2	L_3	L_4	L_5	L_6	L_7
B_1	$\{s_4, s_5, s_6\}$	$\{s_4, s_5, s_6\}$	-	-	$neg\{s_0, s_1, s_2\}$	-	$\{s_4, s_5, s_6\}$

B_2	-	$\{s_2, s_3\}$	$\{s_3\}$	-	$neg\{s_5, s_6\}$	$neg\{s_5, s_6\}$	-
B_3	-	$\{s_4, s_5\}$	$\{s_3, s_4,\}$	-	-	$neg\{s_5, s_6\}$	-
B_4	-	-	$\{s_4, s_5\}$	$\{s_4, s_5\}$	$\{s_2, s_3\}$	$\{s_5, s_6\}$	$\{s_4, s_5, s_6\}$
B_5	s_4, s_5, s_6	-	$neg\{s_5, s_6\}$	$neg\{s_5, s_6\}$	$\{s_4, s_5, s_6\}$	$\{s_4, s_5, s_6\}$	-

Table 2: Hesitant Linguistic Term Sets

Consider the set of linguistic expressions $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$. Let $H_S = \{s_i, s_{i+1}, \ldots, s_j\}$ be a HFLTS, where $s_k \in S = \{s_0, s_1, \ldots\ldots, s_g\}, k \in \{i, \ldots\ldots, j\}$. Let $T = (a_L^i, a_M^i, a_L^{i+1}, a_R^i, a_M^{i+1}, \ldots\ldots, a_L^j, a_R^{j-1}, a_M^j, a_R^j)$ be set of points of all membership functions of the linguistic terms in the HFLTS, that is the set of elements to be aggregated. According to the linguistic expressions considered in this study, $a_L^{i+1} = a_R^i = a_M^{i+1}$ for $i = 1,2, \ldots\ldots, g-1$. The linguistic expressions are converted into fuzzy envelopes using Trapezoid Fuzzy Membership Functions (TFMFs). The parameters of the trapezoidal fuzzy membership function $A = T\ (a, b, c, d)$ are computed as follows [3]:

$$a = \min\{a_L^i, a_M^i, a_M^{i+1}, \ldots\ldots, a_M^j, a_R^j\} = a_L^i$$

$$d = \max\{a_L^i, a_M^i, a_M^{i+1}, \ldots\ldots, a_M^j, a_R^j\} = a_R^j$$

The parameters b and c are computed using OWA operator [14].

i. If $i + j$ is even then

$$b = OWA_{W^2}\left(a_M^i, a_M^{i+1}, \ldots\ldots, a_R^{\frac{i+j-1}{2}}\right) \text{ and}$$

$$c = OWA_{W^1}\left(a_M^j, a_M^{j-1}, \ldots\ldots, a_R^{\frac{i+j+1}{2}}\right)$$

ii. If $i + j$ is odd then

$$b = OWA_{W^2}\left(a_M^i, a_M^{i+1}, \ldots\ldots, a_R^{\frac{i+j}{2}}\right) \text{ and}$$

$$c = OWA_{W^1}\left(a_M^i, a_M^{i+1}, \ldots\ldots, a_R^{\frac{i+j}{2}}\right)$$

The comparative linguistic expressions of the causal relations of HFRM are transformed into trapezoidal fuzzy membership functions are as follows.

$\tilde{R}_{11} = T(0.5\ , 0.85, 1, 1)$
$\tilde{R}_{12} = T(0.5\ , 0.85, 1, 1)$
$\tilde{R}_{15} = T(0, 0, 0.15, 0.5)$
$\tilde{R}_{17} = T(0.5\ , 0.85, 1, 1)$
$\tilde{R}_{22} = T(0.17\ , 0.33, 0.5, 0.67)$
$\tilde{R}_{23} = T(0.33\ , 0.5, 0.5, 0.67)$
$\tilde{R}_{25} = T(0.67, 0.97, 1, 1)$
$\tilde{R}_{26} = T(0.67, 0.97, 1, 1)$
$\tilde{R}_{32} = T(0.5\ , 0.67, 0.83, 1)$
$\tilde{R}_{33} = T(0.33\ , 0.5, 0.67, 0.83)$
$\tilde{R}_{36} = T(0.67, 0.97, 1, 1)$
$\tilde{R}_{43} = T(0.5\ , 0.67, 0.83, 1)$
$\tilde{R}_{44} = T(0.5\ , 0.67, 0.83, 1)$
$\tilde{R}_{45} = T(0.17\ , 0.33, 0.5, 0.67)$
$\tilde{R}_{46} = T(0.67\ , 0.97, 1, 1)$
$\tilde{R}_{47} = T(0.5\ , 0.85, 1, 1)$
$\tilde{R}_{51} = T(0.5\ , 0.85, 1, 1)$
$\tilde{R}_{53} = T(0.67, 0.97, 1, 1)$
$\tilde{R}_{54} = T(0.67, 0.97, 1, 1)$
$\tilde{R}_{55} = T(0.5\ , 0.85, 1, 1)$
$\tilde{R}_{56} = T(0.5\ , 0.85, 1, 1)$

Trapezoidal fuzzy membership functions are converted into crisp values by weighted average defuzzification method within the interval [−1, 1]. The crisp values obtained denote the edge weights of the HFRM. The adjacency matrix of the concepts is taken to be the relational matrix of the HFRM and is given in the table 3. The obtained HFRM model is studied for different input vectors and the equilibrium state values of the factors are obtained.

	L_1	L_2	L_3	L_4	L_5	L_6	L_7
B_1	0.8873	0.8873	0	0	-0.4192	0	0.8873
B_2	0	0.5010	0.5289	0	-0.9313	-0.9313	0
B_3	0	0.7959	0.6424	0	0	-0.9313	0
B_4	0	0	0.7959	0.7959	0.5010	0	0.8873
B_5	0.8873	0	-0.9313	-0.9313	0.8873	0.8873	0

Table 3: Relational Matrix of HFRM

5. Results and Discussion

The input vectors are passed through the dynamical system of HFRM and the resultant vector is thresholded using Sigmoid activation function and updated until the fixed point is obtained [4,12]. In case of FRM, the fixed point is a pair of vectors corresponding to the domain space and range space. For all possible input vectors, the fixed points are presented in table 4. The concepts B_1(Self-awareness and self-control), B_4(Personal Influence) and B_5(Mastery of purpose and Vision) behave in the same manner and turn ON maximum components in resultant state vectors. The concepts B_2(Empathy) and B_3 (Social Expertness) influence the system in the same way. The intrinsic belief domains of emotional intelligence, namely empathy and social expertness turns ON all the principles of leadership except two factors that is required for producing results. The rest of the belief domains also turn ON all the attributes except continuous learning.

Sl. No	Input Vector	Fixed point
1	{1 0 0 0 0}	{(1 0 1 1 1),(1 1 1 0 1 1 1)}
2	{0 1 0 0 0}	{(1 1 1 1 0),(1 1 1 1 0 0 1)}
3	{0 0 1 0 0}	{(1 1 1 1 0),(1 1 1 1 0 0 1)}
4	{0 0 0 1 0}	{(1 0 1 1 1),(1 1 1 0 1 1 1)}
5	{0 0 0 0 1}	{(1 0 1 1 1),(1 1 1 0 1 1 1)}

Table 4: Fixed points of HFRM

6. Conclusion

In this paper, the relational model of emotional intelligence and leadership based on hesitant fuzzy sets is constructed. Also, the causal relationships between the belief domains of emotional intelligence and the principles of leaderships are identified. The hesitant linguistic term sets enable the experts feel free to express their hesitancy and be flexible in their assessment. In HFRM, the loss of information is reduced as all the experts' opinions are taken into account and quantified using comprehensive method of aggregation and mathematical calculations. Hence, this HFRM model is relatively accurate and genuine in identifying and evaluating relationships among the factors compared to regular FRM model as it includes the hesitancy of the experts. The study of the relationship between the belief domains of emotional intelligence and principles of effective leadership using HFRM gives the realistic view of the causal influence between these two constructs by quantifying the uncertainty without loss of much information.

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