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Prime Labeling on Few Trees of Diameter Less Than 6

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Abstract

A graph G which is simple, non-trivial, undirected and finite of size p and order q with the $V(G)$ and $E(G)$ as its vertex and the edge set respectively is said to admit prime labeling if an injective function $\mu^*: V(G) \rightarrow \{1, 2, 3, \ldots p\}$ maps the every vertices is such that the gcd $(\mu^*(u), \mu^*(v)) = 1$. Then G is a Prime graph. Few Trees of diameter 2,3,4 and 5 graphs are established to be prime graphs in this paper,

Keywords: Graph labeling, Prime labeling, Tree, Diameter of graph.

MSC Classification Code: 05C78

1. Introduction

In nineties graph labeling concept was introduced . For last 60 years, over 200 types of graph labeling have been studied with well almost 2500 articles published. Under some specified rules, graph labeling is the allotment of natural numbers to vertices, edges, or both. A graph with ŋ vertices admits a prime label if any two adjacent vertices can be labeled with the 1st n natural numbers in such a way that their labels are relatively prime.

Gallian's 2016 paper [2] provides an in-depth examination of graph labeling. Rosa pioneered vertex labeling in graph theory in 1967 [6]. In 1982, Tout, Dabboucy, and Howalla. [8] proposed Prime graph labeling. we investigate the prime labeling on few Trees of diameter graphs with diameter less than or equal to 5 in this work.

2. Definitions

Definition 2.1: "Graph labeling" is the process of assigning values to the vertices or edges of a graph depending on certain conditions.

Definition 2.2: A prime labeling of a graph is an injective function $\mu^*: V(G) \rightarrow \{1, 2, 3, .. p\}$ such that the gcd $(\mu^*(u), \mu^*(v)) = 1$ for each adjacent vertices u and v. A prime graph is a graph that permits prime labeling.

Definition 2.3: The Connected graph G without any cycle is called Tree.

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Definition 2.4: The maximum length of path in a graph G is called diameter of graph.

Definition 2.5:

Let $Ţ_1^2$ is the tree of diameter 2 acquired by connecting 'ŋ' leaves to the internal vertex of the path Ƥ$_3$.

Definition 2.6:

We define the trees whose diameter is 3 denoted by $Ţ_s^3$, 1≤ s ≤ 4 are as follows.

1. The tree $Ţ_1^3$ is acquired by connecting ''ŋ' number of path P_3. to the mid vertex of the path Ƥ$_3$.
2. The tree $Ţ_2^3$ is tree acquired by connecting 'ŋ' leaves through a bridge to the midvertex of the path Ƥ$_3$.
3. The tree $Ţ_3^3$ is tree acquired by connecting 'ŋ' leaves to the 1st internal vertex of the path Ƥ$_4$.
4. The tree $Ţ_4^3$ is tree acquired by connecting 'ɱ', leaves to the 1st and the 2nd internal vertices of the path Ƥ$_4$.

Definition 2.7:

We define the trees whose diameter is 4 denoted by$Ţ_s^4$, 1 ≤ s ≤ 13 are as follows.

1. A 'ŋ' number of path P_4 is connected to the mid vertex of the path P_3 results a tree $Ţ_1^4$.

2. A 'ŋ' number of path P_3 through a bridge is connected to the mid vertex of the path P_3 results a tree $Ţ_2^4$

3. A 'ŋ' number of leaves through the path P_3 is connected to the mid vertex of the path P_3 results a tree $Ţ_3^4$

4. A 'ŋ' number of path P_3 is connected to the 1^{st} internal vertex of the path P_4 results a tree $Ţ_4^4$

5. A 'ŋ' number of leaves through a bridge is connected to the 1^{st} internal vertex of the path P_4 results a tree$Ţ_5^4$

6. A 'ŋ' number of path P_4 is connected to the midvertex of the path P_3 results a tree$Ţ_6^4$

7. A 'ɱ' number of leaves is connected to the 1^{st} internal vertex and attaching 'ŋ' pendant edges through a bridge to the second internal vertex of the path P_4 results a tree$Ţ_7^4$

8. A 'ŋ' number of leaves is connected to the 1^{st} internal vertex of the path P_5 results a tree $Ţ_8^4$

9. A 'ŋ' number of leaves is connected to the 2nd internal vertex of the path P_5 results a tree$Ţ_9^4$

10. A 'ɱ' number of leaves is connected to the 1^{st} and 2nd internal vertices of the path P_5 results a tree $Ţ_{10}^4$

11. A 'ɱ' number of leaves is connected to the 1^{st} and 3rd internal vertices of the path P_5 results a tree $Ţ_{11}^4$

12. A 'ŋ' number of path P_3 is connected to the middle vertex of the path P_5 results a tree $Ţ_{12}^4$

13. A'ŋ' pendant leaves is connected through a bridge to the middle vertex of the path P_5 results a tree $Ţ_{13}^4$

Definition 2.8:

The trees of diameter 5 denoted by $Ţ_s^5$, 1 ≤ s ≤ 9 are defined as follows.

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1. The Graph acquired by connecting 'ŋ' number of path P_5 to the middle vertex of the path P_3 it is denoted by $Ʈ_1^5$.
2. The Graph acquired by connecting 'ŋ' number of path P_4 through a bridge to the middle vertex of the path P_3 it is denoted by $Ʈ_2^5$.
3. The Graph acquired by connecting 'ŋ' number of path P_3 through a path of length 2 to the middle vertex of the path P_3 it is denoted by $Ʈ_3^5$.
4. The Graph acquired by connecting 'ŋ' number of leaves to the middle vertex of the path P_3 through a path of length 3 it is denoted by $Ʈ_4^5$.
5. The Graph acquired by connecting 'ŋ' number of path P_4 to the 1^{st} internal vertex of the path P_4 it is denoted by $Ʈ_5^5$.
6. The Graph acquired by connecting 'ŋ' number of path P_3 to the 1^{st} internal vertex of the path P_4 through a bridge it is denoted by $Ʈ_6^5$.
7. The Graph acquired by connecting 'ŋ' number of leaves through a path of length 2 to the 1^{st} internal vertex of the path P_4 it is denoted by $Ʈ_7^5$.
8. The Graph acquired by connecting 'ŋ' number of path P_3 to the 1^{st} internal vertex of the path P_5 it is denoted by $Ʈ_8^5$.
9. The Graph acquired by connecting 'ŋ' number of leaves through a bridge to the 1^{st} internal vertex of the path P_5 it is denoted by $Ʈ_9^5$.

3. Main Results

Theorem 3.1: The Tree $Ʈ_1^2$ is Prime graph.

Proof:

Let $V(G) = \{v_ɨ:\ 1 \le ɨ \le ŋ\} \cup \{p_{1,}p_2, p_3\}\ and$

$E(G) = \{p_{1,}p_2\} \cup \{p_2, p_3\} \cup \{p_2 v_ɨ:\ 1 \le ɨ \le ŋ\}$

$|V| = ŋ + 3.$ $\quad |E| = ŋ + 2.$

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*\ (p_1) = 2,\quad \mu^*\ (p_2) = 1,\quad \mu^*\ (p\ _3) = 3,$

$\mu^*\ (v_ɨ) = 3 + ɨ\ ;\qquad for\ \ 1 \le ɨ \le ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_{1,}p_2\ ) = 1.$

b) The gcd $(p_2, p_3\ ) = 1.$

c) The gcd $(p_2 v_ɨ) = 1;\qquad for\ 1 \le ɨ \le ŋ$

Hence by the definition 2.2, It is clear that The Tree $Ʈ_1^2$ is Prime graph.

Theorem 3.2: The Tree $Ʈ_1^3$ is Prime graph with path P_3.

Proof:

Let $V(G) = \{p_{1,}p_2, p_3\} \cup \{v_{\text{i}}:\ 1 \le \text{i} \le \eta\} \cup \{u_{\text{i}}:\ 1 \le \text{i} \le \eta\}$ and

$E(G) = \{p_1\ p_2\} \cup \{p_2\ p_3\} \cup \{p_2 v_{\text{i}}:\ 1 \le \text{i} \le \eta\ \} \cup \{v_{\text{i}} u_{\text{i}}:\ 1 \le \text{i} \le \eta\}$

$|V| = 2\eta + 3.$ $\qquad |E| = 2\eta + 2.$

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*\ (p_1) = 2,\qquad \mu^*\ (p_2) = 1,\qquad \mu^*\ (p_3) = 3,$

$\mu^*\ (v_{\text{i}}) = 4 + (\text{i} - 1)2\ ;\qquad for\ \ 1 \le \text{i} \le \eta$

$\mu^*\ (u_{\text{i}}) = 5 + (\text{i} - 1)2\ ;\qquad for\ \ 1 \le \text{i} \le \eta$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_1\ p_2) = 1.$
b) The gcd $(p_2\ p_3) = 1.$
c) The gcd $(p_2 v_{\text{i}}) = 1;\qquad for\ \ 1 \le \text{i} \le \eta.$
d) The gcd $(v_{\text{i}} u_{\text{i}}) = 1;\qquad for\ \ 1 \le \text{j} \le \eta.$

Hence by the definition 2.2, It is clear that the graph Ţ_1^3 is Prime graph.

Theorem 3.3: The Tree Ţ_2^3 is Prime graph.

Proof:

Let $V(G) = \{p_{1,}p_2, p_3\} \cup \{p\} \cup \{v_{\text{i}}:\ 1 \le \text{i} \le \eta\}$ and

$E(G) = \{p_1\ p_2\} \cup \{p_2\ p_3\} \cup \{p_2 p\} \cup \{p v_{\text{i}}:\ 1 \le \text{i} \le \eta\}$

$|V| = \eta + 4.$ $\qquad |E| = \eta + 3.$

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*\ (p_1) = 2,\qquad \mu^*\ (p_2) = 3,\qquad \mu^*\ (p_3) = 4,\qquad \mu^*\ (p) = 1,$

$\mu^*\ (v_{\text{i}}) = 4 + \text{i}\ ;\qquad for\ \ 1 \le \text{i} \le \eta$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_1\ p_2) = 1.$
b) The gcd $(p_2\ p_3) = 1.$
c) The gcd $(p_2 p) = 1.$
d) The gcd $(p v_{\text{i}}) = 1\ ;\qquad for\ 1 \le \text{i} \le \eta\ .$

Hence by the definition 2.2, It is clear that the graph Ţ_2^3 is Prime graph.

Theorem 3.4: The Tree Ţ_3^3 is Prime graph.

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Proof:

Let $V(G) = \{v_ɨ:\ 1 \le ɨ \le ŋ\} \cup \{p_1, p_2, p_3, p_4\}$ and

$E(G) = \{p_ɨ p_{ɨ+1};\ \text{ for } 1 \le ɨ \le 3\} \cup \{p_2 v_ɨ:\ 1 \le ɨ \le ŋ\}$

$|V| = ŋ + 4.\qquad |E| = ŋ + 3.$

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*(p_1) = 2,\qquad \mu^*(p_2) = 1,\qquad \mu^*(p_3) = 3,\qquad \mu^*(p_4) = 4,$

$\mu^*(v_ɨ) = 4 + ɨ\ ;\qquad for\ 1 \le ɨ \le ŋ$

The following observations can be made based on the above labeling pattern.

d) The gcd $(p_ɨ, p_{ɨ+1}) = 1,\qquad for\ 1 \le ɨ \le 3.$

e) The gcd $(p_2 v_ɨ) = 1.\qquad for\ 1 \le ɨ \le ŋ$

Hence by the definition 2.2, It is clear that the graph $Ţ_3^3$ is Prime graph.

Theorem 3.5: The Tree $Ţ_4^3$ is Prime graph.

Proof:

Let $V(G) = \{p_1, p_2, p_3, p_4\} \cup \{v_ɨ:\ 1 \le ɨ \le ŋ\} \cup \{u_ɨ:\ 1 \le ɨ \le m̧\}$ and

$E(G) = \{p_ɨ p_{ɨ+1};\ \text{ for } 1 \le ɨ \le 3\} \cup \{p_2 v_ɨ:\ 1 \le ɨ \le ŋ\} \cup \{p_3 u_ɨ:\ 1 \le ɨ \le m̧\}$

$|V| = ŋ + m̧ + 4.\qquad |E| = ŋ + m̧ + 3.$

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*(p_1) = 4,\qquad \mu^*(p_2) = 1,\qquad \mu^*(p_3) = 2,\qquad \mu^*(p_4) = 3,$

$\mu^*(v_ɨ) = 6 + (ɨ - 1)2\ ;\qquad for\ 1 \le ɨ \le ŋ$

$\mu^*(u_ɨ) = 5 + (ɨ - 1)2\ ;\qquad for\ 1 \le ɨ \le m̧$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_ɨ, p_{ɨ+1}) = 1;\qquad for\ 1 \le ɨ \le 3$

b) The gcd $(p_2 v_ɨ) = 1;\qquad for\ 1 \le ɨ \le ŋ$

c) The gcd $(p_3 u_ɨ) = 1;\qquad for\ 1 \le ɨ \le m̧$

Hence by the definition 2.2, It is clear that the graph $Ţ_4^3$ is Prime graph.

Theorem 3.6: The Tree $Ţ_1^4$ is Prime graph.

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Proof:

Let $V(G) = \{p_1, p_2, p_3\} \cup \{u_{ɨ}:\ 1 \le ɨ \le ŋ\} \cup \{v_{ɨ}:\ 1 \le ɨ \le ŋ\} \cup \{w_{ɨ}:\ 1 \le ɨ \le ŋ\}$

$$E(G) = \{p_1\ p_2\} \cup \{p_2\ p_3\} \cup \{p_2 u_{ɨ}:\ 1 \le ɨ \le ŋ\} \cup \{u_{ɨ} v_{ɨ}:\ 1 \le ɨ \le ŋ\} \cup \{v_{ɨ} w_{ɨ}:\ 1 \le ɨ \le ŋ\}$$

$|V| = 3ŋ + 3.$ $\quad |E| = 3ŋ + 2.$

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*(p_1) = 2,\quad \mu^*(p_2) = 1,\quad \mu^*(p_3) = 3,$

$\mu^*(u_{ɨ}) = 4 + (ɨ - 1)3\ ;\quad for\ 1 \le ɨ \le ŋ$

$\mu^*(v_{ɨ}) = 5 + (ɨ - 1)3\ ;\quad for\ 1 \le ɨ \le ŋ$

$\mu^*(w_{ɨ}) = 6 + (ɨ - 1)3\ ;\quad for\ 1 \le ɨ \le ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_1\ p_2) = 1.$.
b) The gcd $(p_2\ p_3) = 1.$
c) The gcd $(p_2 u_{ɨ}) = 1$; $for\ 1 \le ɨ \le ŋ$
d) The gcd $(u_{ɨ} v_{ɨ}) = 1$; $for\ 1 \le ɨ \le ŋ$
e) The gcd $(v_{ɨ} w_{ɨ}) = 1$; $for\ 1 \le ɨ \le ŋ$

Hence by the definition 2.2, It is clear that the graph $Ţ_1^4$ is Prime graph.

Theorem 3.7: The Tree $Ţ_2^4$ is Prime graph.

Proof:

Let $V(G) = \{p_1, p_2, p_3\} \cup \{p\} \cup \{v_{ɨ}:\ 1 \le ɨ \le ŋ\} \cup \{u_{ɨ}:\ 1 \le ɨ \le ŋ\}$ and

$$E(G) = \{p_1\ p_2\} \cup \{p_2\ p_3\} \cup \{p_2\ p\} \cup \{p v_{ɨ}:\ 1 \le ɨ \le ŋ\} \cup \{v_{ɨ} u_{ɨ}:\ 1 \le ɨ \le ŋ\}$$

$|V| = 2ŋ + 4.$ $\quad |E| = 2ŋ + 3.$

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*(p_1) = 2,\quad \mu^*(p_2) = 3,\quad \mu^*(p_3) = 4,\quad \mu^*(p) = 1,$

$\mu^*(u_{ɨ}) = 6 + (ɨ - 1)2,\quad for\ 1 \le ɨ \le ŋ$

$\mu^*(v_{ɨ}) = 5 + (ɨ - 1)2,\quad for\ 1 \le ɨ \le ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_1\ p_2) = 1.$

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b) The gcd $(p_2\ p_3) = 1$.

c) The gcd $(p_2\ p) = 1$.

d) The gcd $(pv_ɨ) = 1$; $for\ 1 \leq ɨ \leq ŋ$.

e) The gcd $(v_ɨ u_ɨ) = 1$; $for\ 1 \leq ɨ \leq ŋ$.

Hence by the definition 2.2, It is clear that the graph $Ţ_2^4$ is Prime graph.

Theorem 3.8: The Tree $Ţ_3^4$ is Prime graph.

Proof:

Let $V(G) = \{p_{1,}p_2, p_3\} \cup \{p, v\} \cup \{v_ɨ:\ 1 \leq ɨ \leq ŋ\}$and

$E(G) = \{p_1\ p_2\} \cup \{p_2\ p_3\} \cup \{p_2\ p\} \cup \{pv\} \cup \{vv_ɨ:\ 1 \leq ɨ \leq ŋ\}$

$|V| = ŋ + 5$. $|E| = ŋ + 4$.

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*(p_1) = 4$, $\mu^*(p_2) = 3$, $\mu^*(p_3) = 5$, $\mu^*(p) = 2$,

$\mu^*(v) = 1$,

$\mu^*(v_ɨ) = 5 + ɨ$; $for\ 1 \leq ɨ \leq ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_1\ p_2) = 1$.

b) The gcd $(p_2\ p_3) = 1$.

c) The gcd $(p_2\ p) = 1$.

d) The gcd $(pv) = 1$.

e) The gcd $(vv_ɨ) = 1$; $for\ 1 \leq ɨ \leq ŋ$.

Hence by the definition 2.2, It is clear that the graph $Ţ_3^4$ is Prime graph.

Theorem 3.9: The Tree $Ţ_4^4$ is Prime graph.

Proof:

Let $V(G) = \{p_ɨ:\ 1 \leq ɨ \leq 4\} \cup \{v_ɨ:\ 1 \leq ɨ \leq ŋ\} \cup \{u_ɨ:\ 1 \leq ɨ \leq ŋ\}$ and

$E(G) = \left\{p_{ɨ,}p_{ɨ+1};\ \text{for } 1 \leq ɨ \leq 3\right\} \cup \{p_2 v_ɨ:\ 1 \leq ɨ \leq ŋ\} \cup \{v_ɨ u_ɨ:\ 1 \leq ɨ \leq ŋ\}$

$|V| = 2ŋ + 4$. $|E| = 2ŋ + 3$.

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*(p_1) = 2$, $\mu^*(p_2) = 1$, $\mu^*(p_3) = 3$, $\mu^*(p_4) = 4$,

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$\mu^*\ (u_{ɨ}) = 6 + (ɨ - 1)2$; $\quad for\ 1 \le ɨ \le ŋ$

$\mu^*\ (v_{ɨ}) = 5 + (ɨ - 1)2$; $\quad for\ 1 \le ɨ \le ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_{ɨ,}p_{ɨ+1}) = 1$; $\quad for\ 1 \le ɨ \le 3$.

b) The gcd $(p_2 v_{ɨ}) = 1$; $\quad for 1 \le ɨ \le ŋ$

c) The gcd $(v_{ɨ} u_{ɨ}) = 1$; $\quad for 1 \le ɨ \le ŋ$

Hence by the definition 2.2, It is clear that the graph $Ţ_4^4$ is Prime graph.

Theorem 3.10: The Tree $Ţ_5^4$ is Prime graph.

Proof:

Let $V(G) = \{p_{ɨ}:\ 1 \le ɨ \le 4\ \}\} \cup \{p\} \cup \{v_{ɨ}:\ 1 \le ɨ \le ŋ\}$ and

$E(G) = \{p_{ɨ,}p_{ɨ+1};\ \text{for } 1 \le ɨ \le 3\} \cup \{p_2 p\} \cup \{p v_{ɨ}:\ 1 \le ɨ \le ŋ\ \}$

$|V| = ŋ + 5. \qquad |E| = ŋ + 4.$

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\ \}$ as.

$\mu^*\ (p_1) = 2, \quad \mu^*\ (p_2) = 3, \quad \mu^*\ (p_3) = 4, \quad \mu^*\ (p_4) = 5, \quad \mu^*\ (p) = 1,$

$\mu^*\ (v_{ɨ}) = 5 + ɨ$; $\quad for\ 1 \le ɨ \le ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_{ɨ,}p_{ɨ+1}) = 1$; $\quad for\ 1 \le ɨ \le 3$.

b) The gcd $(p_2 p) = 1$.

c) The gcd $(p v_{ɨ}) = 1$; $\quad for\ 1 \le ɨ \le ŋ$.

Hence by the definition 2.2, It is clear that the graph $Ţ_5^4$ is Prime graph.

Theorem 3.11: The Tree $Ţ_6^4$ is Prime graph.

Proof:

Let $V(G) = \{p_{1,}p_2, p_3 p_4\} \cup \{v_{ɨ}:\ 1 \le ɨ \le ŋ\} \cup \{u_{ɨ}:\ 1 \le ɨ \le m\!\!\!\!\!\!\,ɉ\ \} \cup \{w_{ɨ}:\ 1 \le ɨ \le m\!\!\!\!\!\!\,ɉ\ \}$ and

$E(G) = \{p_{ɨ,}p_{ɨ+1};\ \text{for } 1 \le ɨ \le 3\ \} \cup \{p_2 u_{ɨ}:\ 1 \le ɨ \le ŋ\ \} \cup$

$\{p_3 v_{ɨ}:\ 1 \le ɨ \le m\!\!\!\!\!\!\,ɉ\ \} \cup \{v_{ɨ} w_{ɨ}:\ 1 \le ɨ \le m\!\!\!\!\!\!\,ɉ\}$

$|V| = ŋ + 2m\!\!\!\!\!\!\,ɉ + 4. \qquad |E| = ŋ + 2m\!\!\!\!\!\!\,ɉ + 3.$

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\ \}$ as.

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$\mu^*\ (p_1) = 4, \quad \mu^*\ (p_2) = 1, \quad \mu^*\ (p_3) = 2, \quad \mu^*\ (p_4) = 3,$

$\mu^*\ (v_ɨ) = 5 + (ɨ - 1)2\ ; \qquad for\ \ 1 \le ɨ \le ŋ$

$\mu^*\ (w_ɨ) = 6 + (ɨ - 1)2\ ; \qquad for\ \ 1 \le ɨ \le ŋ$

$\mu^*\ (u_ɨ) = 2ɱ + 4 + ɨ\ ; \qquad for\ \ 1 \le ɨ \le ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_{ɨ,}p_{ɨ+1}) = 1\ ;\quad for\ 1 \le ɨ \le 3$.
b) The gcd $(p_2 u_ɨ) = 1\ ;\quad for\ 1 \le ɨ \le ŋ$
c) The gcd $(p_3 v_ɨ) = 1\ ;\quad for\ 1 \le ɨ \le ɱ$
d) The gcd $(v_ɨ w_ɨ) = 1\ ;\quad for\ 1 \le ɨ \le ɱ$

Hence by the definition 2.2, It is clear that the graph $Ţ_6^4$ is Prime graph.

Theorem 3.12: The Tree $Ţ_7^4$ is Prime graph.

Proof:

Let $V(G) = \{p_ɨ\!:\ 1 \le ɨ \le 4\} \cup \{p\} \cup \{v_ɨ\!:\ 1 \le ɨ \le ŋ\} \cup \{u_ɨ\!:\ 1 \le ɨ \le ɱ\}$ and

$E(G) = \{p_{ɨ,}p_{ɨ+1};\ \text{for } 1 \le ɨ \le 3\} \cup \{p_2\ v_ɨ\!:\ 1 \le ɨ \le ŋ\} \cup$

$\{p_3 p\} \cup \{p u_ɨ\!:\ 1 \le ɨ \le ɱ\}$

$|V| = ŋ + ɱ + 5. \qquad |E| = ŋ + ɱ + 4.$

We have defined the following $\mu^*\!: V(G) \to \{1,2,3, \ldots \ldots p\}$ as.

$\mu^*\ (p_1) = 5, \quad \mu^*\ (p_2) = 2, \quad \mu^*\ (p_3) = 3, \quad \mu^*\ (p_4) = 4,$

$\mu^*\ (p) = 1,$

$\mu^*\ (v_ɨ) = 7 + (ɨ - 1)2\ ; \qquad for\ \ 1 \le ɨ \le ŋ$

$\mu^*\ (u_ɨ) = 6 + (ɨ - 1)2\ ; \qquad for\ \ 1 \le ɨ \le ɱ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_{ɨ,}p_{ɨ+1}) = 1\ ;\quad for\ 1 \le ɨ \le 3.$
b) The gcd $(p_2\ v_ɨ) = 1\ ;\quad for\ 1 \le ɨ \le ŋ$
c) The gcd $(p_3 p) = 1\ ;$
d) The gcd $(p u_ɨ) = 1\ ;\quad for\ 1 \le ɨ \le ɱ.$

Hence by the definition 2.2, It is clear that the graph $Ţ_7^4$ is Prime graph.

Theorem 3.13: The Tree $Ţ_8^4$ is Prime graph.

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Proof:

Let $V(G) = \{p_ɨ\colon\ 1 \le ɨ \le 5\} \cup \{v_ɨ\colon\ 1 \le ɨ \le ŋ\}$ and

$E(G) = \{p_{ɨ,}p_{ɨ+1};\ \text{for } 1 \le ɨ \le 4\} \cup \{p_2 v_ɨ\colon\ 1 \le ɨ \le ŋ\}$

$|V| = ŋ + 5.$ $|E| = ŋ + 4.$

We have defined the following $\mu^*\colon V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*(p_1) = 2,$ $\mu^*(p_2) = 1,$ $\mu^*(p_3) = 3,$ $\mu^*(p_4) = 4,$ $\mu^*(p_5) = 5,$

$\mu^*(v_ɨ) = 5 + ɨ$; $for\ 1 \le ɨ \le ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_{ɨ,}p_{ɨ+1}) = 1$; $for\ 1 \le ɨ \le 4.$

b) The gcd $(p_2\ v_ɨ) = 1$; $for\ 1 \le ɨ \le ŋ$

Hence by the definition 2.2, It is clear that the graph $Ţ_8^4$ is Prime graph.

Theorem 3.14: The Tree $Ţ_9^4$ is Prime graph.

Proof:

Let $V(G) = \{p_ɨ\colon\ 1 \le ɨ \le 5\} \cup \{v_ɨ\colon\ 1 \le ɨ \le ŋ\}$ and

$E(G) = \{p_{ɨ,}p_{ɨ+1};\ \text{for } 1 \le ɨ \le 4\} \cup \{p_3 v_ɨ\colon\ 1 \le ɨ \le ŋ\}$

$|V| = ŋ + 5.$ $|E| = ŋ + 4.$

We have defined the following $\mu^*\colon V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*(p_1) = 2,$ $\mu^*(p_2) = 3,$ $\mu^*(p_3) = 1,$ $\mu^*(p_4) = 4,$ $\mu^*(p_5) = 5,$

$\mu^*(v_ɨ) = 5 + ɨ$; $for\ 1 \le ɨ \le ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_{ɨ,}p_{ɨ+1}) = 1$; $for\ 1 \le ɨ \le 4.$

b) The gcd $(p_2\ v_ɨ) = 1$; $for\ 1 \le ɨ \le ŋ$

Hence by the definition 2.2, It is clear that the graph $Ţ_9^4$ is Prime graph.

Theorem 3.15: The Tree $Ţ_{10}^4$ is Prime graph,

Proof:

Let $V(G) = \{p_ɨ\colon\ 1 \le ɨ \le 5\} \cup \{v_ɨ\colon\ 1 \le ɨ \le ŋ\} \cup \{u_ɨ\colon\ 1 \le ɨ \le ɱ\}$ and

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$E(G) = \{p_{ɨ,}p_{ɨ+1};\ \text{for } 1 \le ɨ \le 4\} \cup \{p_2 v_ɨ:\ 1 \le ɨ \le ŋ\} \cup \{p_3 u_ɨ:\ 1 \le ɨ \le ɱ\}$

$|V| = ŋ + ɱ + 5.$ $|E| = ŋ + ɱ + 4.$

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*(p_1) = 3,$ $\mu^*(p_2) = 2,$ $\mu^*(p_3) = 1,$ $\mu^*(p_4) = 4,$ $\mu^*(p_5) = 5,$

$\mu^*(v_ɨ) = 7 + (ɨ - 1)2$; $for\ 1 \le ɨ \le ŋ$

$\mu^*(u_ɨ) = 6 + (ɨ - 1)2$; $for\ 1 \le ɨ \le ɱ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_{ɨ,}p_{ɨ+1}) = 1$; $for\ 1 \le ɨ \le 4.$
b) The gcd $(p_2\ v_ɨ) = 1$; $for\ 1 \le ɨ \le ŋ$
c) The gcd $(p_3\ u_ɨ) = 1$; $for\ 1 \le ɨ \le ɱ$

Hence by the definition 2.2, It is clear that the graph $Ţ_{10}^4$ is Prime graph.

Theorem 3.16: The Tree $Ţ_{11}^4$ is Prime graph,

Proof:

Let $V(G) = \{p_ɨ:\ 1 \le ɨ \le 5\} \cup \{v_ɨ:\ 1 \le ɨ \le ŋ\} \cup \{u_ɨ:\ 1 \le ɨ \le ɱ\}$ and

$E(G) = \{p_{ɨ,}p_{ɨ+1};\ \text{for } 1 \le ɨ \le 4\} \cup \{p_2 v_ɨ:\ 1 \le ɨ \le ŋ\} \cup \{p_4 u_ɨ:\ 1 \le ɨ \le ɱ\}$

$|V| = ŋ + ɱ + 5.$ $|E| = ŋ + ɱ + 4.$

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*(p_1) = 5,$ $\mu^*(p_2) = 2,$ $\mu^*(p_3) = 3,$ $\mu^*(p_4) = 1,$ $\mu^*(p_5) = 4,$

$\mu^*(v_ɨ) = 7 + (ɨ - 1)2$; $for\ 1 \le ɨ \le ŋ$

$\mu^*(u_ɨ) = 6 + (ɨ - 1)2$; $for\ 1 \le ɨ \le ɱ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_{ɨ,}p_{ɨ+1}) = 1$; $for\ 1 \le ɨ \le 4.$
b) The gcd $(p_2\ v_ɨ) = 1$; $for\ 1 \le ɨ \le ŋ$
c) The gcd $(p_3\ u_ɨ) = 1$; $for\ 1 \le ɨ \le ɱ$

Hence by the definition 2.2, It is clear that the graph $Ţ_{11}^4$ is Prime graph.

Theorem 3.17: The Tree $Ţ_{12}^4$ is Prime graph.

Proof:

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Let $V(G) = \{p_ɨ :\ 1 \le ɨ \le 5\} \cup \{u_ɨ :\ 1 \le ɨ \le ŋ\} \cup \{v_ɨ :\ 1 \le ɨ \le ŋ\}$ and

$$E(G) = \{p_{ɨ\,,}p_{ɨ+1};\ \text{for } 1 \le ɨ \le 3\} \cup \{p_3 u_ɨ :\ 1 \le ɨ \le ŋ\} \cup \{u_ɨ v_ɨ :\ 1 \le ɨ \le ŋ\}$$

$|V| = 2ŋ + 5.$ $\qquad |E| = 2ŋ + 4.$

We have defined the following $\mu^* : V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*(p_1) = 5,\quad \mu^*(p_2) = 2,\quad \mu^*(p_3) = 1,\quad \mu^*(p_4) = 3,\quad \mu^*(p_5) = 4,$

$\mu^*(u_ɨ) = 6 + (ɨ - 1)2\ ;\qquad for\ 1 \le ɨ \le ŋ$

$\mu^*(v_ɨ) = 7 + (ɨ - 1)2\ ;\qquad for\ 1 \le ɨ \le ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_{ɨ\,,}p_{ɨ+1}) = 1$; $\quad for\ 1 \le ɨ \le 3.$
b) The gcd $(p_3\ u_ɨ) = 1$; $\quad for\ 1 \le ɨ \le ŋ$
c) The gcd $(u_ɨ v_ɨ) = 1$; $\quad for\ 1 \le ɨ \le ŋ$

Hence by the definition 2.2, It is clear that the graph $Ţ^4_{12}$ is Prime graph.

Theorem 3.18: The Tree $Ţ^4_{13}$ is Prime graph.

Proof:

Let $V(G) = \{p_ɨ :\ 1 \le ɨ \le 5\}\} \cup \{p\} \cup \{v_ɨ :\ 1 \le ɨ \le ŋ\}$ and

$$E(G) = \{p_{ɨ\,,}p_{ɨ+1};\ \text{for } 1 \le ɨ \le 4\} \cup \{p_3 p\} \cup \{p u_ɨ :\ 1 \le ɨ \le ŋ\}$$

$|V| = ŋ + 6.$ $\qquad |E| = ŋ + 5.$

We have defined the following $\mu^* : V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*(p_ɨ) = i + 1,\qquad for\ 1 \le ɨ \le 5.$

$\mu^*(u_ɨ) = 6 + ɨ\ ;\qquad for\ 1 \le ɨ \le ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_{ɨ\,,}p_{ɨ+1}) = 1,\quad for\ 1 \le ɨ \le 4.$
b) The gcd $(p_3 p) = 1.$
c) The gcd $(p u_ɨ) = 1,\quad for\ 1 \le ɨ \le ŋ.$

Hence by the definition 2.2, It is clear that the graph $Ţ^4_{13}$ is Prime graph.

Theorem 3.19: The Tree $Ţ^5_1$ is Prime graph.

Proof:

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Let $V(G) = \{p_{1,}p_2, p_3\} \cup \{u_ɨ:\ 1 \leq ɨ \leq ŋ\} \cup \{v_ɨ:\ 1 \leq ɨ \leq ŋ\} \cup$

$\{w_ɨ:\ 1 \leq ɨ \leq ŋ\} \cup \{x_ɨ:\ 1 \leq ɨ \leq ŋ\}$

$E(G) = \{p_1\ p_2\} \cup \{p_2\ p_3\} \cup \{p_2 u_ɨ:\ 1 \leq ɨ \leq ŋ\} \cup \{u_ɨ v_ɨ:\ 1 \leq ɨ \leq ŋ\} \cup \{v_ɨ w_ɨ:\ 1 \leq ɨ \leq ŋ\} \cup \{w_ɨ x_ɨ:\ 1 \leq ɨ \leq ŋ\}$

$|V| = 4ŋ + 3.$ $|E| = 4ŋ + 2.$

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*\ (p_1) = 2,$ $\mu^*\ (p_2) = 1,$ $\mu^*\ (p_3) = 3,$

$\mu^*\ (u_ɨ) = 4 + (ɨ - 1)4\ ;$ $for\ \ 1 \leq ɨ \leq ŋ$

$\mu^*\ (v_ɨ) = 5 + (ɨ - 1)4\ ;$ $for\ \ 1 \leq ɨ \leq ŋ$

$\mu^*\ (w_ɨ) = 6 + (ɨ - 1)4\ ;$ $for\ \ 1 \leq ɨ \leq ŋ$

$\mu^*\ (x_ɨ) = 7 + (ɨ - 1)4\ ;$ $for\ \ 1 \leq ɨ \leq ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_1\ p_2) = 1.$.
b) The gcd $(p_2\ p_3) = 1.$
c) The gcd $(p_2 u_ɨ) = 1;$ $for\ 1 \leq ɨ \leq ŋ$
d) The gcd $(u_ɨ v_ɨ) = 1;$ $for\ 1 \leq ɨ \leq ŋ$
e) The gcd $(v_ɨ w_ɨ) = 1;$ $for\ 1 \leq ɨ \leq ŋ$
f) The gcd $(w_ɨ x_ɨ) = 1.$ $for\ 1 \leq ɨ \leq ŋ$

Hence by the definition 2.2, It is clear that the graph $Ţ_1^5$ is Prime graph.

Theorem 3.20: The Tree $Ţ_2^5$ is Prime graph.

Proof:

Let $V(G) = \{p_{1,}p_2, p_3\} \cup \{p\} \cup \{v_ɨ:\ 1 \leq ɨ \leq ŋ\} \cup \{u_ɨ:\ 1 \leq ɨ \leq ŋ\} \cup \{w_ɨ:\ 1 \leq ɨ \leq ŋ\}$ and

$E(G) = \{p_1\ p_2\} \cup \{p_2\ p_3\} \cup \{p_2\ p\} \cup \{p u_ɨ:\ 1 \leq ɨ \leq ŋ\} \cup \{u_ɨ v_ɨ:\ 1 \leq ɨ \leq ŋ\} \cup \{v_ɨ w_ɨ:\ 1 \leq ɨ \leq ŋ\}$

$|V| = 3ŋ + 4.$ $|E| = 3ŋ + 3.$

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*\ (p_1) = 2,$ $\mu^*\ (p_2) = 3,$ $\mu^*\ (p_3) = 4,$ $\mu^*\ (p) = 1,$

$\mu^*\ (u_ɨ) = 5 + (ɨ - 1)3\ ;$ $for\ \ 1 \leq ɨ \leq ŋ$

$\mu^*\ (v_ɨ) = 6 + (ɨ - 1)3\ ;$ $for\ \ 1 \leq ɨ \leq ŋ$

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$\mu^*\ (w_{ɨ}) = 7 + (ɨ - 1)3$; $\quad for\ \ 1 \leq ɨ \leq ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_1\ p_2) = 1$.
b) The gcd $(p_2\ p_3) = 1$.
c) The gcd $(p_2\ p) = 1$.
d) The gcd $(pu_{ɨ}) = 1$; $\quad for\ 1 \leq ɨ \leq ŋ$.
e) The gcd $(u_{ɨ}v_{ɨ}) = 1$; $\quad for\ 1 \leq ɨ \leq ŋ$.
f) The gcd $(v_{ɨ}w_{ɨ}) = 1$; $\quad for\ 1 \leq ɨ \leq ŋ$

Hence by the definition 2.2, It is clear that the graph $Ţ_2^5$ is Prime graph.

Theorem 3.21: The Tree $Ţ_3^5$ is Prime graph.

Proof:

Let $V(G) = \{p_{1,}p_2, p_3\} \cup \{p, v\} \cup \{u_{ɨ}:\ 1 \leq ɨ \leq ŋ\} \cup \{v_{ɨ}:\ 1 \leq ɨ \leq ŋ\}\ and$

$E(G) = \{p_1\ p_2\} \cup \{p_2\ p_3\} \cup \{p_2\ p\} \cup \{pv\} \cup \{vu_{ɨ}:\ 1 \leq ɨ \leq ŋ\} \cup \{u_{ɨ}v_{ɨ}:\ 1 \leq ɨ \leq ŋ\}$

$|V| = 2ŋ + 5. \qquad |E| = 2ŋ + 4.$

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*\ (p_1) = 4, \qquad \mu^*\ (p_2) = 3, \qquad \mu^*\ (p_3) = 5, \qquad \mu^*\ (p) = 2,$

$\mu^*\ (v) = 1,$

$\mu^*\ (u_{ɨ}) = 6 + (ɨ - 1)2$; $\quad for\ \ 1 \leq ɨ \leq ŋ$

$\mu^*\ (v_{ɨ}) = 7 + (ɨ - 1)2$; $\quad for\ \ 1 \leq ɨ \leq ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_1\ p_2) = 1$.
b) The gcd $(p_2\ p_3) = 1$.
c) The gcd $(p_2\ p) = 1$,
d) The gcd $(pv) = 1$.
e) The gcd $(vu_{ɨ}) = 1$; $\quad for\ 1 \leq ɨ \leq ŋ$.
f) The gcd $(u_{ɨ}v_{ɨ}) = 1$; $\quad for\ 1 \leq ɨ \leq ŋ$.

Hence by the definition 2.2, It is clear that the graph $Ţ_3^5$ is Prime graph.

Theorem 3.22: The Tree $Ţ_4^5$ is Prime graph.

Proof:

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Let $V(G) = \{p_{1,}p_2, p_3\} \cup \{p, u, v\ ,\} \cup \{u_ɨ:\ 1 \leq ɨ \leq ŋ\ \}$and

$E(G) = \{p_1\ p_2\ \} \cup \{p_2\ p_3\ \} \cup \{p_2\ p\ \} \cup \{pu\} \cup \{uv\} \cup \{vu_ɨ:\ 1 \leq ɨ \leq ŋ\ \}$

$|V| = ŋ + 6.\qquad |E| = ŋ + 5.$

We have defined the following $\mu^*: V(G) \rightarrow \{1,2,3, \dots \dots p\ \}$ as.

$\mu^*\ (p_i) = i + 3, \quad for\ \ 1 \leq ɨ \leq 3$

$\mu^*\ (p) = 3, \qquad \mu^*\ (u) = 2, \qquad \mu^*\ (v) = 1,$

$\mu^*\ (u_ɨ) = 7 + i\ \ ; \qquad\qquad for\ \ 1 \leq ɨ \leq ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_1\ p_2) = 1$.
b) The gcd $(p_2\ p_3) = 1$.
c) The gcd $(p_2\ p\ ) = 1$.
d) The gcd $(\ pu\ ) = 1$.
e) The gcd $(\ uv\ ) = 1$.
f) The gcd $(vu_ɨ) = 1\ ;\qquad for\ 1 \leq ɨ \leq ŋ$.

Hence by the definition 2.2, It is clear that the graph $Ţ_4^5$ is Prime graph.

Theorem 3.23: The Tree $Ţ_5^5$ is Prime graph.

Proof:

Let $V(G) = \{p_ɨ:\ 1 \leq ɨ \leq 4\} \cup \{v_ɨ:\ 1 \leq ɨ \leq ŋ\ \} \cup \{u_ɨ:\ 1 \leq ɨ \leq ŋ\ \} \cup \{w_ɨ:\ 1 \leq ɨ \leq ŋ\}\ and$

$E(G) = \left\{p_{ɨ\ ,}p_{ɨ+1};\ \text{ for } 1 \leq ɨ \leq 3\ \right\} \cup \{p_2u_ɨ:\ 1 \leq ɨ \leq ŋ\ \ \} \cup \{u_ɨv_ɨ:\ 1 \leq ɨ \leq ŋ\ \} \cup \{v_ɨw_ɨ:\ 1 \leq ɨ \leq ŋ\ \}$

$|V| = 3ŋ + 4.\qquad |E| = 3ŋ + 3.$

We have defined the following $\mu^*: V(G) \rightarrow \{1,2,3, \dots \dots p\ \}$ as.

$\mu^*\ (p_1) = 2, \qquad \mu^*\ (p_2) = 1, \qquad \mu^*\ (p_3) = 3, \qquad \mu^*\ (p_4) = 4,$

$\mu^*\ (u_ɨ) = 5 + (ɨ - 1)3\ \ ; \qquad for\ \ 1 \leq ɨ \leq ŋ$

$\mu^*\ (v_ɨ) = 6 + (ɨ - 1)3\ \ ; \qquad for\ \ 1 \leq ɨ \leq ŋ$

$\mu^*\ (w_ɨ) = 7 + (ɨ - 1)3\ \ ; \qquad for\ \ 1 \leq ɨ \leq ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $\left(p_{ɨ\ ,}p_{ɨ+1}\right) = 1\ ;\qquad for\ 1 \leq ɨ \leq 3.$
b) The gcd $(p_2u_ɨ) = 1\ \ ;\qquad for 1 \leq ɨ \leq ŋ$

c) The gcd $(u_ɨ v_ɨ) = 1.$ $for 1 \leq ɨ \leq ŋ$

d) The gcd $(v_ɨ w_ɨ) = 1.$ $for\ 1 \leq ɨ \leq ŋ$

Hence by the definition 2.2, It is clear that the graph $Ţ_5^5$ is Prime graph.

Theorem 3.24: The Tree $Ţ_6^5$ is Prime graph.

Proof:

Let $V(G) = \{p_ɨ:\ 1 \leq ɨ \leq 4\}\} \cup \{p\} \cup \{u_ɨ:\ 1 \leq ɨ \leq ŋ\} \cup \{v_ɨ:\ 1 \leq ɨ \leq ŋ\}\ and$

$E(G) = \{p_{ɨ\,,}p_{ɨ+1};\ \text{for } 1 \leq ɨ \leq 3\} \cup \{p_2 p\} \cup \{pu_ɨ:\ 1 \leq ɨ \leq ŋ\} \cup \{u_ɨ v_ɨ:\ 1 \leq ɨ \leq ŋ\}$

$|V| = 2ŋ + 5.$ $|E| = 2ŋ + 4.$

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*\ (p_i) = i + 1,,$ $for\ \ 1 \leq ɨ \leq 4$

$\mu^*\ (p) = 1,$

$\mu^*\ (u_ɨ) = 6 + (ɨ - 1)3\ ;$ $for\ \ 1 \leq ɨ \leq ŋ$

$\mu^*\ (v_ɨ) = 7 + (ɨ - 1)3\ ;$ $for\ \ 1 \leq ɨ \leq ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_{ɨ\,,}p_{ɨ+1}) = 1\ ;$ $for\ 1 \leq ɨ \leq 3.$

b) The gcd $(p_2 p) = 1.$

c) The gcd $(pu_ɨ) = 1\ ;$ $for\ 1 \leq ɨ \leq ŋ.$

d) The gcd $(u_ɨ v_ɨ) = 1\ ;$ $for\ 1 \leq ɨ \leq ŋ.$

Hence by the definition 2.2, It is clear that the graph $Ţ_6^5$ is Prime graph.

Theorem 3.25: The Tree $Ţ_7^5$ is Prime graph.

Proof:

Let $V(G) = \{p_ɨ:\ 1 \leq ɨ \leq 4\}\} \cup \{p\} \cup \{u\} \cup \{v_ɨ:\ 1 \leq ɨ \leq ŋ\}$ and

$E(G) = \{p_{ɨ\,,}p_{ɨ+1};\ \text{for } 1 \leq ɨ \leq 3\} \cup \{p_2 p\} \cup \{pu\} \cup \{uv_ɨ:\ 1 \leq ɨ \leq ŋ\}$

$|V| = ŋ + 6.$ $|E| = ŋ + 5.$

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^*\ (p_1) = 4,$ $\mu^*\ (p_2) = 3,$ $\mu^*\ (p_3) = 5,$ $\mu^*\ (p_4) = 6,$

$\mu^*\ (p) = 2,$ $\mu^*\ (u) = 1,$

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$\mu^* (u_ɨ) = 6 + i$; $for\ 1 \le ɨ \le ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_{ɨ,} p_{ɨ+1}) = 1$; $for\ 1 \le ɨ \le 3$.

b) The gcd $(p_2 p) = 1$.

c) The gcd $(pu) = 1$. .

d) The gcd $(uv_ɨ) = 1$; $for\ 1 \le ɨ \le ŋ$.

Hence by the definition 2.2, It is clear that the graph $Ţ_7^5$ is Prime graph.

Theorem 3.26: The Tree $Ţ_8^5$ is Prime graph.

Proof:

Let $V(G) = \{p_ɨ\text{: } 1 \le ɨ \le 5\} \cup \{u_ɨ\text{: } 1 \le ɨ \le ŋ\} \cup \{v_ɨ\text{: } 1 \le ɨ \le ŋ\}$ and

$E(G) = \{p_{ɨ,} p_{ɨ+1};\ \text{for } 1 \le ɨ \le 4\} \cup \{p_2 u_ɨ\} \cup \{u_ɨ v_ɨ\text{: } 1 \le ɨ \le ŋ\}$

$|V| = 2ŋ + 5$. $|E| = 2ŋ + 4$.

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\}$ as.

$\mu^* (p_i) = i,\ for\ 3 \le ɨ \le 5$ $\mu^* (p_2) = 1$, $\mu^*(p_1) = 2$,

$\mu^* (u_ɨ) = 6 + (ɨ - 1)2$; $for\ 1 \le ɨ \le ŋ$

$\mu^* (v_ɨ) = 7 + (ɨ - 1)2$; $for\ 1 \le ɨ \le ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_{ɨ,} p_{ɨ+1}) = 1$; $for\ 1 \le ɨ \le 4$.

b) The gcd $(p_2 u_ɨ) = 1$.

c) The gcd $(u_ɨ v_ɨ) = 1$; $for\ 1 \le ɨ \le ŋ$.

Hence by the definition 2.2, It is clear that the graph $Ţ_8^5$ is Prime graph.

Theorem 3.27: The Tree $Ţ_9^5$ is Prime graph.

Proof:

Let $V(G) = \{p_ɨ\text{: } 1 \le ɨ \le 5\} \cup \{u_ɨ\text{: } 1 \le ɨ \le ŋ\}\ and$

$E(G) = \{p_{ɨ,} p_{ɨ+1};\ \text{for } 1 \le ɨ \le 4\} \cup \{p_2 p\} \cup \{pu_ɨ\ \ for\ 1 \le ɨ \le ŋ\}$

$|V| = ŋ + 6$. $|E| = ŋ + 5$.

We have defined the following $\mu^*: V(G) \to \{1,2,3, \dots \dots p\}$ as

$\mu^* (p_i) = i + 1$; $for\ 1 \leq ɨ \leq 5$. $\mu^* (p) = 1$,

$\mu^* (u_ɨ) = 6 + i$; $for\ 1 \leq ɨ \leq ŋ$

The following observations can be made based on the above labeling pattern.

a) The gcd $(p_{ɨ,} p_{ɨ+1}) = 1$; $for\ 1 \leq ɨ \leq 4$.

b) The gcd $(p_2 p) = 1$.

c) The gcd $(p u_ɨ) = 1$; $for\ 1 \leq ɨ \leq ŋ$.

Hence by the definition 2.2, It is clear that the graph $Ţ_9^5$ is Prime graph.

Conclusion

We studied the existence of prime labeling for few Trees of diameter graphs in this paper such as and proved to be prime graphs. Investigating the presence of prime labeling on some other types of graphs are our next projects.

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