JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3333-3337
https://publishoa.com
ISSN: 1309-3452

Steiner certified domination in fuzzy middle and splitting graphs

Priscilla Pacifica. G[1], JenitAjitha. J[2]

[1]Assistant Professor, Department of Mathematics, St.Marys' College (Autonomous), Thoothukudi, Tamilnadu, India
[2]Research Scholar (Reg no 19222212092011), Department of Mathematics, St. Marys' College (Autonomous), Thoothukudi, (Affiliated to Manonmaniam Sundaranar University, Tirunelveli), Tamilnadu, India

Received 2022 March 25; **Revised** 2022 April 28; **Accepted** 2022 May 15.

Abstract

In this article some new results on fuzzy steiner certified domination are established. Bounds on fuzzy steiner certified domination number of fuzzy middle graphs and fuzzy splitting graphs of some standard fuzzy graphs are acquired.

AMS Subject Classification 2010: 05C72, 05C69, 51E10

Key words : fuzzy steiner certified domination, fuzzy steiner certified domination number, fuzzy splitting graphs, fuzzy middle graphs

1. Introduction

In crisp graphs, the study of certified domination has been instigated by M.Dettlaff et.al in 2018 [3]. The concept of steiner domination in crisp graphs was studied from [2],[4],[5] and [7]. Domination in fuzzy graphs has been studied from [1] and the notion of fuzzy graph theory has been studied from [6].Fuzzy steiner domination number of a fuzzy graph G is the fuzzy cardinality of a minimum steiner certified dominating set of G. For a connected fuzzy graphG(V, σ, μ), a subset of nodes C of V(G) is said to be steiner certified dominating set if it it is both steiner set as well as certified dominating set. The fuzzy Steiner interval, FI(S) of a non empty subset of nodes S is defined as the set of all nodes which lie in some steiner tree of S. If FI(S)=V(G) then S is called a fuzzy Steiner set of G. A set of nodes C is said to be certified if each node in the set has either zero or two neighbours in $V(G) - C$. A non-empty subset S of V is called a fuzzy Steiner dominating set if S is a fuzzy dominating set and a fuzzy Steiner set of G. The minimum fuzzy cardinality of a minimal steiner certified dominating set is called fuzzy steiner certified dominating number denoted by $\gamma_{scer}^{f}(G)$ and the corresponding set of nodes is called γ_{scer}^{f}-set. The maximum fuzzy cardinality of a minimal steiner certified dominating set is called upper fuzzy steiner certified dominating number denoted by $\Gamma_{scer}^{f}(G)$.

2. Steiner Certified Domination in fuzzy middle graphs

2.1 Definition

Let $G(V,\sigma,\mu)$ be a fuzzy graph with node set V and arc set E. The fuzzy middle graph of G denoted by $M^f(G)(V',\rho,\lambda)$ is defined as follows. It has node set $V' = V_1 \cup V_2$ and arc set $E' = E_1 \cup E_2$ where $V_1 = V$ and $V_2 = E$ and

$$E_1 = \{e_1e_2/e_1 \& e_2 \ are\ adjacent\ arcs\ in\ G\}$$

$E_2 = \{ue/u \in V, e \in E\ and\ e\ is\ incident\ with\ u\}$. Also ρ and λ are defined as

$$\rho(z) = \begin{cases}\sigma(z) & if\ z \in V_1\\ \mu(z) & if\ z \in V_2\end{cases} \lambda(e) = \begin{cases}\mu(x) \wedge \mu(y) & if\ e = xy \in E_1\\ \sigma(u) \wedge \mu(y) & if\ e = uy \in E_2\end{cases}$$

2.2 Theorem

For a fuzzy path graph P_n^f, the steiner certified domination number of the fuzzy middle graph is $\gamma_{scer}^{f}\left(M^f(P_n^f)\right) = p$ where p is the order of $M^f(P_n^f)$.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3333-3337
https://publishoa.com
ISSN: 1309-3452

Proof :

A fuzzy path graph P_n^f with 'n' nodes has n-1 arcs. Let $u_1, u_2, \ldots . u_n$ be the nodes and $e_1, e_2, \ldots . e_{n-1}$ be the arcs of P_n^f. Then the fuzzy middle graph $M^f(P_n^f)$ has 2n-1 nodes. Now the nodes $u_1, u_2, \ldots . u_n$ are all extreme nodes in $M^f(P_n^f)$. Since any steiner set must contain all the extreme nodes, these nodes must be in any steiner set of $M^f(P_n^f)$. Also any spanning tree of $C = \{u_1, u_2, \ldots . u_n\}$ must contain the nodes $e_1, e_2, \ldots . e_{n-1}$. Hence C is a steiner dominating set. But C is not certified because the each of the leaves has only one neighbour in $V(M^f(P_n^f)) - C$ and also all the supports must lie in any certified dominating set. If these two end nodes are included in C, then the resulting set is certified but not a steiner set. Hence only $V(M^f(P_n^f))$ is the only steiner certified dominating set. Thus if p is the order of $M^f(P_n^f)$, then $\gamma_{scer}^f(M^f(P_n^f)) = p$.

2.3 Theorem

$\gamma_{scer}^f(M^f(C_n^f)) \leq \frac{p}{2}ifC_n^f$ is a cycle fuzzy graph with n nodes and p is the order of the fuzzy middle graph.

Proof :

The fuzzy middle graph $M^f(C_n^f)$ has 2n nodes. Let $u_1, u_2, \ldots . u_n$ be the nodes and $e_1, e_2, \ldots . e_n$ be the arcs of C_n^f. Any steiner certified dominating set contains the nodes $u_1, u_2, \ldots . u_n$ because these nodes are extreme nodes. Now each node $e_j, 1 \leq j \leq n$ lies in some spanning tree of the nodes $C = \{u_1, u_2, \ldots . u_n\}$. Hence these set of nodes forms a steiner set. Each u_i is adjacent to two of the nodes in $e_1, e_2, \ldots . e_n$. Thus the set C is a steiner certified dominating set. Since each node in C is an extreme node C is the minimal such set with n nodes. If p is the order of $M^f(C_n^f)$ then C has order atmost$\frac{p}{2}$. Hence the result.

2.4 Theorem

$\gamma_{scer}^f(M^f(K_n^f)) = p'$where$p'$ is the order of the complete fuzzy graph with n nodes K_n^f and $\gamma_{scer}^f(M^f(K_2^f)) \leq 2$.

Proof :

There are n nodes and $\frac{n(n-1)}{2}$ arcs in K_n^f. The fuzzy middle graph $M^f(K_n^f)$ has $\frac{n(n+1)}{2}$ nodes. Let $u_1, u_2, \ldots . u_n$ be the nodes and $e_1, e_2, \ldots . e_k$ where $k = \frac{n(n-1)}{2}$ be the arcs of K_n^f. Now each u_i is an extreme node in $M^f(K_n^f)$.

Consider the set of nodes $C = \{u_1, u_2, \ldots . u_n\}$. Since each u_i is incident with n-1 arcs in K_n^f, in $M^f(K_n^f)$, u_i is adjacent to n-1 nodes in the set of nodes $e_1, e_2, \ldots . e_k$. Also C is a steiner dominating set. Hence C is a steiner certified dominating set. Since all the nodes in C are extreme nodes, removal of any node results in a set of nodes which does not form a steiner set and hence C is the minimum such set with n nodes which are the nodes of K_n^f. Therefore if p' is the order of K_n^f, then $\gamma_{scer}^f(M^f(K_n^f)) = p'$.

For n=2, u_1 and u_2 are adjacent to exactly one node each. Therefore there is only one steiner certified dominating set which is $V(M^f(K_2^f))$. Thus $\gamma_{scer}^f(M^f(K_2^f)) \leq 2$.

2.5 Theorem

If $K_{m,n}^f$ is the complete bipartite fuzzy graph, then $\gamma_{scer}^f(M^f(K_{m,n}^f)) = p'$ where p' is the order of $K_{m,n}^f$.

Proof :

Let $u_1, u_2, \ldots . u_m$, $v_1, v_2, \ldots . v_n$ are the nodes and $e_1, e_2, \ldots . e_{mn}$ are the arcs of $K_{m,n}^f$. Any complete bipartite fuzzy graph has with m+n nodes has mn arcs. For $m \geq 2, n \geq 2$ each u_i is adjacent to n nodes in the set $e_1, e_2, \ldots . e_{mn}$ and each v_j is adjacent to m nodes in the above set.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3333-3337
https://publishoa.com
ISSN: 1309-3452

As in the proof of previous theorems, all the nodes of $K_{m,n}^f$ are extreme nodes in $M^f(K_{m,n}^f)$ and by similar argument these nodes forms the minimum steiner certified dominating set. Hence the theorem.

3. Steiner Certified Domination in fuzzy splitting graphs

3.1 Definition

If $G(V,\sigma,\mu)$ is a connected fuzzy graph with node set V and arc set E, then the fuzzy splitting graph of G is denoted by $S^f(G)$ is attained by introducing for each node 'u' ,a node u' such that $N^f(u) = N^f(u')$ where $N^f(u)$ is the fuzzy neighbourhood of u. Also $\sigma(u') = \sigma(u)$and $\mu(u'v) = \sigma(u') \wedge \sigma(v) \forall v \in N^f(u)$.

3.2 Theorem

If P_n^f is the fuzzy path with n nodes, the steiner certified domination number of the fuzzy splitting graph is $\gamma_{scer}^f\left(S^f(P_n^f)\right) = p$ where p is the order of $S^f(P_n^f)$, $n = 2 \; or \; n > 3$. For n=3, $\gamma_{scer}^f\left(S^f(P_3^f)\right) \leq 4$.

Proof :

Let $u_1, u_2, \dots . u_n$ be the nodes of the fuzzy path and $u_1', u_2', \dots . u_n'$ be the corresponding new nodes of the splitting graph of P_n^f. For n=2, there are four nodes of which two are leaves and other two are their supports. Hence any steiner certified dominating set must contain all the nodes. For n=3, the set of nodes $C = \{u_1', u_2', u_3', u_2\}$ is a steiner certified dominating set and this set is the minimum. Hence $\gamma_{scer}^f\left(S^f(P_3^f)\right) \leq 4$.

For $n > 3$, Anysteiner set must contain the nodes $u_1', u_2', \dots . u_n'$. But the end nodes u_1' and u_n' has only one neighbour among the remaining nodes and hence this set is not certified. Now if these two nodes are included then the resulting set does not form a steiner set. Also if any one of the remaining nodes is included in this set, then the set is not certified. Hence there is no non-trivial steiner certified dominating set. Therefore the only steiner certified dominating set is $V(S^f(P_n^f))$. Hence the theorem.

3.3 Theorem

$\gamma_{scer}^f\left(S^f(C_n^f)\right) \leq \frac{p}{2}$where$S^f(C_n^f)$ is the fuzzy splitting graph of the cycle fuzzy graph C_n^f and p is the order of the fuzzy splitting graph with $n \geq 3$.

Proof :

Let $u_1, u_2, \dots . u_n$ and $u_1', u_2', \dots . u_n'$ be the nodes of $S^f(C_n^f)$ such that $u_1, u_2, \dots . u_n$ are the nodes of C_n^f and $u_1', u_2', \dots . u_n'$ are the new nodes respectively. Let $C = u_1', u_2', \dots . u_n'$. This set is certified because each node u_i' is adjacent to two of the nodes in $V(S^f(C_n^f)) - C$. Any spanning tree of C must contain some nodes from the set $\{u_1, u_2, \dots . u_n\}$ and any node u_i must be in some spanning tree of C. Hence C is a steiner certified dominating set with nodes.

Claim 1: C is minimal

Consider $C - \{u_i'\}$where $1 \leq i \leq n$. Since no two nodes in C are adjacent and each u_i' can be reached from any node u_j by one or two arcs, each spanning tree of $C - \{u_i'\}$ does not traverse through u_i'. Hence $C - \{u_i'\}$ is not a steiner set. Hence claim 1.

Claim 2 : C is minimum

Suppose there exists a set of nodes S such that S has less number of nodes than C. Since C is minimal, it is clear that $S \not\subseteq C$. If $S \subseteq \{u_1, u_2, \dots . u_n\}$ then any spanning tree of S does not traverse through the nodes of C which gives a contradiction to S is a steiner set. Therefore S must contain atleast one node from C and atleast one node from the set $\{u_1, u_2, \dots . u_n\}$. Since S has less number of nodes than C, there exist a node u_i' for some i where $1 \leq i \leq n$ such that $u_i' \notin S$. It can be observed that for any two nodes sayu_i' and u_j' there are either two three arcs in the shortest path

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3333-3337
https://publishoa.com
ISSN: 1309-3452

connecting $u_i{}'$and $u_j{}'$. But there are either one or two arcs in the shortest path connecting any node $u_i{}'$ and any node u_j. Thus $u_i{}'$ will not be in any spanning tree of S. This shows that S cannot be a steiner set which is a contradiction. Hence any steiner set must include all the nodes in C. Hence claim 2.

Therefore C is the minimum steiner certified set with n nodes. Since $S^f(C_n^f)$ has 2n nodes, if p is the order of $S^f(C_n^f)$then $\gamma_{scer}^f\left(S^f(C_n^f)\right) \leq \frac{p}{2}$.

3.4 Theorem

If G is either a complete fuzzy graph or a complete bipartite fuzzy graph then the steiner certified domination number of splitting graph of G is $\gamma_{scer}^f(S^f(G)) \leq \frac{p}{2}$ where p is the order of the splitting graph.

Proof :

Assume that G is a complete fuzzy graph with n nodes. Let $u_1, u_2, \ldots. u_n$ and $u_1{}', u_2{}', \ldots. u_n{}'$ be the nodes of $S^f(G)$. Here the nodes $C = u_1{}', u_2{}', \ldots. u_n{}'$ are extreme nodes and so any steiner certified dominating set must contain these nodes. Also since any node u_{i} is in some spanning tree of C, C is a steiner dominating set. Any node $u_j{}'$ is adjacent to atleast two nodes in the set $\{u_1, u_2, \ldots. u_n\}$. Thus C is a steiner certified dominating set. Since each node in C is extreme, C is the minimum such set with n nodes. Hence the result holds for G.

Now let us assume that G is a complete bipartite fuzzy graph. There are no extreme nodes in the splitting graph of G. Suppose $u_1, u_2, \ldots. u_m, v_1, v_2, \ldots. v_n$ and $u_1', u_2', \ldots. u_m', v_1{}', v_2{}', \ldots. v_n{}'$ are the nodes of $S^f(G)$.

Let $C = \{u_1', u_2', \ldots. u_m', v_1', v_2', \ldots. v_n'\}$. As in the proof of theorem 3.3, the nodes of C must be in any steiner set and C is the minimum steiner certified dominating set with n nodes. Hence the theorem holds for G.

3.5 Theorem

$\gamma_{scer}^f\left(S^f(F_n^f)\right) \leq \frac{p}{2}$ where p is the order of $S^f(F_n^f)$, F_n^f is the friendship fuzzy graph.

Proof :

It is known that the friendship fuzzy graph is created by connecting 'n' duplicates of cycle fuzzy graph with 3 nodes with single node. F_n^fhas 2n+1 nodes and the fuzzy splitting graph of F_n^f has 4n+2 nodes. Let u be the common node of F_n^f. Assume that $v_1, v_2, \ldots. v_{2n}$ are the other nodes of $F_n^f$and $u', v_1{}', v_2{}', \ldots. v_{2n}{}'$ are the new nodes. Since $v_1{}', v_2{}', \ldots. v_{2n}{}'$ are extreme nodes any steiner set contains all these nodes. Now consider the set $C = \{v_1', v_2', \ldots. v_{2n}', u'\}$. C is certified because every v_i' is adjacent to two nodes in $V(S^f(F_n^f)) - C$ and u' is adjacent to atleast two of the nodes $v_1, v_2, \ldots. v_{2n}$. Any spanning tree of C traverse through u and since u' is not adjacent to any of the nodes in C it traverse through atleast one node from $v_1, v_2, \ldots. v_{2n}$. So C is a steiner set. Hence C is a steiner certified dominating. Since all the nodes in C are extreme, C is the minimum set. Hence the result.

3.6 Theorem

If W_n^f is the wheel fuzzy graph, $\gamma_{scer}^f\left(S^f(W_n^f)\right) \leq \frac{p}{2}$ where p is the order of $S^f(W_n^f)$.

Proof :

In a wheel fuzzy graph W_n^f, there are n+1 nodes and in its splitting fuzzy graph there are 2n+2 nodes. Assume that u is the apex node $v_1, v_2, \ldots. v_n$ of W_n^f are the rim nodes of $F_n^f$and $u', v_1{}', v_2{}', \ldots. v_n{}'$ are the corresponding new nodes. Here $u', v_1{}', v_2{}', \ldots. v_n{}'$ are not extreme nodes. By similar argument as in proof of theorem 3.3, all these nodes are essential for any steiner set. Also as in the proof of previous theorem, the set $C = \{u', v_1', v_2', \ldots. v_n'\}$ is a steiner certified dominating set. Since u' is adjacent to only the nodes $v_1, v_2, \ldots. v_n$ any spanning tree of $C - \{u'\}$ does not contain u'.

$C - \{v_i'\}$is not a steiner set for any i, because any spanning tree of $C - \{v_i'\}$ does not contain v_i'. Therefore C is the minimum steiner certified dominating set with n+1 nodes. Hence the theorem.

4. Conclusion

This article acquires the bounds on fuzzy steiner certified domination numbers of fuzzy middle graphs of path, cycle, complete, complete bipartite fuzzy graphs and fuzzy splitting graphs of path, cycle, complete, complete bipartite, wheel graph and friendship graphs.

References

1. Benjier H.Arriola, A note on some domination of fuzzy graphs, Far East Journal of Applied Mathematics,(2017) 113-123.
2. G. Chartrand, P. Zhang, The Steiner number of a graph, Discrete Math., 242 (2002), pp. 41-54
3. M.Dettlaff, M.Lemanska, J.Topp, R.Ziemann, P.Zylinski, ‘Certified Domination’, AKCE International Journal of Graphs and Combinatorics(2020), https://doi.org/10.1016/j.akcej.2018.09.04.
4. J. John, G. Edwin and P. Arul Paul Sudhahar, The Steiner Domination Number of a Graph, International Journal of Mathematics and Computer Application Research, Vol 3(3), (2013), 37 - 42,
5. K.Ramalakshmi,K.Palani,On Steiner Domination number of graphs, International Journal of Mathematics Trends and Technology-Volume 4 Number 2-January 2017.
6. M.S Sunitha and Sunil Mathew, ‘Fuzzy Graph Theory: A Survey, Annals of pure and applied mathematics’,(2013),92-110
7. S.K.Vaidya and S.H.Harkar, ‘Steiner Domination number of some graphs’, International Journal of Mathematics and Scientific Computing(ISSN : 2231-5330), Vol 5,No.1, 2015