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Power Domination Number of Some Convex Polytopes

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Abstract

The power system monitoring problem demands that a minimum number of phase measurement units to be put in an electric power system. A set $S \subseteq V(G)$ is a power dominating set of a graph $G=(V,E),$ if every vertex and every edge in the graph is monitored by *S*. The power domination number $\gamma_p(G)$ is the minimum cardinality of a power dominating set of *G*. In this paper, we investigate the power domination number of convex polytopes D_n, S_n, U_n, T_n and R_n''.

Keywords: Convex polytopes, Power domination number, Power domination.

Mathematics Subject classification: 05C69

1. Introduction

A set $S \subseteq V(G)$ is a dominating set of graph *G*, if $N[S]=V(G)$. The problem of finding minimum dominating set is an important problem that has been elaborately studied. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of *G*.

Electric power companies are expected to continuously check the state of their system. A set of factors determine the status of an electrical power network. A phase measurement unit (PMU) is a measurement device installed on a node that can monitor the node's voltage and the current phase of the edge linked to the node, as well as provide system-wide failure warnings. Because PMUs are expensive, it is critical to keep the number of PMUs required to monitor the entire system to a minimum. Haynes et al. formulated this problem as a graph domination problem in [7]. An electric power system can be modelled as a graph $G=(V,E),$ where a vertex represents an electrical node and an edge represents a transmission line joining two electrical nodes.

Definition 1[7]. Let $G=(V,E)$ be a graph, and let $S \subseteq V(G)$. We define the sets $M^i(S)$ of vertices monitored by *S* at level *i*, $i \geq 0,$ inductively as follows:

1. $M^0(S)=N[S]$
2. $M^{i+1}(S)=M^i(S)\cup\{w:\exists v\in M^i(S), N(v)\cap(V(G)\setminus M^i(S))=w\}$

If $M^\infty(S)=V(G),$ then the set *S* is called a power dominating set of *G* and the power domination number of *G*, denoted by $\gamma_p(G)$ is the minimum cardinality of a power dominating set of *G*.

A convex polytope *P* is defined to be the convex hull of finite number of points in E^d. A 2-dimensional polytope is called a convex polygon. A 3-polytope is called a convex polyhedron. It was introduced by Baca[1]. In [2] the classes

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of convex polytopes Q_n and R_n were introduced. In [4,5] it was shown that metric dimension of convex polytopes, T_n, S_n, R_n, U_n are equal to 3. In this paper, we investigate the power domination number of convex polytopes D_n, U_n, S_n, T_n and R''_n. In the next section we determine the power domination number of convex polytopes D_n.

2 Convex Polytope D_n

The graph of convex polytope D_n has vertex set $V(D_n) = \{a_j, b_j, c_j, d_j : j = 0, 1, \cdots, n-1\}$ and edge set $E(D_n) = \{a_j a_{j+1}, d_j d_{j+1}, a_j b_j, b_j c_j, c_j d_j, b_{j+1} c_j : j = 0,1 \cdots n-1\}$.

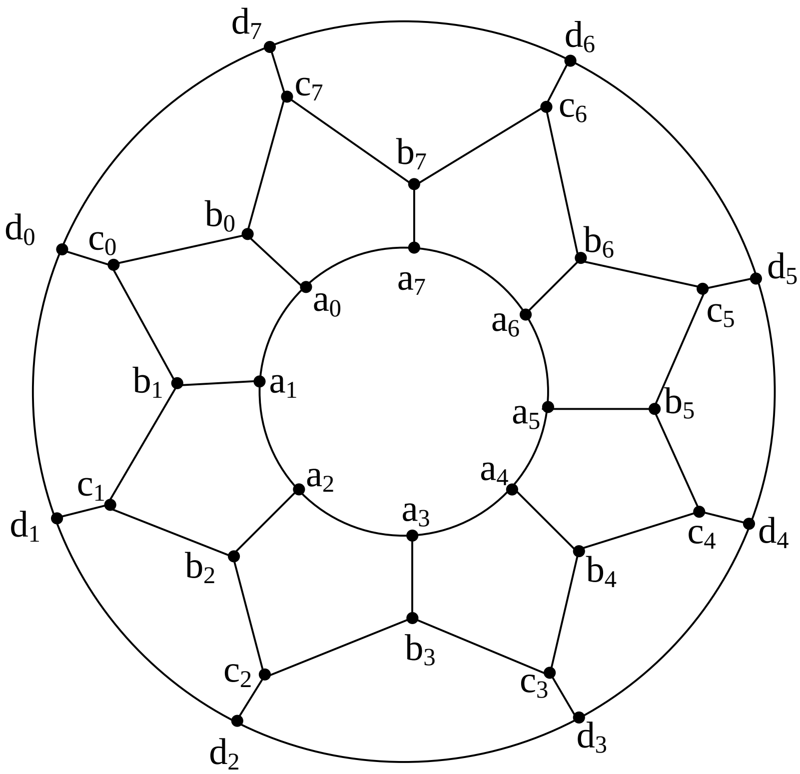

Fig-1. Convex polytope D_8

Theorem 2.1. If $G = D_n$ then $\gamma_p(G) = 2$.

Proof. Let *G* be a convex polytope D_n. Clearly a single vertex cannot monitor the entire graph *G*. Hence $\gamma_p(G) \geq 2$. Consider the set $S = \{a_j, c_{j+1}\}, 0 \leq j \leq n-1 (\mathrm{mod}\, n)$ in *V*(*G*), we claim that S is a power dominating set of *G*.

Thus $M^0(S) = N[S] = \{a_{j-1}, a_j, a_{j+1}, b_j, b_{j+1}, b_{j+2}, c_{j+1}, d_{j+1}\}$ where $0 \leq j \leq n-1 (\mathrm{mod}\, n)$. We observe that each of the vertices $\{a_{j+1}, b_{j+1}\}$ in $M^0(S)$ is adjacent to exactly one unmonitored vertex namely, $\{a_{j+2}, c_j\}$ respectively and hence $\{a_{j+2}, c_j\}$ is monitored. Thus $M^1(S) = \{a_{j-1}, a_j, a_{j+1}, b_j, b_{j+1}, b_{j+2}, c_{j+1}, d_{j+1}, a_{j+2}, c_j\}$ $0 \leq j \leq n-1 (\mathrm{mod}\, n)$. Proceeding inductively, for

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each vertex $x \in M^j(S)$ such that $\left|N[x] \setminus M^j(S)\right| \leq 1$ at each inductive step *j*, $j \geq 2,$ find that all the vertices of *G* are observed. Hence the proof.

In the following section we compute the power domination number of convex polytope U_n

3 Convex Polytope U_n

The graph of convex polytope U_n has a vertex set $V = \{a_j, b_j, c_j, d_j, e_j : j = 0,1 \cdots n-1\}$ and edge set $E = \{a_j\, a_{j+1}, b_j b_{j+1}, a_j b_j, b_j c_j, c_j d_j, c_{j+1} d_j, d_j e_j, e_j e_{j+1} : j = 0,1 \cdots, n-1\}.$

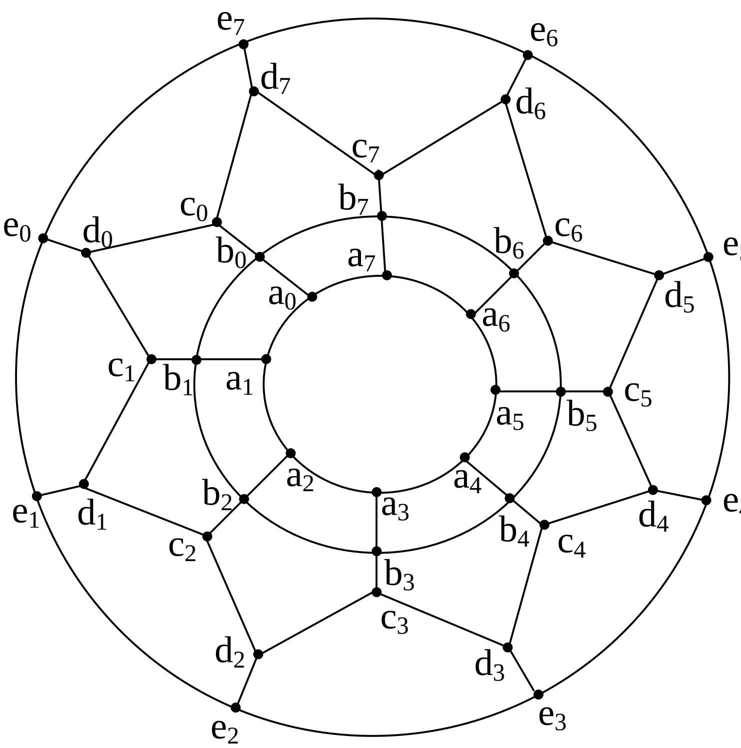

Fig-2 Convex polytope U_8

Theorem 3.1. If $G = U_n$ then $\gamma_p(G) = 3.$

Proof. Let *G* be a convex polytope U_n. Consider the set $S = \{a_j, c_{j+1}, e_{j+2}\}$ $0 \leq j \leq n-1 \ (\mathrm{mod}\, n)$ in *V*(*G*), we claim that *S* is a power dominating set of *G* with $|S| = 3.$ Thus $M^0(S) = N[S] = \{a_j, a_{j+1}, a_{j-1}, b_j, c_{j+1}, d_{j+1}, d_j, b_{j+1}, e_{j+2}, d_{j+2}, e_{j+1}, e_{j+3}\}$ $0 \leq j \leq n-1 \ (\mathrm{mod}\, n).$

Observe that each of the vertices $\{d_j, d_{j+1}, b_{j+1}, a_{j+1}\}$ in $M^0(S)$ is adjacent to exactly one unmonitored vertex namely, $\{e_j, a_{j+2}, b_{j+2}, c_{j+2}\}$ respectively and hence $\{e_j, a_{j+2}, b_{j+2}, c_{j+2}\}$ is monitored.

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Thus $M^1(S)=\{a_j,a_{j+1},a_{j-1},b_j,c_{j+1},d_{j+1},d_j,b_{j+1},e_{j+2},d_{j+2},e_{j+1},e_{j+3},e_j,a_{j+2},b_{j+2},c_{j+2}\}$ Where $0\le j\le n-1\ (\mathrm{mod}\, n)$. Proceeding inductively, for every vertex $v\in M^j(S)$ such that $\left|N[v]\setminus M^j(S)\right|\le 1$ at each inductive step *j*, $j\ge 2$, find that all the vertices of *G* are monitored. Further, observe that no set of two vertices will monitor the entire graph. Hence the proof.

In the next section we determine the power domination number of convex polytopes. S_n.

4 Convex Polytope S_n

The graph of convex polytope S_n has vertex set $V(S_n)=\{a_j,b_j,c_j,d_j : j=0,1,\cdots n-1\}$ and the edge set $E(S_n)=\{a_ja_{j+1},b_jb_{j+1},a_jb_j,b_jc_j,c_jd_j,a_{j+1}b_j,d_jd_{j+1},c_jc_{j+1}: j=0,1\cdots n-1\}$.

Theorem 4.1. If $G=S_n$ then $\gamma_p(G)=3$.

Proof. Let *G* be a convex polytope S_n. Consider the set $S=\{a_j,c_{j+1},d_{j+2}\}\ 0\le j\le n-1\ (\mathrm{mod}\, n)$ in *V*(*G*), we claim that *S* is a power dominating set of *G* with $|S|=3$. Thus $M^0(S)=N[S]=\{a_j,a_{j-1},a_{j+1},b_j,b_{j-1},c_{j+1},d_{j+1},c_j,c_{j+2},d_{j+2},d_{j+3}\}\quad 0\le j\le n-1\ (\mathrm{mod}\, n)$. Observe that each of the vertices a_{j+1},d_{j+1} in $M^0(S)$ is adjacent to exactly one unmonitored vertex a_{j+2},d_j respectively and hence a_{j+2},d_j is monitored.

Thus $M^1(S)=\{a_j,a_{j-1},a_{j+1},b_j,b_{j-1},c_{j+1},d_{j+1},c_j,c_{j+2},d_{j+2},d_{j+3},a_{j+2},d_j\}\quad 0\le j\le n-1(\mathrm{mod}\, n)$. Proceeding inductively, for every vertex $v\in M^j(S)$ such that $\left|N[v]\setminus M^j(S)\right|\le 1$ each inductive step *j*, $j\ge 2$ find that all the vertices of *G* are monitored. Further, observe that no set of two vertices will monitor the entire graph. Hence the proof.

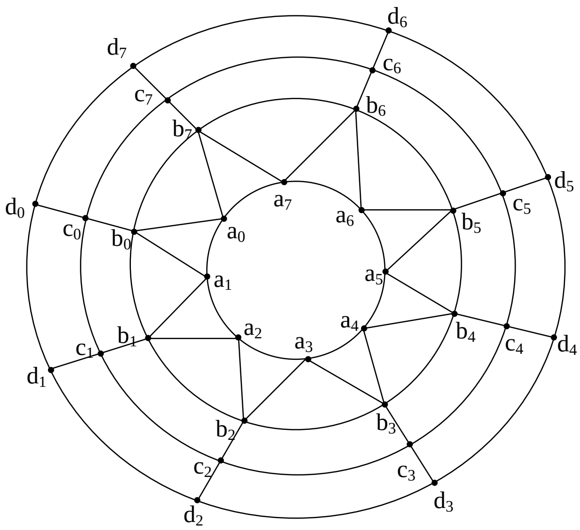

Fig-3 Convex polytope S_8

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In the next section we determine the power domination number of convex polytopes T_n.

5 Convex Polytope T_n

The graph of convex polytops T_n has vertex set $V(T_n) = \{a_j, b_j, c_j, d_j : j = 0, 1, \cdots n-1\}$ and edge set $E(T_n) = \{a_j a_{j+1}; b_j b_{j+1}; a_j b_j; b_j c_j; c_j d_j; a_{j+1} b_j; d_j d_{j+1}; c_j c_{j+1}; c_{j+1} d_j : j = 0, 1, \cdots n-1\}$.

Theorem 5.1. If $G = T_n$ then $\gamma_p(G) = 2$.

Proof. Let *G* be a convex polytope T_n. Clearly a single vertex cannot monitor the entire graph *G*. Hence $\gamma_p(G) \geq 2$. Consider the set $S = \{a_j, c_{j+1}\}$, $0 \leq j \leq n-1 \pmod n$ in *V*(*G*), we claim that S is a power dominating set of *G*.

Thus $M^0(S) = N[S] = \{a_{j-1}, b_{j-1}, b_j, a_j, a_{j+1}, c_{j+1}, d_j, d_{j+1}, c_j, c_{j+2}, b_{j+1}\}$ $0 \leq j \leq n-1 (\bmod n)$. Observe that each of the vertices $\{a_{j+1}, b_j, d_j, d_{j+1}\}$ in $M^0(S)$ is adjacent to exactly one unmonitored vertices namely, $\{d_{j-1}, d_{j+2}, b_{j-1}, a_{j+2}\}$ are respectively, and hence $\{d_{j-1}, d_{j+2}, b_{j-1}, a_{j+2}\}$ are monitored.

Therefore the set $M^1(S) = \{a_{j-1}, b_{j-1}, b_j, a_j, a_{j+1}, c_{j+1}, d_j, d_{j+1}, c_j, c_{j+2}, b_{j+1}, d_{j-1}, d_{j+2}, b_{j-1}, a_{j+2}\}$ $0 \leq j \leq n-1 (\bmod n)$. Proceeding inductively, for every vertex $u \in M^j(S)$ such that $|N(u) \setminus M^j(S)| \leq 1$ at each inductive step *j*, $j \geq 2$ find that all the vertices of *G* are monitored.

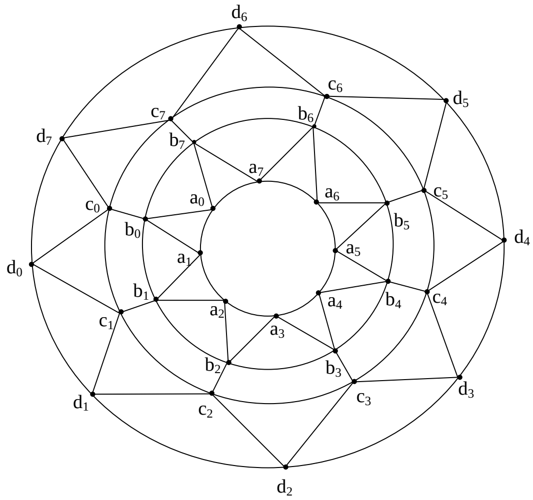

Fig-4 Convex polytope T_8

In the following section we determine the power domination number of convex polytopes R''_n.

6 Convex Polytope R''_n

The graph of convex polytope R''_n has a vertex set $V = \{a_j, b_j, c_j, d_j, e_j, f_j : j = 0, 1 \cdots n-1\}$ and edge set $E = \{a_j a_{j+1}, a_j b_j, b_j c_j, b_{j+1} c_j, c_j d_j, d_j e_j, d_{j+1} e_j, e_j f_j, f_j f_{j+1} : j = 0, 1 \cdots n-1\}$.

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Theorem 6.1. If $G = R''_n$ then $\gamma_p(G) = 3$.

Proof. Let *G* be a convex Polytope R''_n. Consider the set $S = \{a_j, c_{j+1}, e_{j+3}\}$ $0 \le j \le n-1 \,(\bmod n)$ in *V*(*G*), we claim that *S* is a power dominating set of *G* with $|S| = 3$. Then $M^0(S) = N[S] = \{a_j, a_{j-1}, a_{j+2}, c_{j+1}, d_{j+1}, b_{j+1}, b_{j+2}, e_{j+3}, f_{j+3}, d_{j+3}, d_{j+2}, b_j\}$ $0 \le j \le n-1 \,(\bmod n)$. Now each of the vertices $\{a_{j+1}, b_{j+1}\}$ in $M^0(S)$ is adjacent to exactly one unmonitored vertices namely, $\{c_j, a_{j+2}\}$ are respectively, and hence $\{c_j, a_{j+2}\}$ are monitored.

Therefore the set $M^1(S) = \{a_j, a_{j-1}, a_{j+2}, c_{j+1}, d_{j+1}, b_{j+1}, b_{j+2}, e_{j+3}, f_{j+3}, d_{j+3}, d_{j+2}, b_j,, c_j, a_{j+2}\}$ $0 \le j \le n-1 (\bmod n)$. Proceeding inductively, for every vertex $x \in M^j(S)$ such that $|N(x) \setminus M^j(S)| \le 1$ at each inductive step *j*, $j \ge 2$, find that all the vertices of *G* are monitored. Further, observe that no set of two vertices will monitor the entire graph. Hence the proof.

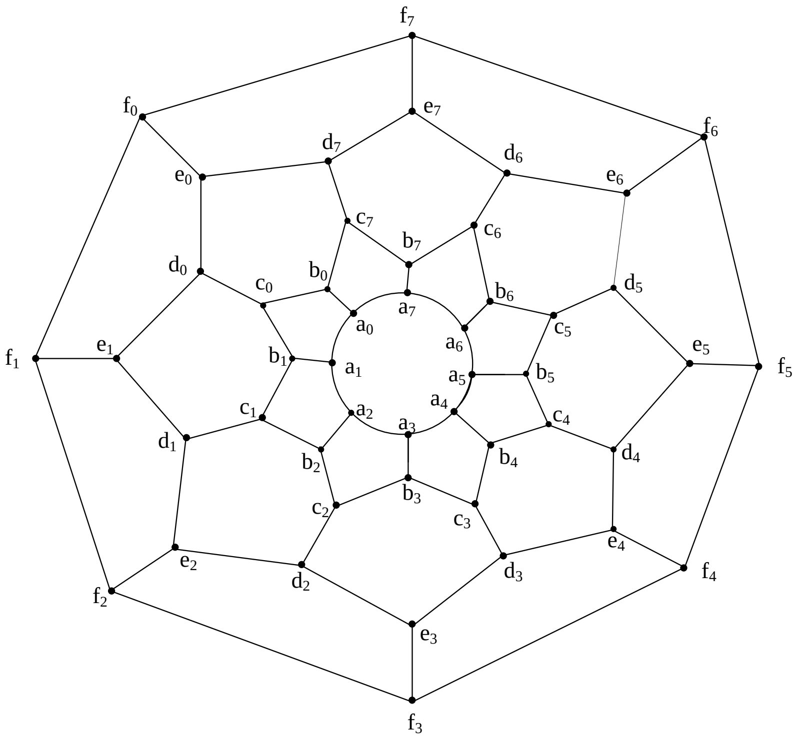

Fig-5 Convex polytope R''_8

.**7. Conclusion.** The power domination number of *G*, denoted by $\gamma_p(G)$ is the minimum cardinality of a power dominating set of *G*. In this article, we computed the power domination number of Convex polytopes D_n, S_n, U_n, T_n and R''_n.

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