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General Position Problem of Middle, Splitting and Shadow Graph of Path, Cycle and Star

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Abstract

For a given graph *G*, the general position problem is to find the *general position number* of *G* which is the maximum number of vertices of *G* such that no three vertices lie on a common geodesic and is denoted by $gp(G)$. In this paper, the *general position number* for Middle, Splitting and Shadow graph of path, cycle and star are computed.

AMS Subject Classification: 05C12

Keywords: General position problem, Middle graph, Splitting Graph, Shadow graph

1 Introduction

A set $S \subseteq V(G)$ is called a general position set of *G* if *S* contains no three vertices of *G* that lie on a common geodesic. The general position number of *G* is the cardinality of the maximum general position set of *G* and is denoted by $gp(G)$. The classical no-three-in-line problem was first introduced by Dudeney [1] and the general position problem was first introduced in [4] motivated by the General Position Subset Selection Problem in Discrete Geometry [3, 8] which is a problem to find a largest subset of point in general position. The general position problem was also proven to be NP-complete in [4]. In this paper, we compute the *gp-number* of Middle, Splitting and Shadow graph of Path, Cycle and Star.

2 Preliminaries

In this paper, we use simple connected graphs. The shortest path between any two vertices *u* and *v* of a graph *G* is known as *geodesic* or *isometric path.* A general position set is a set $S \subseteq V(G)$ such that no three vertices of *S* lie on a common isometric path in *G*. A *max-gp set* of *G* is a general position set of maximum cardinality and this cardinality is called the *gp-number* of *G* and is denoted by $gp(G)$. In [4], it is proved that $gp(P_n) = 2$ for $n \geq 2$ and $gp(C_3) = 3$, $gp(C_4) = 2$ and $gp(C_n) = 3$ for $n \geq 5$.

3 General Position Number of Middle Graph of P_n, C_n and $K_{1,n}$

Definition 3.1 (Middle Graph) The *Middle graph* of a connected graph *G* is denoted by $M(G)$ and is a graph whose vertex set is $V(G) \cup E(G)$ and any two vertices in $V[M(G)]$ are adjacent if

- **(i)** They are adjacent edges of *G* or
- **(ii)** One is a vertex of *G* and the other is an edge incident with it.

Theorem 3.2 Let $M(P_n)$ be the Middle graph of a path $P_n, n \geq 2$. Then, $gp[M(P_n)] = n$.

Proof. Let $v_1, v_2, \dots v_n$ and $u_1, u_2, \dots u_{n-1}$ be the vertices of $M(P_n)$ corresponding to the vertices and edges of P_n respectively.

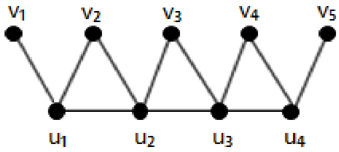

Figure 4.1 Middle graph of path P_5

Consider the set $S = \{v_1, v_2, \dots v_n\}$. Clearly, this set is a general position set of $M(P_n)$. Hence, $gp[M(P_n)] \geq n$. If T is any general position set of $M(P_n)$ such that $T \cap S \neq \emptyset$ then $|T \cap \{u_1, u_2, \dots u_{n-1}\}| \leq 1$. If $u_i \in T$, $1 \leq i \leq n-1$, then obviously either $T \cap \{v_1, v_2, \dots v_i\} = \emptyset$ or $T \cap \{v_{i+1}, v_{i+2}, \dots v_n\} = \emptyset$. Hence, $gp[M(P_n)] \leq n$. This completes the proof.

Theorem 3.3 Let $M(C_n)$ be the Middle graph of a cycle $C_n, n \geq 3$. Then, $gp[M(C_n)] = n$.

Proof. Let $v_1, v_2, \dots v_n$ and $u_1, u_2, \dots u_n$ be the vertices of $M(C_n)$ corresponding to the vertices and edges of C_n respectively. Consider the set $S = \{v_{1,} v_2, \dots v_n\}$. Hence, $gp[M(C_n)] \geq n$ follows from the fact that S is a general position set.

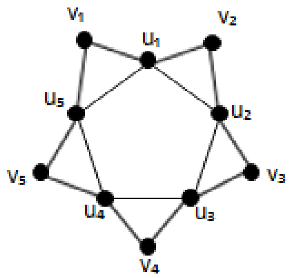

Figure 4.2 Middle graph of cycle C_5

If T is any general position set of $M(C_n)$ then $|T \cap \{u_1, u_2, \dots u_n\}| \leq 3$, since $gp(C_n) = 3$ for $n \geq 5$. Further we observe that if $u_i \in T$, then either $v_i \notin T$ or $v_{i+1} \notin T$, $1 \leq i \leq n$. Hence, $gp[M(C_n)] \leq n$, which completes the proof.

Theorem 3.4 Let $M(K_{1,n})$ be the Middle graph of a star $K_{1,n}, n \geq 2$. Then, $gp\left[M\left(K_{1,n}\right)\right] = n + 1$.

Proof. Let $v, v_1, v_2, \dots v_n$ and $u_1, u_2, \dots u_n$ be the vertices of $M(K_{1,n})$ corresponding to the vertices and edges of $K_{1,n}$ respectively.

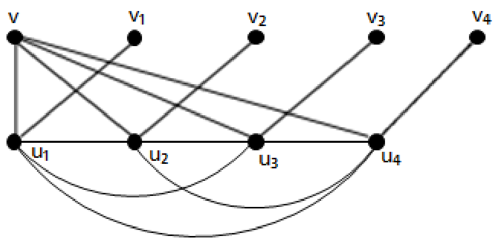

Figure 4.3 Middle graph of star $K_{1,4}$

Consider the set $S = \{v, v_1, v_2, \dots v_n\}$. It follows that the set S is a general position set of $M(K_{1,n})$. Hence, $gp\left[M\left(K_{1,n}\right)\right] \geq n + 1$. Observe that no more vertices can be added to S since each u_i lies in $(v - v_i)$ and $(v_i - v_{i+1})$ geodesic, $1 \leq i \leq n$. Hence, $gp\left[M\left(K_{1,n}\right)\right] = n + 1$, which completes the proof.

4 General Position Number of Splitting Graph of P_n, C_n and $K_{1,n}$

Definition 4.1 (Splitting Graph) The *Splitting graph* of a connected graph G is obtained by adding a new vertex u_i for each vertex $v_i \in V(G)$ and joining u_i to all $v_j \in V(G)$ that are adjacent to v_i. It is denoted by $S(G)$.

Theorem 4.2 Let $S(P_n)$ be the Splitting graph of a path $P_n, n \geq 3$. Then, $gp[S(P_n)] = 4$.

Proof. Let $v_1, v_2, \dots v_n$ and $u_1, u_2, \dots u_n$ be the vertices of $S(P_n)$ corresponding to the vertices of P_n and the newly added vertices respectively.

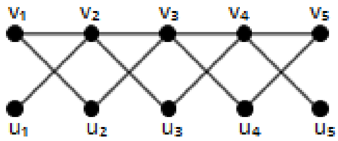

Figure 4.4 Splitting graph of path P_5

For $n = 3$, $\{v_1, v_3, u_1, u_3\}$ is the required general position set. Now, let $n \geq 4$. Consider the set $S = \{u_1, u_2, u_3, u_4\}$. We observe that S is a general position set of $S(P_n)$. Hence, $gp[S(P_n)] \geq 4$. Consider the three isometric paths $\mathcal{P}_1 = v_1 - v_2 - \cdots v_n$, $\mathcal{P}_2 = u_1 - v_2 - u_3 - v_4 \dots - u_n / \mathcal{P}_2 = u_1 - v_2 - u_3 - v_4 \dots - v_n$ if n is odd/even and $\mathcal{P}_3 = v_1 - u_2 - v_3 - u_4 \dots v_n / \mathcal{P}_3 = v_1 - u_2 - v_3 - u_4 \dots u_n$ if n is odd/even in $S(P_n)$. Let T be any general position set of $S(P_n)$. Suppose $|T| > 4$, then obviously at least three vertices of T will lie on any one of the isometric paths $\mathcal{P}_1$, $\mathcal{P}_2$ or $\mathcal{P}_3$, which is a contradiction since $gp(P_n) = 2,\ n \geq 2$. Hence, $gp[S(P_n)] \leq 4$. This completes the proof.

Theorem 4.3 Let $S(C_n)$ be the Splitting graph of a cycle $C_n, n \geq 3$. Then, $gp[S(C_n)] = n$.

Proof. Let $v_1, v_2, \dots v_n$ and $u_1, u_2, \dots u_n$ be the vertices of $S(C_n)$ corresponding to the vertices of C_n and the newly added vertices respectively. Consider the set $S = \{u_{1,}u_2, \dots u_n\}$. Clearly, this set is a general position set of $S(C_n)$. Hence, $gp[S(C_n)] \geq n$. Let T be any general position set of $S(C_n)$ then $|T \cap \{v_1, v_2, \dots v_n\}| \leq 3$, since $gp(C_n) = 3$ for $n \geq 5$. Further we observe that if $v_i \in T$, then either $u_i \notin T$ or $u_{i+1} \notin T$, $1 \leq i \leq n$. Hence, $gp[S(C_n)] \leq n$, which completes the proof.

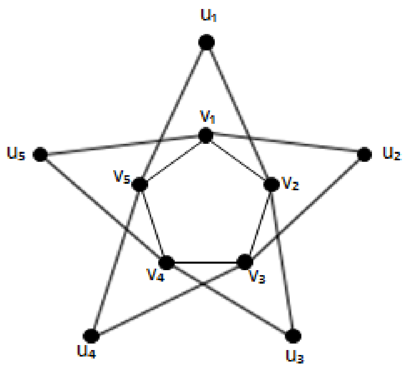

Figure 4.5 Splitting graph of cycle C_5

Theorem 4.4 Let $S(K_{1,n})$ be the Splitting graph of a star $K_{1,n}, n \geq 2$. Then, $gp[S(K_{1,n})] = 2n$.

Proof. Let $v, v_1, v_2, \dots v_n$ and $u, u_1, u_2, \dots u_n$ be the vertices of $S(K_{1,n})$ corresponding to the vertices of $K_{1,n}$ and the newly added vertices respectively.

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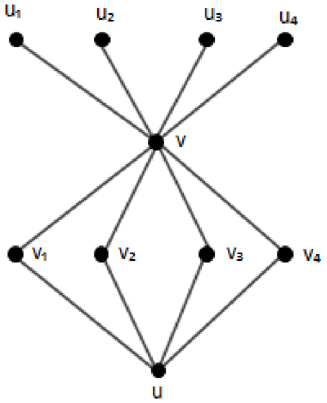

Figure 4.6 Splitting graph of star $K_{1,4}$

Consider the set $S = \{v_1, v_2, \dots v_n, u_1, u_2, \dots u_n\}$. This set is a general position set of $S(K_{1,n})$. Hence, $gp[S(K_{1,n})] \geq 2n$. Consider the isometric paths $\mathcal{P}_i = u_i - v - v_i - u$, $1 \leq i \leq n$ in $S(K_{1,n})$. Let T be any general position set of $S(K_{1,n})$. Suppose $|T| > 2n$, then at least three vertices of T will lie on any one of the isometric paths $\mathcal{P}_i$, which is a contradiction. Hence, $gp[S(K_{1,n})] \leq 2n$. This completes the proof.

5 General Position Number of Shadow Graph of P_n, C_n and $K_{1,n}$

Definition 5.1 (Shadow Graph) The *Shadow graph* of a connected graph G is constructed from G by taking two copies of G namely G and G' and by joining each vertex v in G to the neighbors of the corresponding vertex u in G'. It is denoted by $D_2(G)$

Theorem 5.2 Let $D_2(P_n)$ be the Shadow graph of a path $P_n, n \geq 3$. Then, $gp[D_2(P_n)] = 4$.

Proof. Let $v_1, v_2, \dots v_n$ and $u_1, u_2, \dots u_n$ be the vertices of $D_2(P_n)$ corresponding to the vertices of P_n and the newly added vertices respectively.

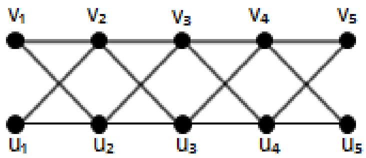

Figure 4.7 Shadow graph of path P_5

Consider the set $S = \{v_1, v_n, u_1, u_n\}$. We can clearly observe that S is a general position set of $D_2(P_n)$. Hence, $gp[D_2(P_n)] \geq 4$. Consider the isometric paths $\mathcal{P}_1 = v_1 - v_2 - \cdots v_n$, $\mathcal{P}_2 = u_1 - u_2 - \cdots u_n$ in $D_2(P_n)$. It can be observed that $|S \cap \mathcal{P}_1| = 2$ and $|S \cap \mathcal{P}_2| = 2$. Let T be any general position set of $D_2(P_n)$. Suppose, $|T| > 4$, then $|T \cap \mathcal{P}_1| > 2$ and $|T \cap \mathcal{P}_2| > 2$, which is a contradiction. Hence, $gp[D_2(P_n)] \leq 4$, which completes the proof.

Theorem 5.3 Let $D_2(C_n)$ be the Shadow graph of a cycle C_n. Then, $gp[D_2(C_3)] = 3$, $gp[D_2(C_4)] = 4$ and $gp[D_2(C_n)] = 6$ for $n \geq 5$.

Proof. Let $v_1, v_2, \dots v_n$ and $u_1, u_2, \dots u_n$ be the vertices of $D_2(C_n)$ corresponding to the vertices of C_n and the newly added vertices respectively. For $n = 3,4$, the proof is obvious. Consider the case when $n \geq 5$. Let $S = \{v_1, v_3, v_{n-1}, u_1, u_3, u_{n-1}\}$. This set is a general position set of $D_2(C_n)$. Hence, $gp[D_2(C_n)] \geq 6$. Consider the isometric cycles $C_1 = \{v_1, v_2, \dots v_n\}$ and $C_2 = \{u_1, u_2, \dots u_n\}$ in $D_2(C_n)$. Let T be an arbitrary general position set of $D_2(C_n)$. Suppose $|T| > 6$, then obviously at least three vertices of T will lie on any one of the isometric paths in C_1 or C_2, which is a contradiction. Hence, $gp[D_2(C_n)] \leq 6$. This completes the proof.

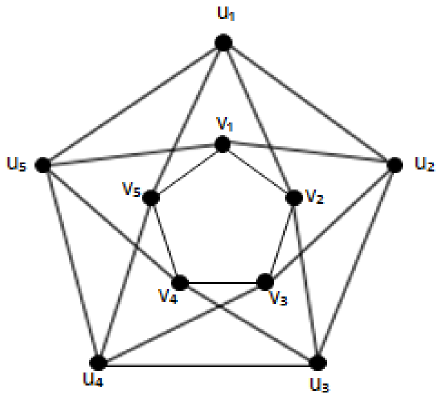

Figure 4.8 Shadow graph of cycle C_5

Theorem 5.4 Let $D_2(K_{1,n})$ be the Shadow graph of a star $K_{1,n}, n \geq 2$. Then, $gp[D_2(K_{1,n})] = 2n$.

Proof. Let $v, v_1, v_2, \dots v_n$ and $u, u_1, u_2, \dots u_n$ be the vertices of $D_2(K_{1,n})$ corresponding to the vertices of $K_{1,n}$ and the newly added vertices respectively.

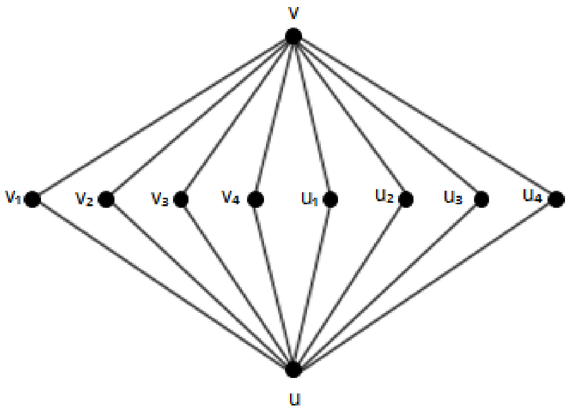

Figure 4.9 Shadow graph of star $K_{1,4}$

Consider the set $S = \{v_1, v_2, \dots v_n, u_1, u_2, \dots u_n\}$. Clearly, this set is a general position set of $D_2(K_{1,n})$. Hence, $gp[D_2(K_{1,n})] \geq 2n$. Let $T \subseteq V[D_2(K_{1,n})]$. Suppose $|T| > 2n$, then obviously either $v \in T$ or $u \in T$, in which case $|T \cap S| \leq 1$. Hence, $gp[D_2(K_{1,n})] \leq 2n$. This completes the proof.

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