

Distance Closed G-Eccentric Domination in Fuzzy Graphs

A. Mohamed Ismayil

Associate Professor
 Department of Mathematics
 Jamal Mohamed College
 (Affiliated to Bharathidasan University)
 Tiruchirappalli -620 020, Tamil Nadu, INDIA.

S. Muthupandiyan*

Research Scholar
 Department of Mathematics
 Jamal Mohamed College
 (Affiliated to Bharathidasan University)
 Tiruchirappalli -620 020, Tamil Nadu, INDIA.

*Corresponding Author. E-mail:muthupandiyanmaths@gmail.com

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Abstract. A dominating set D of the vertex set $V(G)$ in a fuzzy graph $G(\tau, \omega)$ is said to be a g-eccentric dominating set if for every $b \in V - D$, $\exists$ at least one g-eccentric vertex a of b in D . A subset $D \subseteq V(G)$ of a fuzzy graph $G(\tau, \omega)$ is said to be distance closed set if for every vertex $a \in D$ and for every vertex $c \in V - D$, $\exists$ at least one vertex $b \in D$ such that $d_g(a, b)(< D >) = d_g(a, c)(in G)$. A subset D of a vertex set $V(G)$ in a fuzzy graph $G(\tau, \omega)$ is said to be distance closed g-eccentric dominating set if $< D >$ is distance closed and D is g-eccentric dominating set. The lowest cardinality of the distance closed g-eccentric dominating set is called the distance closed g-eccentric domination number of G . This article discusses the distance closed g-eccentric dominating set and its number in fuzzy graphs. Bounds for the distance closed g-eccentric domination number are found for several fuzzy graph types. Several theorems, results, and observations are presented on distance closed g-eccentric dominating sets and numbers in fuzzy graph

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I Introduction

Rosenfeld [7] is a man who invented the concept of fuzzy graphs (abbreviated as FG). In the year 2010, T.N. Janakiraman et al. [3] introduced the distance closed domination in graph. In 2010, T.N. Janakiraman et al. [4] developed the notion of eccentric domination in graph. Linda.J.P and M.S.Sunitha [5] proposed the concept of g-eccentric nodes, g-boundary nodes, and g-interior nodes of a FG in 2012. The concept of a graph's distance closed eccentric domination number was initially introduced by M. Bhanumathi and Sudha Senthil [2] in 2016. Mohamed Ismayil and Muthupandiyan proposed g-eccentric domination in FG in 2020 [6].

The concepts of a distance closed g-eccentric point set, a distance closed g-eccentric dominating set, and corresponding FG numbers are introduced in this article. Theorems and proofs on distance closed g-eccentric dominating sets are presented. For some typical FG, bounds on distance closed g-eccentric domination number are found.

For unknown graph and FG notions, the reader should refer [1, 7, 8]. In this research, only connected FG are consider.

Definition 1.1. [6,8] A FG $G(\tau, \omega)$ is characterized with two functions τ on V and ω on $\subseteq V \times V$, where $\tau : V \rightarrow [0,1]$ and $\omega : E \rightarrow [0,1]$ such that $\omega(a, b) \leq \tau(a) \wedge \tau(b), \forall a, b \in V$. We anticipate that V is a nonempty finite set, ω is reflexive and symmetric functions. We indicate the crisp graph $G^* = (\tau^*, \omega^*)$ of the fuzzy graph $G(\tau, \omega)$ where

$\tau^* = \{a \in V : \tau(a) > 0\}$ and $\omega^* = \{(a, b) \in E : \omega(a, b) > 0\}$. The order and size of a fuzzy graph $G(\tau, \omega)$ are denoted and defined by $p = \sum_{a \in V} \tau(a)$ and $q = \sum_{ab \in E} \omega(a, b)$ respectively.

Definition 1.2. [6, 8] An edge (a, b) is strong (or strong arc) in a FG $G(\tau, \omega)$ if $\omega(a, b) \geq \omega^\infty(a, b) = \text{CONN}_{G-(a,b)}(a, b)$. A path P in a FG of length n is a sequence of distinct nodes $a_0, a_1, \dots, a_n$ such that $\omega(a_{i-1}, a_i) > 0, i = 1, 2, \dots, n$ and the strength of the path P is $s(P) = \min\{\omega(a_{i-1}, a_i), i = 1, 2, \dots, n\}$.

Definition 1.3. [6, 8] Let $G(\tau, \omega)$ be a fuzzy graph. If (a, b) is strong then b is called strong neighbors of a . The strong neighborhood of a is the collection of all of its strong neighbors and represented by $N_s(a)$. The closed strong neighborhood of a is $N_s[a] = N_s(a) \cup \{a\}$. The strong degree of a vertex $b \in \tau^*$ is defined as the sum of membership values of all strong edges occurring at b and it is denoted by $d_s(b)$. Also it is defined by $d_s(b) = \sum_{a \in N_s(b)} \omega(a, b)$ where $N_s(b)$ denotes the set of all strong neighbors of b .

Definition 1.4. [1, 6] The distance between two vertices in a graph $G(V, E)$ is the number of edges in a shortest path (graph geodesic) connecting them, denoted by $d(a, b)$. A strong path P in $G(\tau, \omega)$ from a to b is called geodesics if there is no shorter strong path from a to b and a length of $a - b$ geodesic is the geodesic distance (g-distance) from a to b denoted by $d_g(a, b)$.

Definition 1.5. [5, 6] The geodesic eccentricity (g-eccentricity) $e_g(a)$ of a vertex $a \in V$ in a connected FG $G(\tau, \omega)$ is characterized by $e_g(a) = \max\{d_g(a, b), b \in V\}$. The least g-eccentricity among the vertices of G is called g-radius and indicated by $r_g(G) = \min\{e_g(a), a \in V\}$ and the greatest g-eccentricity among the vertices of G is called g-diameter and indicated by $d_g(G) = \max\{e_g(a), a \in V\}$. A vertex b is g-central vertex if $e_g(b) = r_g(G)$. Moreover, a vertex b in G is g-peripheral vertex if $e_g(b) = d_g(G)$.

Definition 1.6. [5] Let $a, b \in V(G)$ be any two vertices in a FG $G(\tau, \omega)$. A vertex a at g-distance $e_g(b)$ from b is a g-eccentric point of b . The g-eccentric set of a vertex b is defined and the domination number is symbolised by $E_g(b) = \{a : d_g(a, b) = e_g(b)\}$.

Definition 1.7. [6] The set $S \subseteq V$ in a FG $G(\tau, \omega)$ is g-eccentric point set if for each $b \in V - S$, there exists at least one g-eccentric point a of b in S .

Definition 1.8. [6, 4] A dominating set $D \subseteq V(G)$ in a FG $G(\tau, \omega)$ is said to be a g-eccentric dominating set if each vertex $b \notin D$, then $\exists$ at least a g-eccentric vertex a of b in D . The least scalar cardinality taken over all g-eccentric dominating set is called g-eccentric domination number and the domination number is symbolised by $\gamma_{ged}(G)$.

Definition 1.9. [2] A subset $D \subseteq V(G)$ of a FG $G(\tau, \omega)$ is said to be distance closed set (DC-set) if for each vertex $a \in D$ and for each vertex $c \in V - D$, $\exists$ at least one vertex $b \in D$ such that $d_g(a, b) (\text{in } < D >) = d_g(a, c) (\text{in } G)$ and respectively.

II Distance Closed g-Eccentric Point Set in Fuzzy Graph

In this part, the distance closed g-eccentric point set and its numbers are defined in FG. Specific results, observations, and bounds on the distance closed g-eccentric number have been achieved for some classes of FG.

Definition 2.1. A sub set $S \subseteq V(G)$ of a FG $G(\tau, \omega)$ is said to be distance closed g-eccentric point set (DCgEP-set) if (i) $< S >$ is distance closed (ii) S is g-eccentric point set (gEP-set).

Definition 2.2. The least cardinality of all the distance closed g-eccentric point set S of a FG $G(\tau, \omega)$ is distance closed g-eccentric number and is accompanied by $e_{dcg}(G)$. The greatest cardinality of all the distance closed g-eccentric point set S of a fuzzy graph $G(\tau, \omega)$ is the upper distance closed g-eccentric number and is accompanied by $E_{dcg}(G)$.

Note 2.3. For any FG, the distance closed set have at least two vertices. Hence $e_{dcg} \leq 2$.

Example 2.4.

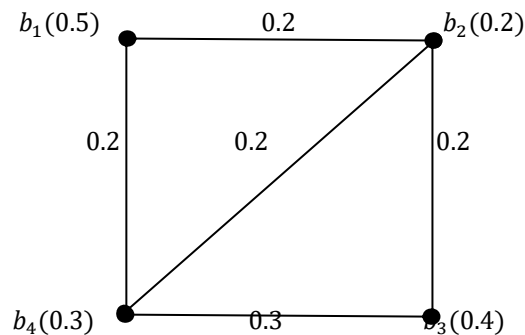


Figure 1

In the figure (1), we see that the DCgEP-sets $S_1 = \{b_2, b_3\}$, $S_2 = \{b_3, b_4\}$, $S_3 = \{b_1, b_2\}$ and $S_4 = \{b_1, b_4\}$. Hence, $e_{dcg}(G) = 0.6$ and $E_{dcg}(G) = 0.8$.

Observation 2.5.

1. If S is a DCgEP-set, then $S^0 \supset S$ is also an DCgEP-set.
2. If S is a minimal DCgEP-set, then $S^0 \subset S$ is not a DCgEP-set.
3. In T_τ , every DCgEP-set contains at least one pendent vertex.
4. For any FG $G(\tau, \omega)$, $e_{dcg}(G) \leq E_{dcg}(G)$.
5. The complement of an DCgEP-set need not be a DCgEP-set (See Figure 2).

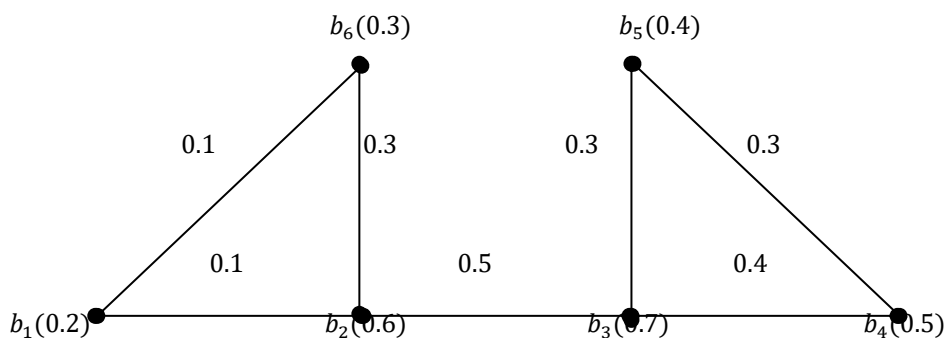


Figure 2

Example 2.6. In the figure (2), the set $S = \{b_1, b_4, b_5, b_6\}$ is DCgEP-set, but the complement of S is $\{b_2, b_3\}$ which is not a DCgEP-set.

Observation 2.7.

1. $e_{dcg}(K_\tau) \leq 2, |\tau^*| \geq 3$.
2. $e_{dcg}(S_\tau) \leq 2, |\tau^*| \geq 3$.

III Distance Closed g-Eccentric Dominating set in Fuzzy Graph

The distance closed g-eccentric dominating set and its number, as well as theorems relating to the number of distance closed g-eccentric dominating sets in FG are explored in this section.

Definition 3.1. A sub set $D \subseteq V(G)$ of a FG $G(\tau, \omega)$ is said to be a distance closed g-eccentric dominating set (DCgED-set) if

1. $\langle D \rangle$ is distance closed and
2. D is g-eccentric dominating set (gED-set).

The DCgED-set of a FG $G(\tau, \omega)$ is described as minimal if there does not exist any DCgED-set $S^0 \subset S$ in G . The least cardinality taken over all the minimal DCgED-set is called the distance closed g-eccentric domination number and is denoted as $\gamma_{dcged}(G)$. The greatest cardinality taken over all the minimal DCgED-set is called the upper distance closed g-eccentric domination number and is typified by $\Gamma_{dcged}(G)$.

Example 3.2.

In the figure (1), the DCgED-sets are $D_1 = \{b_3, b_2\}$, $D_2 = \{b_1, b_2\}$, $D_3 = \{b_3, b_4\}$ and $D_4 = \{b_1, b_4\}$. All the sets D_1, D_2, D_3 and D_4 are minimal DC-set and g-ED-set. Hence, $\gamma_{dcged}(G) = 0.6$ and $\Gamma_{dcged}(G) = 0.8$.

Note 3.3. The minimum DCgED-set in a FG is denoted by γ_{dcged} -set

Note 3.4. Every DCgED-set contains at least 2 vertices.

Observation 3.5.

1. Clearly, $0 < \gamma_{dcged}(G) \leq p$.
2. If D is a minimal DCgED-set, then the subset $D^0 \subset D$ is not a DCgED-set.
3. If D is a DCgED-set then the set $D'' \supset D$ is also a DCgED-set.
4. For any connected FG $G(\tau, \omega)$, $\gamma_{dcged}(G) \leq \Gamma_{dcged}(G)$.
5. If D is a DCgED-set then the complement $V - D$ is need not be a DCgED-set.

Example 3.6. In the figure (2), the set $S = \{b_1, b_4, b_5, b_6\}$ is DCgED-set, but the complement of S is $\{b_2, b_3\}$, not a DCgED-set.

Theorem 3.7. $\gamma_{dcged}(K_\tau) \leq 2, |\tau^*| \geq 3$.

Proof. Let $K_\tau = G(\tau, \omega)$ be a complete FG where $|\tau^*| \geq 3$, then $r_g(G) = d_g(G) = 1$. By note 3.4 let $D = \{a, b\}$. Here a or b dominates other vertices and is also a g-eccentric point of other vertices. Since $e_g(a) = e_g(b) = 1$, D is a DC-set. Therefore, D is a DCgED-set. Hence, $\gamma_{dcged}(K_\tau) \leq 2$.

Theorem 3.8 $\gamma_{dcged}(K_{\tau_1, \tau_2}) \leq 3, |\tau_1^*| = 1$ and $|\tau_2^*| = n, n \geq 3$.

Proof. Let $K_{\tau_1, \tau_2} = G(\tau, \omega)$ be a star FG where $|\tau_1^*| = 1$, and $|\tau_2^*| = n, n \geq 3$. Let $D = \{a, b, c\}$. Here a be a g-central vertex such that $d_s(a) = p$ in G . Clearly $r_g(G) = 1$ and $d_g(G) = 2$ and also a dominates all vertices in $V - D$ and every point of $V - D$ has a g-EP in D . Here any two non adjacent vertices b, c of eccentricity 2 together with a will form a DCgED-set. Hence, $\gamma_{dcged}(K_{\tau_1, \tau_2}) \leq 3$.

Theorem 3.9 $\gamma_{dcged}(K_{\tau_1, \tau_2}) \leq 4, |\tau_1^*| = m$ and $|\tau_2^*| = n$ and $m, n \geq 3$.

Proof. Let $G(\tau, \omega) = (K_{\tau_1, \tau_2})$ be a complete bipartite FG where $|\tau_1^*| = m$ and $|\tau_2^*| = n$ and $m, n \geq 3$ and $\tau = \tau_1 \cup \tau_2$ such that each element $|\tau_1^*|$ is adjacent to each element of $|\tau_2^*|$. Here every vertices of $G(\tau, \omega)$ has g-eccentricity equal to two with $r_g(G) = d_g(G) = 2$. Let $D = \{a, b, f, g\}$, $a, f \in m$ and $b, g \in n$, u dominates all the vertices of n and it is g-eccentricity of $m - \{u\}$, similarly b dominates all the vertices of m and it is the g-eccentric vertex of $n - \{b\}$. Hence, D is a minimum DCgED-set and hence, $\gamma_{dcged}(K_{\tau_1, \tau_2}) \leq 4$.

Note 3.10.

1. If $m = n = 1$, then $\gamma_{dcged}(K_{\tau_1, \tau_2}) \leq 2$.
2. If $m = n = 2$, then $\gamma_{dcged}(K_{\tau_1, \tau_2}) \leq 3$.
3. If $m = 1, n = 2$ or $m = 2, n = 1$, then $\gamma_{dcged}(K_{\tau_1, \tau_2}) \leq 3$.

Theorem 3.11. $\gamma_{dcged}(W_\tau) \leq 4, |\tau^*| \geq 4$.

Proof. Let $G(\tau, \omega) = W_\tau, |\tau^*| \geq 4$. Let $D = \{a, b, f, c\}$ where a, b and f are any three adjacent non g-central vertices and c is the g-central vertex. Clearly, $r_g(G) = 1$ and $d_g(g) = 2$. Hence, D is the DCgED-set. Therefore, $\gamma_{dcged}(W_\tau) \leq 4$.

IV Bounds for Distance Closed g-eccentric domination number of Fuzzy graph.

Bounds on the distance closed g-eccentric domination number are examined in this section.

Theorem 4.1. Let $K_\tau, |\tau^*| = n, n$ is even, and $G(\tau, \omega)$ be a FG obtained from complete FG K_τ by deleting edges of a linear factor, then $\gamma_{dcged}(G) = p$.

Proof. Let D be a γ_{dcged} -set. We know that $\gamma_{dcged}(G) \leq p$. If $\gamma_{dcged}(G) = p$, then $\gamma_{dcged}(G) < p$. This implies that $\exists a \in D$ such that $e_g(a/G) < e_g(a/G)$. Therefore, D is not a DC-set which is a contradiction to our assumption. Hence, $\gamma_{dcged}(G) = p$.

Theorem 4.2. If $G(\tau, \omega)$ be a self centered FG with a dominating edge which is not in a triangle, then $\gamma_{dcged}(G) \leq 4$.

Proof. Let $ab \in E(G)$ be a dominating edge of a FG $G(\tau, \omega)$ which is not in a triangle and let $ca, bf \in E(G)$, then $D = \{a, b, c, f\}$ is a γ_{dcged} -set. Hence, $\gamma_{dcged}(G) \leq 4$.

Theorem 4.3. Let $G(\tau, \omega)$ be a self centered FG then $\gamma_{dcged}(G) \leq 1 + 2\delta_s(G)$.

Proof. Let $b \in V(G)$ such that $d_s(b) = \delta_s(G)$. Let $\{b\} \cup N_s(b)$ is a gED-set. Consider $f \in N_s(b)$ such that f has no g-eccentric vertex in $N_s(b)$. Then f has g-eccentric vertex g in $N_2(b)$. Let $S \subseteq N_2(b)$ such that vertices in $N_s(b)$ have their g-eccentric vertices in D . Thus $D = \{v\} \cup N_s(b) \cup S$ form a DCgED-set. Hence, $\gamma_{dcged}(G) \leq 1 + \delta_s(G) + \delta_s(G) = 1 + 2\delta_s(G)$.

Theorem 4.4. Let $G(\tau, \omega)$ be FG with $r_g(G) > 2$. Then $\gamma_{dcged}(G) \leq p - \Delta_s(G) + 2$.

Proof. Let $a \in V(G)$ such that $d_g(a) = \Delta_s(G)$. Since $r_g(G) > 2$, vertices in $N_s(a)$ have their g-eccentric vertices in $V - N_s(a)$ only. Hence, $V - N_s(a)$ is a g-ED-set of $G(\tau, \omega)$. Consider, $N_s(a)$, if $N_s(a)$ is complete, $(V - N_s(a)) \cup \{b\}$, where $b \in N_s(a)$ and degree ≥ 2 is a DCgED-set. If $N_s(a)$ is not complete, let $f, g \in N_s(a)$ such that f and g are not adjacent in $N_s(a)$. Then $(V - N_s(a)) \cup \{f, g\}$ is a DCgED-set. Hence, $\gamma_{dcged}(G) \leq p - \Delta_s(G) + 2$.

Observation 4.5.

1. A vertex subset D of a FG $G(\tau, \omega)$ is a DCgED-set if and only if

- (i) $e_g(a / < D >) \geq e_g(a / G)$ for $a \in D$.
- (ii) Every node with g-eccentricity at most $d_g(G) - 1$ is a cut vertex, where $d_g(G)$ is the diameter of G .
- 2. A FG with $\gamma_{dcged}(G) = p$ is called a 0-distance closed g-eccentric dominating FG.
- 3. $\gamma_{dcged}(G) \leq 2$ iff $G(\tau, \omega)$ is complete.
- 4. If $G(\tau, \omega) \neq K_\tau$, then $\gamma_{dcged}(G) \geq 3$.

Theorem 4.6. In a FG $G(\tau, \omega)$, $r_g(G) = 1$, $d_g(G) = 2$ and $\gamma_{ged}(G) \leq 2$, then $\gamma_{dcged}(G) \leq 3$.

Proof. Given in a FG $G(\tau, \omega)$, $r_g(G) = 1$, $d_g(G) = 2$ and $\gamma_{ged}(G) \leq 2$. Let $D = \{f, g\}$ be a g-ED set, $\gamma_{ged}(G) \leq 2$, $E_1 = \{b \in V(G) \mid e_g(b) = 1\}$ and $E_2 = \{b \in V(G) \mid e_g(b) = 2\}$.

Case (i): Since, $r_g(G) \neq g_g(G)$, $e_g(f) = e_g(g) = 1$, is not possible.

Case(ii): $e_g(f) = 1$, $e_g(g) = 2$. In this case, f is g-eccentric to all other vertices of E_2 . Let $c \in E_2$, c is not adjacent to b . Let $D = \{f, g, c\}$ is a γ_{dcged} -set. Hence, $\gamma_{dcged}(G) \leq 3$.

Case(iii): $e_g(f) = e_g(g) = 2$. All the other vertices of E_2 are adjacent to either f or b . If there is a vertex $h \in E_2$ such that it is adjacent to both f and g then f, g are not g-eccentric to h . Hence D is not a γ_{ged} -set, which is a contradiction. Therefore, there is no h in E_2 which is adjacent to both f, g .

Sub Case (i): f and g are adjacent in $G(\tau, \omega)$. Take $D = \{a, f, g\}$ where $e_g(a) = 1$, then D is a γ_{dcged} -set. Hence, $\gamma_{dcged}(G) \leq 3$.

Sub Case (ii): f and g are not adjacent in $G(\tau, \omega)$. In this case take $D = \{f, g, h\}$, where $e_g(h) = 2$, then D is a γ_{dcged} -set. Hence, $\gamma_{dcged}(G) \leq 3$. Thus we see that, a FG with radius one and diameter 2, then $\gamma_{dcged}(G) \leq 3$ if $\gamma_{ged}(G) \leq 2$.

Corollary 4.7.

- 1. If $r_g(G) = 1$ and $G(\tau, \omega)$ has a pendent vertex, then $\gamma_{dcged}(G) \leq 3$.

Theorem 4.8. There is no FG $G(\tau, \omega)$ such that both G and $\bar{G}$ are 0-DCgED-graphs.

Proof. Since all the 0-distance closed dominating FG $G(\tau, \omega)$ are with $d_g(G) \geq 3$ and there is no FG for which both G and $\bar{G}$ are with $d_g(G) \geq 3$, we have the result.

Observation 4.9.

- 1. If $G = \overline{K_\tau} + K_1 + K_1 + \overline{K_\tau}$, $|\tau *| = 2$, then $\gamma(G) \leq 2$, $\gamma_{ged}(G) \leq 4$, $\gamma_{dcl}(G) \leq 4$, $\gamma_{dcged}(G) \leq 4$.
- 2. If $G = K_\tau + K_1 + K_1 + K_\tau$, $|\tau *| \geq 2$, then $\gamma(G) \leq 2$, $\gamma_{ged}(G) \leq 2$, $\gamma_{dcl}(G) \leq 4$, $\gamma_{dcged}(G) \leq 4$.

Theorem 4.10. For a bi central fuzzy tree T_τ with $r_g(T_\tau) = 2$, then $\gamma_{dcged}(T_\tau) \leq 4$.

Proof. All the four vertices of a bi central path form a DCgED- set. Hence the theorem follows.

Observation 4.11. For a bi central fuzzy tree T_τ ,

- 1. $\gamma_{dcged}(T_\tau) \leq \gamma_{ged}(T_\tau) + 1$, if there exists atleast one peripheral vertex with support vertex u , $d_g(u) \leq 2$.

2. $\gamma_{dcged}(T_\tau) = \gamma_{ged}(T_\tau)$, if strong degree of every support vertex is greater than two.

Observation 4.12. For a unicentral fuzzy tree T_τ ,

1. $\gamma_{dcged}(T_\tau) \leq \gamma_{ged}(T_\tau) + 2$, if there exists atleast one peripheral vertex with support vertex $u, d_g(u) \leq 2$.
2. $\gamma_{dcged}(T_\tau) = \gamma_{ged}(T_\tau)$, if strong degree of every support vertex is greater than two.

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