JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3479-3487
https://publishoa.com
ISSN: 1309-3452

$(\boldsymbol{\alpha},\boldsymbol{\beta})-\boldsymbol{Q}-$Fuzzy Subgroup and Its Properties

Dr. R. Jahir Hussain[1], S.Palaniyandi[2]

[1] Associate Professor,PG & Research Department of Mathematics,

[2] Research Scholar, PG & Research Department of Mathematics,

Jamal Mohamed College (Autonomous),

Affiliated to Bharathidasan University

Tiruchirappalli, Tamilnadu, India.

Received: 2022 March 15; **Revised:** 2022 April 20; **Accepted:** 2022 May 10.

ABSTRACT:

In this paper, we introduce the concept of $(\alpha,\beta)-Q-$fuzzy subgroup of a group, $(\alpha,\beta)-Q-$fuzzy normal subgroup and both right and left cosets and discuss some of its properties. The main objective of this paper, we extend the concept of Q-fuzzy subgroup.

Keywords:-Fuzzy set, fuzzy subgroup, Q-fuzzy subgroup, $\alpha-Q-$fuzzy subgroup,$(\alpha,\beta)-Q-$fuzzy set,$(\alpha,\beta)-Q-$fuzzy subgroup.

1. INTRODUCTION

The concept of fuzzy set was coined by Zadeh.L.A [8].The fuzzy set hasused in many research area.In the view of group theory, Rosenfeld [5] invented the idea of the fuzzy subgroups. Biswas.R [1] gave the concept of anti -fuzzy subgroups.Solairaju.A and Nagarajan.R [7] were introduced the new structure of Q-fuzzy groups. Sharma .P.K.[6] was introduced the concept of $(\alpha,\beta)-$fuzzy subgroup.Muwafaq M. Salih and Delbrin H. Ahmed [3] were initiated the idea of α-Q-fuzzy Subgroups. In this paper, we introduce the concept of $(\alpha,\beta)-Q-$fuzzy subgroup and $(\alpha,\beta)-Q-$fuzzy both left and right cosets and discuss some of its properties.

2. PRELIMINARIES

2.1 Definition [8]

Let X be a non-empty set. A fuzzy subset A of Xis $A: X \to [0,1]$.

2.2 Definition [5]

Let G be a group and Abe a fuzzy subset of a group G. Then$A$is called fuzzy subgroup of a group G if forall $x,y \in G$,

i) $A(xy) \geq \min\{A(x),A(y)\}$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3479-3487
https://publishoa.com
ISSN: 1309-3452

ii) $A(x^{-1}) = A(x)$

2.3 Definition [1]

Let G be a group and Abe a fuzzy subset of a group G. Then A is called anti fuzzy subgroup of a group G if forall $x, y \in G$,

i) $A(xy) \leq max\{A(x), A(y)\}$
ii) $A(x^{-1}) = A(x)$

2.4Definition [7]

A Q- fuzzy set$A$ is called Q-fuzzy group of a group G if for all$x, y \in G$, and $q \in Q$

i) $A(xy, q) \geq \min\{A(x, q), A(y, q)\}$
ii) $A(x^{-1}, q) = A(x, q)$

2.5 Definition [3]

Let A be a Q-fuzzy subgroup of a group G and α∈[0, 1]. Then, A^{α} is called α-Q-fuzzy subgroup of G, if for all $x, y \in G$, and $q \in Q$the following conditions hold:

(i) $A^{\alpha}(xy, q) \geq \min\{A^{\alpha}(x, q), A^{\alpha}(y, q)\}$

(ii) $A^{\alpha}(x^{-1}, q) = A^{\alpha}(x, q)$

2.6Definition [6]

A^{α} and A_{β} are α-fuzzy set and β- anti fuzzy set of the set X (w.r.t. the fuzzy set A), then the fuzzy set $A^{(\alpha, \beta)}$ defined by $A^{(\alpha,\beta)}(x) = \min\{A^{\alpha}(x), A^{c}_{\beta}(x)\}$ for every $x \in X$ is called (α, β)- fuzzy set of X(w.r.t. the fuzzy set A) where $\alpha, \beta \in [0, 1]$ such that $\alpha + \beta \leq 1$.

3. $(\boldsymbol{\alpha}, \boldsymbol{\beta}) - \boldsymbol{Q}$ −FUZZY SUBGROUP AND ITS PROPERTIES

3.1 Definition

$A^{\alpha}$and $A_{\beta}$are $\alpha - Q$ −fuzzy set and β- anti Q −fuzzy set of the set X (w.r.t. the fuzzy set A), then the Q −fuzzy set $A^{(\alpha, \beta)}$is defined by $A^{(\alpha,\beta)}(x, q) = \min\{A^{\alpha}(x, q), A^{c}_{\beta}(x, q\}$ for all$x \in X$ and $q \in Q$is called $\alpha, \beta - Q -$fuzzy set of X

Remark

(i) $A^{(1,0)}(x, q) = \min\{A^{1}(x, q, A0cx, q\}$
$= \min\{A(x, q), 0\}$
$= 0$

(ii) $A^{(0,1)}(x, q) = \min\{A^{0}(x, q, A1cx, q\}$
$= \min\{0, A^{c}(x, q)\}$
$= 0$

3.2 Definition

Let A be a $(\alpha, \beta) - Q$ −fuzzy set of a group G(w.r.t. the fuzzy set A), A is said to be $(\alpha, \beta) - Q$ −fuzzy subgroup $[(\alpha, \beta) - QFSG]$ of the group G if it is satisfied the following conditions

(i) $A^{(\alpha,\beta)}(xy, q) \geq \min\{A^{(\alpha,\beta)}(x, q), A^{(\alpha,\beta)}(y, q)\}$
(ii) $A^{(\alpha,\beta)}(x^{-1}, q) = A^{(\alpha,\beta)}(x, q), \forall x, y \in G \ and \ q \in Q$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3479-3487
https://publishoa.com
ISSN: 1309-3452

Equivalently, we have $A^{(\alpha,\beta)}(xy^{-1}, q) \geq \min\{A^{(\alpha,\beta)}(x, q), A^{(\alpha,\beta)}(y, q)\}$, $\forall x, y \in G \text{ and } q \in Q$

3.3 Theorem

If A is a $(\alpha, \beta) - Q -$fuzzy subgroup of a group G, then we have

(i) $A^{(\alpha,\beta)}(e, q) \geq A^{(\alpha,\beta)}(x, q)$, where e is the identity of the group G and
(ii) If $A^{(\alpha,\beta)}(xy^{-1}, q) = A^{(\alpha,\beta)}(e, q) \Rightarrow A^{(\alpha,\beta)}(x, q) = A^{(\alpha,\beta)}(y, q)$

Proof:

(i) $A^{(\alpha,\beta)}(e,q) = \min\{A^{\alpha}(e,q), A^{c}_{\beta}(e,q)\}$

$$\geq \min\{A^{\alpha}(x,q), A^{c}_{\beta}(x,q)\}$$

$$= A^{(\alpha,\beta)}(x,q)$$

$$A^{(\alpha,\beta)}(x,q) = A^{(\alpha,\beta)}(xy^{-1}y,q)$$

$$\geq \min\{A^{(\alpha,\beta)}(xy^{-1},q), A^{(\alpha,\beta)}(y,q)\}$$

$$= \min\{A^{(\alpha,\beta)}(e,q), A^{(\alpha,\beta)}(y,q)\}$$

$$= A^{(\alpha,\beta)}(y,q)$$

(ii) $A^{(\alpha,\beta)}(y,q) = A^{(\alpha,\beta)}(yxx^{-1},q)$

$$\geq \min\{A^{(\alpha,\beta)}(yx^{-1},q), A^{(\alpha,\beta)}(x,q)\}$$

$$= \min\{A^{(\alpha,\beta)}((xy^{-1})^{-1},q), A^{(\alpha,\beta)}(x,q)\}$$

$$= \min\{A^{(\alpha,\beta)}(xy^{-1},q), A^{(\alpha,\beta)}(x,q)\}$$

$$= \min\{A^{(\alpha,\beta)}(e, q), A^{(\alpha,\beta)}(x,q)\} \qquad = A^{(\alpha,\beta)}(x,q)$$

Hence $A^{(\alpha,\beta)}(x,q) = A^{(\alpha,\beta)}(y,q)$

3.4 Theorem

Let A be a $\alpha - Q -$ fuzzy subgroup as well as $\beta -$ anti $Q -$ fuzzy subgroup of a group G, then A is also $(\alpha, \beta) - Q -$ fuzzy subgroup of a group G.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3479-3487
https://publishoa.com
ISSN: 1309-3452

Proof :

Let x, y be any element of the group G and $q \in Q$

Then$A^{(\alpha,\beta)}(xy^{-1}, q) = \min\{A^{\alpha}(xy^{-1},\ q), A^{c}_{\beta}(xy^{-1},\ q)\}$

$$\geq \min\{\min\{A^{\alpha}(x,\ q), A^{\alpha}(y,\ q)\},\ \min\{A^{c}_{\beta}(x,\ q), A^{c}_{\beta}(y,\ q)\}\}$$

$$= \min\{\min\{A^{\alpha}(x,\ q), A^{c}_{\beta}(x,\ q)\}, \min\{A^{\alpha}(y,\ q)\ , A^{c}_{\beta}(y,\ q)\}\}$$

$$= \min\{A^{(\alpha,\beta)}(x, q), A^{(\alpha,\beta)}(y, q)\}$$

Thus$A^{(\alpha,\beta)}(xy^{-1}, q) \geq \min\{A^{(\alpha,\beta)}(x, q),\ \ A^{(\alpha,\beta)}(y, q)\}$

Hence A is a $(\alpha, \beta) -\ Q -$ fuzzy subgroup of a group G.

3.5Example

Let $G = \{e, a, b, ab\}$, where $a^2 = b^2 = e$ and $ab = ba$ be the Klein four group.

$Q -$fuzzy set A of G is defined by

$$A = \{< (e, a), 0.1 >, < (a, q), 0.4 >, < (b, q), 0.4 >, < (ab, q), 0.3 >\}$$

Clearly, A is not a $Q -$ fuzzy subgroup of a group G.

Take $\alpha = 0.05$

Then $(x, q) > \alpha$, $\forall \quad x \in G\ and\ q \in Q$

So that $A^{\alpha}(x,\ q) \geq \min\{A(x, q), \alpha)\} = \alpha, \forall \quad x \in G\ and\ q \in Q$

Therefore$A^{\alpha}(xy^{-1},\ q) \geq \min\{A(x, q), A(y, q)\}$ hold.

Therefore A is a $\alpha - Q -$ fuzzy subgroup of a group G.

Again Let $\beta = 0.4$

Then $A_{\beta}(x, q) = \max\{A(x, q), 1 - \beta\}$

$$= \max\{A(x, q), 0.6\}$$

$$= 0.6, \forall \quad x \in G\ and\ q \in Q.$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3479-3487
https://publishoa.com
ISSN: 1309-3452

Thus $A_\beta(xy^{-1},q) \leq \max\{A(x,q),A(y,q)\}$hold.

Therefore A is a $\beta - anti\ Q -$ fuzzy subgroup of a group G.

Further

$$A^{(0.05,0.4)}(x,q) = \min\{A^{0.05}(x,q), A^c_{0.4}(x,q)\}$$

$$= \min\{0.05, 0.4\}$$

$$= 0.05, \forall \quad x \in G \ and \ q \in Q$$

Hence A is $(0.05, 0.4) - Q -$ fuzzy subgroup of a group G.

3.6 Theorem

The intersection of two $(\alpha,\beta) - Q -$ fuzzy subgroups of a group G is also $(\alpha,\beta) - Q -$fuzzy subgroup of a group G.

Proof:

Let A and B be two $(\alpha,\beta) - Q -$ fuzzy subgroups of a group G and for all $x \in G \ and \ q \in Q$

$$(A \cap B)^{(\alpha,\beta)}(xy,q) = (A^{(\alpha,\beta)} \cap B^{(\alpha,\beta)})(xy,q)$$

$$= \min\{A^{(\alpha,\beta)}(xy,q), B^{(\alpha,\beta)})(xy,q)\}$$

$$\geq \min\{\min\{A^{(\alpha,\beta)}(x,q), A^{(\alpha,\beta)}(y,q)\}, \min\{B^{(\alpha,\beta)}(x,q), B^{(\alpha,\beta)}(y,q)\}$$

$$= \min\{\min\{A^{(\alpha,\beta)}(x,q), B^{(\alpha,\beta)}(x,q), \quad A^{(\alpha,\beta)}(y,q), B^{(\alpha,\beta)}(y,q)\}\}$$

$$= \min\{(A \cap B)^{(\alpha,\beta)}(x,q), (A \cap B)^{(\alpha,\beta)}(y,q)\}$$

Thus

$$(A \cap B)^{(\alpha,\beta)}(xy,q) \geq \min\{(A \cap B)^{(\alpha,\beta)}(x,q), (A \cap B)^{(\alpha,\beta)}(y,q)\}$$

Moreover

$$(A \cap B)^{(\alpha,\beta)}(x^{-1},q) = (A^{(\alpha,\beta)} \cap B^{(\alpha,\beta)})(x^{-1},q)$$

$$= \min\{(A^{(\alpha,\beta)}(x^{-1},q), B^{(\alpha,\beta)})(x^{-1},q)$$

$$= \min\{(A^{(\alpha,\beta)}(x,q), B^{(\alpha,\beta)})(x,q)$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3479-3487
https://publishoa.com
ISSN: 1309-3452

Hence $(A \cap B)^{(\alpha,\beta)}(x^{-1}, q) = (A \cap B)^{(\alpha,\beta)}(x,\ q)$

Consequently $A \cap B$ is $(\alpha, \beta) -\ Q -$ fuzzy subgroups of a group G.

3.7 Example

Let $Z = \{0, \pm 1, \pm 2, \ldots\}$ be a greoup under addition and $q \in Q$. Define the two $Q -$ fuzzy subsets A and B of Z as follows

$A(x,q) = \begin{cases} 0.6 & if\ x \in 3Z \\ 0.04 & otherwise \end{cases}$ and $B(x,q) = \begin{cases} 0.4 & if\ x \in 2Z \\ 0.06 & otherwise \end{cases}$

Taking $\alpha = 0.4$ and $\beta = 0.6$

Now $(A \cup B)(x,q) = \max\{A(x,q), B(x,q)\}$

Therefore

$$A \cup B = \begin{cases} 0.6 & if\ x \in 3Z \\ 0.4 & if\ x \in 2Z \\ 0.06 & if\ x \notin 2Z\ \ or\ \ x \notin 3Z \end{cases}$$

Take $x = 21$ and $y = 4$

Then $(A \cup B)(x,q) = 0.6$ and $(A \cup B)(y,q) = 0.4$

But $(A \cup B)(x - y, q) = 0.04$

In addition, $\min\{(A \cup B)(x,q), (A \cup B)(y,q)\} = \min\{0.6, 0.4\} = 0.4$

Clearly $(A \cup B)(x - y, q) < \min\{(A \cup B)(x,q), (A \cup B)(y,q)\}$

Consequently ,$A \cup B$ is not $(0.4, 0.6)$- $Q -$ fuzzy subgroup of Z.

Hence the union of two $(\alpha, \beta) - Q -$ fuzzy subgroups of Z is not $(\alpha, \beta) - Q -$ fuzzy subgroups of Z.

3.8 Definition

Let A and B be two $(\alpha, \beta) -\ Q -$ fuzzy subgroups of a group G_1 and G_2 respectively. Then the product of A and B is defined as $A^{(\alpha,\beta)} \times\ B^{(\alpha,\beta)}\big((x,y), q\big) = \min\{A^{(\alpha,\beta)}(x,q),\ B^{(\alpha,\beta)}(y,q)\}$ for all $x \in G_1$ and $y \in G_2$ and $q \in Q$.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3479-3487
https://publishoa.com
ISSN: 1309-3452

3.9Theorem

Let A and B be two $(\alpha,\beta)-Q-$ fuzzy subgroups of a group G_1 and G_2 respectively. Then $A^{(\alpha,\beta)} \times B^{(\alpha,\beta)}$ is $(\alpha,\beta)-Q-$ fuzzy subgroup of $G_1 \times G_2$.

Proof:

Let $x_1, x_2 \in G_1$ and $y_1, y_2 \in G_2$ then $(x_1,y_1),(x_2,y_2) \in G_1 \times G_2$ and $q \in Q$

Now $A^{(\alpha,\beta)} \times B^{(\alpha,\beta)}((x_1,y_1)(x_2^{-1},y_2^{-1}),q) = A^{(\alpha,\beta)} \times B^{(\alpha,\beta)}((x_1x_2^{-1},y_1y_2^{-1}),q)$

$$= \min\{A^{(\alpha,\beta)}(x_1x_2^{-1},q),\ B^{(\alpha,\beta)}(y_1y_2^{-1},q)\}$$

$$\geq \min\{\min\{A^{(\alpha,\beta)}(x_1,q),\ A^{(\alpha,\beta)}(x_2^{-1},q)\},\min\{B^{(\alpha,\beta)}(y_1,q),\ B^{(\alpha,\beta)}(y_2^{-1},q)\}\}$$

$$= \min\{\min\{A^{(\alpha,\beta)}(x_1,q),\ A^{(\alpha,\beta)}(x_2,q)\},\min\{B^{(\alpha,\beta)}(y_1,q),\ B^{(\alpha,\beta)}(y_2,q)\}\}$$

$$= \min\{\min\{A^{(\alpha,\beta)}(x_1,q),\ B^{(\alpha,\beta)}(y_1,q)\},\min\{A^{(\alpha,\beta)}(x_2,q),\ B^{(\alpha,\beta)}(y_2,q)\}\}$$

$$= \min\{A^{(\alpha,\beta)} \times B^{(\alpha,\beta)}((x_1,y_1),q),\ A^{(\alpha,\beta)} \times B^{(\alpha,\beta)}((x_2,y_2),q)\}$$

Hence

$$A^{(\alpha,\beta)} \times B^{(\alpha,\beta)}((x_1,y_1)(x_2^{-1},y_2^{-1}),q)$$

$$\geq \min\{A^{(\alpha,\beta)} \times B^{(\alpha,\beta)}((x_1,y_1),q),\ A^{(\alpha,\beta)} \times B^{(\alpha,\beta)}((x_2,y_2),q)\}$$

3.10 Definition

Let A be a $(\alpha,\beta)-Q-$ fuzzy subgroup of a group $G$and $\alpha,\beta \in [0,1]$. For any

$m \in G$ *and* $q \in Q$. The $(\alpha,\beta)-Q-$ fuzzy left coset of A in G is represented by $mA^{(\alpha,\beta)}$ as defined as $mA^{(\alpha,\beta)}(x,q) = t_p\{A(m^{-1}x,q),(\alpha,\beta)\}$ for all $m,x \in G$ *and* $q \in Q$.

Similarly, we define the $(\alpha,\beta)-Q-$ fuzzy right coset of A in G is represented by $A^{(\alpha,\beta)}m$ as defined as $A^{(\alpha,\beta)}m(x,q) = t_p\{A(xm^{-1},q),(\alpha,\beta)\}$ for all $m,x \in G$ *and* $q \in Q$.

Remarks

Let A be a $(\alpha,\beta)-Q-$ fuzzy subgroup of a group $G$and $\alpha,\beta \in [0,1]$. Then A is called $(\alpha,\beta)-Q-$ fuzzy normal subgroup of a group G if and only if $mA^{(\alpha,\beta)} = A^{(\alpha,\beta)}m$, for all $m, \in G$.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3479-3487
https://publishoa.com
ISSN: 1309-3452

3.11 Theorem

If A is $(\alpha, \beta) - Q -$ fuzzy normal subgroup of a group G.

Then $A^{(\alpha,\beta)}(y^{-1}xy, q) = A^{(\alpha,\beta)}(x, q)$ or equivalently $A^{(\alpha,\beta)}(xy, q) = A^{(\alpha,\beta)}(yx, q)$,

For all $x , y \in G \ and \ q \in Q$.

Proof:

Since A is $(\alpha, \beta) - Q -$ fuzzy normal subgroup of a group G, $mA^{(\alpha,\beta)} = A^{(\alpha,\beta)}m$, for all $m \in G \ and \ q \in Q$.

Thus $(mA^{(\alpha,\beta)})(y^{-1}, q) = (A^{(\alpha,\beta)}m)(y^{-1}, q)$

$$\Rightarrow t_p\{A(x^{-1}y^{-1}, q), (\alpha, \beta)\} = t_p\{A(y^{-1}x^{-1}, q), (\alpha, \beta)\}$$

Which implies that

$A^{(\alpha,\beta)}((yx)^{-1}, q) = A^{(\alpha,\beta)}((xy)^{-1}, q)asA$ is $(\alpha, \beta) - Q -$ fuzzy normal subgroup of a group G.

So $A^{(\alpha,\beta)}(g^{-1}, q) = A^{(\alpha,\beta)}(g, q)$, for all $g \in G \ and \ q \in Q$.

Consequently $A^{(\alpha,\beta)}(xy, q) = A^{(\alpha,\beta)}(yx, q)$.

3.12 Theorem

Every $Q -$ fuzzy normal subgroup of a group G is a $(\alpha, \beta) - Q -$ fuzzy normal subgroup of a group G.

Proof:

Let A be a $Q -$ fuzzy normal subgroup of a group G.

Then for all $m \in G$, we have $mA = Am$

This implies $(mA)(x, q) = Am(x, q)$ for all $x \in G \ and \ q \in Q$.

So $A(m^{-1}x, q) = A(xm^{-1}, q)$

Hence $t_p\{A(m^{-1}x, q), (\alpha, \beta)\} = t_p\{A(xm^{-1}, q), (\alpha, \beta)\}$

i.e $mA^{(\alpha,\beta)} = A^{(\alpha,\beta)}m$ for all $m \in G$

consequently$A$ is a $(\alpha, \beta) - Q -$ fuzzy normal subgroup of a group G.

Remark

The converse of the above theorem may not be true.

4. CONCLUSION

In this paper, the concepts of (α,β)-Q-fuzzy subset and (α,β)-Q-fuzzy subgroup have been defined and related properties are proven. Also (α,β)-Q-fuzzy left and right cosets are defined with some results.

REFERENCES:

[1] Biswas .R, Fuzzy subgroups and Anti Fuzzy subgroups, Fuzzy sets and Systems, 35(1990) 121-124.

[2]Mohamed Asaad, Groups and Fuzzy subgroups Fuzzy sets and systems 39(1991) 323-328.

[3] Muwafaq M. Salih and Delbrin H. Ahmed, α-Q-fuzzy Subgroups, Academic Journal of Nawroz University (AJNU). 26-31.

[4] Palaniappan .N.Muthuraj. R, Anti fuzzy group and Lower level subgroups, AntarticaJ.Math, 1 (1) (2004), 71-76.

[5] Rosenfeld. A, fuzzy groups, J. math.Anal.Appl. 35 (1971), 512-517.

[6]. Sharma .P.K., (α,β)- Fuzzy subgroups, Fuzzy Sets, Rough Sets and Multivalued Operations and Applications Vol. 4, No. 2, (July-December 2012): 59– 71

[7] Solairaju A. and Nagarajan R., A new Structure and construction of Q-Fuzzy Groups, Advances in Fuzzy Mathematics, Volume 4, Number 1 (2009) pp. 23-29

[8]. Zadeh. L.A., Fuzzy Set, Information and Control 8(1965), 338-353