JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3377-3382
https://publishoa.com
ISSN: 1309-3452

Connected g-Eccentric Domination in Fuzzy Graph

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Received 2022 March 25; **Revised** 2022 April 28; **Accepted** 2022 May 15.

Abstract: A dominating set $D \subseteq P(G)$ in a fuzzy graph $G(\tau,\nu)$ is said to be a g-eccentric dominating set if for each vertex b in $P - D$, $\exists$ at least one g-eccentric vertex a of b in D. A g-eccentric dominating set D of G is said to be connected g-eccentric dominating set if the induced sub graph $< D >$ is connected. This study proposes the connected g-eccentric point set and connected g-eccentric dominating set. Some standard fuzzy graphs have a connected g-eccentric domination number associated with them, which must be established. The connected g-eccentric domination number and its bounds are studied.

Keywords: g-Eccentric dominating set, g-Eccentric domination number, Connected g-eccentric dominating set, Connected g-eccentric domination number.

AMS Subject Classification 2020: 05C05, 05C12, 05C72.

I Introduction

The concept of fuzzy graphs proposed by Rosenfeld [6], in 1975. The g-node and related concepts were first described by Linda and Sunitha in 2010 [4]. The eccentric domination in graph was first introduced in 2010 by Janakiraman et al., [3]. The connected eccentric domination in graphs were the first to introduced by Jahir Hussain and Fathima Begam [2] in 2015. g-Eccentric domination in fuzzy graphs (Simply,FG) was pioneered by Mohamed Ismayil and Muthupandiyan [5] in 2020.

This article presents the connected g-eccentric dominating set and its number in FG. Bounds on the connected g-eccentric domination number for several standard FG were also obtained, as well as some theorems on connected g-eccentric domination in FG that were stated and proved.

Harary [1] and A. Rosenfeld[6], S. Somasundaram and A. Somasundaram [7] are referred to by the names graph and FG theoretic terminology

Definition 1.1[5,7]: A FG $G = (\tau,\nu)$ defined on $G(P,Q)$ is characterizd with two functions τ on P and ν on $Q \subseteq P \times P$, where $\tau : P \to [0,1]$ and $\nu : Q \to [0,1]$ such that $\nu(a,b) \leq \tau(a) \wedge \tau(b), \forall a,b \in P$. We indicate the crisp grpah $G^* = (\tau^*,\nu^*)$ of the fuzzy graph $G(\tau,\nu)$ where $\tau *^* = \{a \in P : \tau(a) > 0\}$ and $\nu^* = \{(a,b) \in Q : \nu(a,b) > 0\}$. The order and size of a FG $G(\tau,\nu)$ are defined by$p = \sum_{a\in P}\tau(a)$ and $q = \sum_{(a,b)\in Q}\nu(a,b)$ respectively.

Definition 1.2 [4, 5]: An edge $\nu(a,b)$ is said to strong (or strong arc) if $\nu(a,b) \geq \nu\infty(a,b) = CONN_{G-(a,b)}(a,b), a \neq b$. A path P in a FG of length n is a sequence of distinct nodes $a_0, a_1, \dots, a_n$ such that $\nu(a_{i-1},a_i) > 0, i = 1,2,..,n$ and the strength of the path P is $s(P) = min\{\nu(a_{i-1},a_i), i = 1,2,,\dots n\}$. A path is strong path if $\nu(a_{i-1},a_i), \forall i$ is strong arc.

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Definition 1.3 [5]:Let $G(\tau,\nu)$ be a FG. If (a,b) is strong then b is called strong neighbors of a. The set of all strong neighbors of a is called the strong neighborhood of a and represented by$N_s(a)$. The closed strong neighborhood $N_s[a] = N_s(a) \cup \{a\}$. The strong degree of a vertex $a \in \tau^*$ is defined as the sum of membership values of all strong arcs incident at a and it is denoted and defined by $d_s(a) = \sum_{b\in N_s(a)} \nu(a,b)$ where $N_s(a)$ denotes the set of all strong neighbors of a. The lowest strong degree of a fuzzy graph $G(\tau,\nu)$ is $\delta_s(G) = \wedge\{d_s(b) : b \in \tau^*\}$ and highest strong degree of G is $\Delta_s(G) = \vee\{d_s(b) : b \in \tau^*\}$.

Definition 1.4 [3]: If $\nu(a,b)$ is strong arc then the geodesic distance(g-distance) of a and b is 1. The g-distance from a to b is defined by $d_g(a,b) = min\{Pi \mid Pi\ are\ different\ strong\ paths\ from\ a\ to\ b\}$. The geodesic eccentricity(g-eccentricity) $e_g(a)$ of a node $a \in P$ in a connected FG $G = (\tau,\nu)$ is characterized by $e_g(a) = max\{d_g(a,b), b \in P\}$. $r_g(G) = min\{e_g(a), a \in P\}$ is g-radius and $d_g(G) = max\{e_g(a), a \in P\}$ is g-diameter . A vertex b is said to be a g-central vertex if $e_g(b) = r_g(G)$. A vertex b is said to be a g-pheriperal vertex if $e_g(b) = d_g(G)$.

Definition 1.5[4]: Let $a, b \in V(G)$ be any two nodes in a FG $G(\tau,\nu)$, a vertex a at a g-distance $e_g(b)$ from b is a g-eccentric point of b. The g-eccentric set of a vertex b is defined and intended through $E_g(b) = \{a/d_g(a,b) = e_g(b)\}$.

Definition 1.6 [5]: A dominating set $D \subseteq P(G)$ in a FG $G = (\tau,\nu)$ is said to be a g-eccentric dominating set, for each vertex $b \in D$, $\exists$ at least a g-eccentric vertex $a \in D$ of b. The lowest scalar cardinality taken on all g-eccentric dominating set is called g-eccentric domination number and is denoted by $\gamma_{ged}(G)$.

Definition 1.7[5]: The set $S \subseteq P$ in a FG $G(\tau,\nu)$ is said to be a g-eccentric point set if for every $a \in P - S$, there exists at least one g-eccentric point b of a in S.

Unless otherwise mentioned, only connected FG's are addressed in this study.

II Connected g-Eccentric point set in Fuzzy Graphs

This section introduces the connected g-eccentric point set and its number of FG. Some observations and example given at the end.

Definition 2.1 A set $S \subseteq P(G)$ of a FG $G(\tau,\nu)$ is said to be connected g-eccentric point set(CgEP-set) if S is g-eccentric point set of a FG $G(\tau,\nu)$ and also the induced subgraph $< S >$ is connected. The connected g-eccentric point set is a minimal connected g-eccentric point set if no proper subset S^0 of S is a connected g-eccentric point set. A lowest connected g-eccentric point set is a minimal connected g-eccentric point set with a lowest cardinality. The connected g-eccentric number(upper connected g-eccentric point number) is the cardinality of the smallest(highest) connected g-eccentric point set and is represented by $e_{cge}(G)(E_{gce}(G))$.

Example 2.1

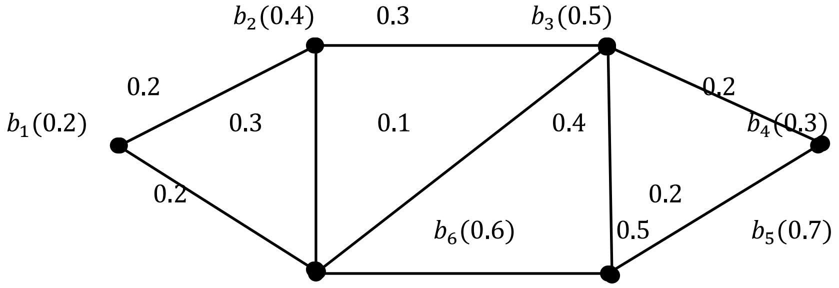

Figure 1

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3377-3382
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ISSN: 1309-3452

In the FG $G(\tau,\nu)$ given in example 2.1, the set $S_1 = \{b_1, b_4\}$ is a g-eccentric point set, but not a connected g-eccentric point se. Also, the set $S_2 = \{b_1, b_2, b_3\}$ is a connected g-eccentric point set. Therefore the connected g-eccentric point number of $G(\tau,\nu)$ is $e_{cge}(G) = 1.1$.

Observation 2.1

(i) $e_{ge}(G) \leq e_{cge}(G)$

(ii) Only the connected FG have connected g-eccentric point set.

(iii) For a complete FG K_τ, $e_{cge}(K_\tau) = \tau_0$, where $\tau_0 = min\{\tau(a), a \in P(G)\}$.

III Connected g-Eccentric Domination number of some standard Fuzzy Graphs

The connected g-eccentric dominating set and its number of FG are introduced in this section. For some standard FG's, some theorems on CgED numbers are stated and proven.

Definition 3.1 A set $D \subseteq P(G)$ of a FG $G(\tau,\nu)$ is a connected g-eccentric dominating set(CgED-set) if D is a g-eccentric dominating set of $G(\tau,\nu)$ and also the induced subgraph $< D >$ is connected. The connected g-eccentric dominating set is a minimal connected g-eccentric dominating set if no proper subset D^0 of D is a connected g-eccentric dominating set. The minimal connected g-eccentric dominating set with lowest cardinality is known as a minimum connected g-eccentric dominating set(γ_{cged}-set). The lowest scalar cardinality taken on all g-eccentric dominating sets is called the g-eccentric domination number and is denoted by $\gamma_{ged}(G)$. The cardinality of highest connected g-eccentric dominating set is known as the upper connected g-eccentric domination number and is denoted by $\Gamma_{cge}(G)$.

Example 3.1 In the example 2.1 of FG $G(\tau,\nu)$, $D_1 = \{b_1, b_4\}$ is a g-ED- set, but not a CgED-set, the set $D_2 = \{b_2, b_3\}$ is connected dominating set but not g-eccentric point set. Therefore, the set $D_3 = \{b_1, b_2, b_3\}$ is a γ_{cged}- set. Therefore the connected g-eccentric domination number is $\gamma_{cge}(G) = 1.1$. And the set $D_4 = \{b_4, b_5, b_6\}$ is a maximum CgED- set. Hence, the upper connected geccentric domination number $\Gamma_{cge}(G) = 1.6$.

Observations 3.1

(i) It is easy to observe that only connected FG have CgED- set.

(ii) Every CgED-set is a g-ED set and every g-ED set is the dominating set. Therefore we have $\gamma(G) \leq \gamma_{ged}(G) \leq \gamma_{cge}(G)$.

(iii) Every CgED-set is the CD-set and every CD-set is the D-set. Therefore, we have $\gamma(G) \leq \gamma_c(G) \leq \gamma_{cge}(G)$.

Observation 3.2 Let $G(\tau,\nu)$ be a connected FG and $H(\tau',\nu')$ be any connected spanning FSG of G. Then every CgED-set of H is also a CgED-set of G and hence $\gamma_{cge}(G) \leq \gamma_{cge}(H)$.

Theorem 3.1 Let K_τ be complete FG, then $\gamma_{cge}(K_\tau) = \tau_0$, where $\tau_0 = min\{\tau(a), a \in P(G)\}$.

Proof: LetK_τ be complete FG. When $G = K_\tau$, $r_g(G) = p - d_g = \tau_0$. Since, each vertex $a \in P(G)$ is adjacent to remaining vertices of $P(G)$ and also each vertex $a \in P(G)$ is a g-eccentric vertex of remaining vertices of $P(G)$. Hence, take a vertex $a \in P(G)$ with lowest cardinality dominates remaining vertices of $P(G)$ and it is also g-eccentric vertices and also it is evident that every trivial FG is connected. So $D = \{a\}$ is a γ_{cge}-set of G. Therefore $G = K_\tau, \gamma_{cge}(K_\tau) = \tau_0, where\ \tau_0 = min\{\tau(a), a \in P(G)\}$.

Theorem 3.2 Let $K_{(\tau_1,\tau_2)}, |\tau_1^*| = 1, |\tau_2^*| \geq 2$ be a star FG , then $\gamma_{cge}(K_{\tau_1,\tau_2}) \leq 2$.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3377-3382
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ISSN: 1309-3452

Proof: Let $KK_{(\tau_1,\tau_2)}, |\tau_1^*| = 1, |\tau_2^*| \geq 2$ be a star FG. Then g-radius $r_g = 1$ and g-diameter $d_g = 2$. Let $D = \{a, b\}$, where b is a g-central vertex. The g-central vertex b dominates all other vertices in $P - D$ and a is a g-eccentric vertex of all the vertices in $P - D$. The induced subgraph D is connected. Therefore, $\gamma_{cge}(K_{(\tau_1,\tau_2)}) \leq 2$.

Theorem 3.3 let $K_{(\tau_1,\tau_2)}$ be a complete bipartite FG, then

$$\gamma_{cge}\left(K_{(\tau_1,\tau_2)}\right) \leq \begin{cases} 1, if\ |\tau_1^*| = |\tau_2^*| = 1 \\ 2, if\ otherwise \end{cases}$$

Proof:let $K_{(\tau_1,\tau_2)}$be a complete bipartite FG.

Case(i): If $|\tau_1^*| = \tau_2^*| = 1$, obviously $\gamma(\tau_1, \tau_2) \leq 1$.

Case(ii): If $|\tau_1^*| = 1, |\tau_2^*| \geq 2$ or $|\tau_1^*| \geq 2, |\tau_2^*| = 1$, then g-radius $r_g = 1$, g-diameter $d_g = 2$ and for $|\tau_1^*| \geq 2, |\tau_2^*| \geq 2$ we have g-radius $r_g =$ g-diameter $d_g = 2$. Take $G(\tau, \nu) = K(\tau_1, \tau_2)$ where $P(G) = P_1 \cup P_2$, $|P_1| = |\tau_1^*|, |P_2| = |\tau_2^*|$ and such that each element of P_1 is adjacent to every vertex of P_2 and vice versa. Let $D = \{a, b\}, a \in P_1$ and $b \in P_2, a$ dominates all the vertices of P_2 and it is an g-eccentric vertex of all vertices of $P_1 - \{a\}$. Similarly b dominates all the vertices of P_1 and it is an g-eccentric vertex of all vertices of $P_2 - \{b\}$. The induced subgraph $< D >$ is connected. Therefore, $\gamma_{cge}\left(K_{(\tau_1,\tau_2)}\right) \leq \begin{cases} 1, if\ |\tau_1^*| = |\tau_2^*| = 1 \\ 2, if\ otherwise \end{cases}$

Corollary 3.1 let $K_{\tau_1,\tau_2}(G)$, $|\tau_1^*| = 1 = |\tau_2^*|$ is same as P_τ, $|\tau^*| = 2$ then $\gamma_{cge}(K_{\tau_1,\tau_2}) = \tau_0$.

Theorem 3.4 Let C_τ be a Cycle FG then $\gamma_{cge}(C_\tau) \leq p - 2$.

Proof: Let $G(\tau, \nu) = C_\tau$, g-radius $r_g \leq \frac{|\tau^*|}{2}$, $|\tau^*|$ is even and $r_g \leq \frac{(|\tau^*|-1)}{2}$, $|\tau^*|$ is odd.

In $C_\tau, r_g(G) = d_g(G)$. Consider the cycle $C_\tau : b, b_2, b_3, \ldots bn, b_{(n+1)} = b_1$. Since in C_τ every vertex is 2-regular, each vertex of $V(C_\tau)$ dominates exactly 2 vertices. The vertex b_1 dominates b_2 and b_n Now include the vertex b_1 in the set D. In order to form a CD-set D, we have to include next consecutive vertex either b_2 or b_n in D, otherwise we can not form a CD-set. Suppose we select $b_2 \in D$, then we have to choose next consecutive vertex b_3 in D. This process is continued until we have $(n - 2)$ vertices of C_τ in D. Therefore, $D = \{b_1, b_2, b_3, \ldots b_{(n-2)}\}$. The vertices $b_{(n-1)} \in P - D$ is dominated by $b_{(n-2)}$ of D and the vertex b_n in $P - D$ is dominated by $b_1 \in D$. Clearly D is the γ_{cd} -set of C_τ. We know that C_τ is a self-centered FG and g-radius= r_g.

Case (i): When $|\tau^*|$ is even then g-eccentric vertex of

$$bi = \begin{cases} b_{(i+r)}\ if\ i \leq r_g \\ b_{(i-r)}\ if\ i < r_g \end{cases}$$

$\therefore$ the g-eccentric point set is equal to $\{b_1, b_2, \cdots b_{r_g}\}$ and connected dominating set is any of the consecutive $n - 2$ vertices. Hence $\gamma_{cge}(C_\tau) \leq p - 2$.

Case (ii): When $|\tau^*|$ is odd then g-eccentric vertex of

$$b_i = \begin{cases} b_{r_g}, b_{r_g+1}\ if\ i \leq r_g \\ b_{i-r_g+1}, b_{i-r_g+1}\ if\ i \geq r_g \end{cases}$$

$\therefore$ the g-eccentric point set is equal to $\{b_1, b_2, \cdots b_{r_g-1}\}$ and connected dominating set is any of the consecutive $n - 2$ vertices. Hence $\gamma_{cge}(C_\tau) \leq p - 2$.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3377-3382
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ISSN: 1309-3452

Theorem 3.5 Let W_τ be a Wheel FG, Then $\gamma_{cge}(W_\tau) \leq \begin{cases} 1, |\tau^*| = 4 \\ 3, |\tau^*| \geq 5 \end{cases}$

Proof: Let $G(\tau, \nu) = W_\tau$.

Case(i):Suppose $| \tau^* | = 4$,take $W_\tau = K_\tau$ then $\gamma_{cge}(W_\tau) = \tau_0 \leq 1$.

Case(ii): Suppose, $n \geq 5$, consider $D = \{a, b, c\}$, where b is a g-central vertex and a, c be any two adjacent non g-central vertices. D is a connected dominating set and also g-eccentric set. Therefore, $\gamma_{cge}(W_\tau) \leq 3$.

Theorem 3.6 Let $P_\tau, |\tau^*| = n$ is a path FG. Then $\gamma_{cge}(P_\tau) = p - \tau_e$, where $\tau_e = max\ \{\tau(a) \mid a\ is\ a\ pendent\ vertex\}$.

Proof: Let $P_\tau, |\tau^*| = n$ is a path FG. Then there exists two pendent vertices say a, b in P_τ. Evidently, $D = P(P_\tau) - (a, b)$ is the γ_{cd}-set. But the g-eccentric vertex of $a \in P(P_\tau) - D$ is $b \in P(P_\tau) - D$ and vice versa. Therefore, we have to add either a or b in D to form the minimum g-ED-set. So that take $D = P(P_\tau) - \{b\}$, then clearly D is the γ_{cged} −set and $|D| = p - \tau_e$, where τ_e is maximum of either a or b. $\therefore \gamma_{cge}(P_\tau) = p - \tau_e$.

IV Bounds on connected g-eccentric Domination in Fuzzy Graph

Bounds on connected g-eccentric dominating set in FG are discussed in this section.

Theorem 4.1 If $G(\tau, \nu)$ is a FG with $d_g(G) = 2$, then $\gamma_{cge}(G) \leq \delta_s(G) + \tau_0$.

Proof: If $G(\tau, \nu)$ is a FG with $d_g(G) = 2$. Let $c \in P(G)$ such that $d_s(c) = \delta_s(G)$. Consider $D = \{c\} \cup N_s(c)$. This is a CgED-set of $G(\tau, \nu)$. The induced subgraph $< D >$ is connected. Therefore, $\gamma_{cge}(G) \leq \delta_s(G) + \tau_0$

Theorem4.2 If the tree T_τ is of $r_g = 2$ with unique g-central vertex a and $d_s(b) \leq 2$ for every $b \in N_s(a)$ then $\gamma_{cge}(G) \leq d_s(a) + 2$.

Proof: Let the tree T_τ is of g-radius 2 with unique g- ccentral vertex a. Then $N_g[a]$ is a CD-set for $G(\tau, \nu)$.

Case(i) If any vertex $b \in N_s(a)$ is a pendent vertex then $N_s[a] - \{b\}$ is a $\gamma_{cd} -$ set. Suppose if there are k pendent vertex in $N_s(a)$, put all that vertex in the set S. Then $N_s[a] - \{S\}$ is the minimum connected dominating set for $G(\tau, \nu)$. Any vertex c in $P - N_s[a]$ is an g-eccentric vertex for all other remaining vertices $P - N_s[a]$ and also for the vertices of S.

Therefore, $N_s[a] - \{s\} + \{c\}$ is a γ_{cged} −set.

$$\gamma_{cge}(G) = |N_s[a] - \{S\} + \{c\}|$$

$$= d_s(a) + 1 - k + 1$$

$$= d_s(a) + 2\,k$$

$$< d_s(a) + 2$$

Case(ii)

If no vertex of $N_s(a)$ is a pendent vertex then $N_s[a]$ is the γ_{cd}-set. Any vertex $w \in P - N_s[a]$ is an g-eccentric vertex for all other vertices of $P - N_s[a]$. Therefore, $N_s[a] + \{c\}$ is $\gamma_{cged} -$ set for $G(\tau, \nu)$.

$$\gamma cge(G) = |Ns[a] + \{c\}|$$

$$= d_s(a) + 1 + 1$$

$$= d_s(a) + 2.$$

Hence, from case(i) and case(ii) $\gamma_{cge}(G) \leq d_s(a) + 2.$, where a is a g-central vertex which is unique and of $r_g = 2$

Conclusion

The connected g-eccentric point set, the connected g-eccentric dominating set and its number, and bounds on the connected g-eccentric dominating number for a few fundamental FG are all discussed in this article.

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