JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3488-3496
https://publishoa.com
ISSN: 1309-3452

Computing an Optimal Solution for LAP through Hungarian Method- A New Approach

Amulu Priya. S[1], Maheswari. V[2], Balaji. V[3]

[1] Department of Mathematics

Research Scholar, Vels Institute of Science, Technology & Advanced Studies, Chennai - 600117, India.

[2] Department of Mathematics

Associate Professor, Vels Institute of Science, Technology & Advanced Studies, Chennai – 600117, India.

[3] Department of Mathematics, Sacred Heart College, Tirupattur – 635601

Author Emails

[a] amulupriyas@gmail.com

[b] maheswari.sbs@velsuniv.ac.in

[c] Pulibala70@gmail.com

Received: 2022 March 15; **Revised:** 2022 April 20; **Accepted:** 2022 May 10.

Abstract: Assignment problems (AP) are well-known topics in engineering and management science and are used frequently to analyze problems. A linear programming problem form was used in transforming assignment problem for crisp problem and in solving as per Hungarian methodology. Assigning n objects over m objects is an exciting problem that arises in different situations. Here we explore the optimal ways for assigning in n objects over m other objects. Numerous numerical applications are needed in production planning, Sales proportions, airline operators, etc. The existing Hungarian methods and maximized (or minimized) assignment methods have been solved using these methods. Entry details in the cost matrix are not always clear. This parameter is uncertain in many applications, and these uncertain parameters are represented by intervals. To solve interval linear assignment problems, we present interval Hungarian method and interval analysis concept.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3488-3496
https://publishoa.com
ISSN: 1309-3452

AMS Subject Classification: MSC90B0

Keywords: Optimal solution, Assignment Problem, Optimization, Hungarian assignment problem, Hungarian Assignment method (HA)

1 Introduction

Assignment Problem on General Intervals

By devising the condition that involves m works comprising of n people who are exhibiting dissimilar skillsets. If a cost for performing j^{th} work which is carried out by k^{th} people is designated as $c_{jk.}$ Now considering the problem in which, the work has been assigned with whom such that the actual cost required to complete the work will be considered minimum. From a mathematical context, we expressed problem presented below:

Min. Z(cost)= c_{jk} x_{jk} ; j=1,2,3…………n; k=1,2,3….,n

Where $x_{jk} = \begin{cases} 1; & \text{wherein, the } j^{th} \text{ person has been assigned with } k^{th} \text{ task/ work} \\ 0; & \text{if } j^{th} \text{ person is not assigned the } k^{th} \text{ task/ work} \end{cases}$

with certain restrictions held under each task/ work

x_{jk} =1; j =1,2,3….,n

i.e., the k^{th} person will only be doing only one task/ work that has been assigned

x_{jk}= 1; = 1,2,3,….,n

i.e., k^{th} task will be carried out only by one individual in this regard

2 Introduction to Interval Hungarian approach

Using general interval arithmetic, this section presents an algorithm for solving an assignment problem

Hungarian method:

Step 1: In the cost matrix, find the midpoints of each interval.

Step 2: Divide all of the entries from the column with interval from the smallest of the mid ranged value.

Step 3: Add/ summing up values of the intervals involving with the smallest mid values to that of the rows without intervals comprising of zero.

Step 4: Make sure each interval containing zero of the cost matrixes is covered by lines via appropriate rows as well as columns.

Step 5: In order to verify optimality, (i)the order of the cost matrix must equal the minimum number of covering lines. (ii) If there are fewer than the number of covering lines from that of the matrix, further move to the next step.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3488-3496
https://publishoa.com
ISSN: 1309-3452

Step 6: Estimate smallest of the midvalues for those intervals that do not have any lines covering them. Add this entry to the crossing containing a zero interval and subtract it from all uncrossed elements. Continue with step 4.

Tabular form of the problem

As shown in the chart, the interval assignment problem has a cost matrix

People or Jobs	1	2
1	C_{11}	C_{21}
2	C_{12}	C_{22}
3	C_{13}	C_{23}
….j	C_{1j}	C_{2j}
….n	C_{1n}	C_{2n}

For example, consider

An A timetable of the airline's seven-day schedule is shown below. During layovers (rest) between flights, crew members must have a minimum of five hours of rest. The shortest layover time away from your quarters is found by choosing the flight pair with the shortest layover time. Every crew will be located at the city where there are the fewest layovers

COMPUTING AN OPTIMAL SOLUTION FOR LAP THROUGH HUNGARIAN METHOD- A NEW APPROACH

Chennai	Flight Number	1	2	3	4
	Departure	7.00 h	8.00 h	13.30 h	18.30 h
	Arrival	8.00 h	9.00 h	2.30 h	19.30 h
Goa	Flight number	101	102	103	104
	Departure	8.00 h	8.30 h	12.00 h	17.30 h
	Arrival	9.15 h	9.45 h	13.15 h	18.45 h

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3488-3496
https://publishoa.com
ISSN: 1309-3452

Please specify where each crew should be based for each pair

Solution

Step1: Formulation of the layover times for crews based in Chennai between flights, Consider 15 min. = 1 part/ unit"

Flight arrive and then departs from Goa, The minimum layover for each flights is h.

		101	102	103	104
	Flight designator. 1	24 h = 96 parts	24hrs+30 min =98 parts	24hrs+4 min =112 parts	9 hrs+30 min =38 parts
Time	Arrival	8.00 h	8.00 h	8.00 h	8.00 h
	Departure	8.00 h	8.30 h	12.00 h	17.30 h

		101	102	103	104
	Flight no 2	23 h = 92 parts	23h+30 min =94 parts	24h+3 min =108 parts	8 h+30 min =34 parts
Time	Arrival	9.00 h	9.00 h	9.00 h	9.00 h
	Departure	8.00 h	8.30 h	12.00 h	17.30 h

		101	102	103	104
	Flight designator. 3	17 h +30 min = 70 parts	18 h =72 parts	21h+30 min =86 parts	24 h+3 h =108 parts
Time	Arriv. time	14.30 h	14.30 h	14.30 h	14.30 h
	Depart. time	8.00 h	8.30 h	12.00 h	17.30 h

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3488-3496
https://publishoa.com
ISSN: 1309-3452

		101	102	103	104
	Flight designator. 4	12 h + 30 min = 50 parts	13h =52 parts	16h+30 min =66 parts	22 h =88 parts
Timing	Arrives	19.30 h	19.30 h	19.30 h	19.30 h
	Depart	8.00 h	8.30 h	12.00 h	17.30 h

Layover times for the crew based in Chennai (Table 1)

Flight designator.	1	2	3	4
101	96	92	70	50
102	98	94	72	52
103	112	108	86	66
104	38	34	108	88

Flight arrived and then departed from Chennai, as minimum place over attributed to be 5 h between its flights.

		1	2	3	4
	Flight designator. 101	21 h + 45 min = 87 parts	22h+45 min =91 parts	28h+ 15 min =113 parts	9 h+ 15 min =37 parts
Time	Arrival	9.15 h	9.15 h	9.15 h	9.15 h
	Departure	7.00 h	8.00 h	13.30 h	18.30 h

		1	2	3	4
	Flight designator. 102	21 h + 15 min = 85 parts	22h+15 min =89 parts	27h+45 min =111 parts	8 h+ 45 min =35 parts

Time	Arrival	9.45h	9.45 h	9.45 h	9.45 h
	Departure	7.00 h	8.00 h	13.30 h	18.30 h

		1	2	3	4
	Flight designator. 103	17 h + 45 min = 71 parts	18h+45 min =75 parts	24h+15 min =97 parts	5 h+ 15 min =21 parts
Time	Arrival	13.15h	13.15 h	13.15 h	13.15 h
	Departure	7.00 h	8.00 h	13.30 h	18.30 h

		1	2	3	4
	Flight no 104	12 h + 15 min = 49 parts	13h+15 min =53 parts	18h+45 min =75 parts	23 h+ 45 min =95 parts
Time	Arrival	18.45 h	18.45 h	18.45 h	18.45 h
	Departure	7.00 h	8.00 h	13.30 h	18.30 h

Layover times wherein the crew of the flights based on Goa (Table 2)

Flight designator.	1	2	3	4
101	87	91	113	37
102	85	89	111	35
103	71	75	97	21
104	49	53	75	95

<u>Step 2</u> : Construct table in case for the minimal layover of times among the flights that is presented with the help of Table 1 and 2

Flight designator.	1	2	3	4
101	87	91	70	37
102	85	89	72	35

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3488-3496
https://publishoa.com
ISSN: 1309-3452

103	71	75	86	21
104	38	34	75	88

Step 3: Cost matrix comprising of the interval entries

Flight designator.	1	2	3	4
101	[86,88]	[90,92]	[69,71]	[36,38]
102	[84,86]	[88,90]	[71,73]	[34,36]
103	[70,72]	[74,76]	[85,87]	[20,22]
104	[37,39]	[33,35]	[74,76]	[87,89]

Step 4: Cost matrix comprising of interval entries

Flight designator.	1	2	3	4
101	<87,1>	<91,1>	<70,1>	<37,1>
102	<85,1>	<89,1>	<72,1>	<35,1>
103	<71,1>	<75,1>	<86,1>	<21,1>
104	<38,1>	<34,1>	<75,1>	<88,1>

Step 5: Subtraction of smallest of the entries in each of the row as well as column, we get

Flight designator.	1	2	3	4
101	<49,1>	<57,1>	<0,1>	<16,1>
102	<45,1>	<53,1>	<0,1>	<12,1>
103	<33,1>	<41,1>	<16,1>	<0,1>
104	<0,1>	<0.1>	<5,1>	<67,1>

Step 6:

Flight designator.	1	2	3	4
101	<4,1>	<12,1>	<0,1>	<4,1>
102	<0,1>	<8,1>	<0,1>	<0,1>
103	<0,1>	<8,1>	<28,1>	<0,1>
104	<0,1>	<0,1>	<50,1>	<100,1>

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3488-3496
https://publishoa.com
ISSN: 1309-3452

Flight designator.1 102

Flight designator.2 104

Flight designator. 3 101

Flight designator.4 103

3 Conclusion

The assignment problems are solved using a variety of classical and heuristic approaches. Many classical methods are implemented using packet programs. Because the number of programs available for solving assignments is limited by the number of components, there is no ready-to-use software that can handle complex problems regardless of their size (such as complex problems, problems that do not have mathematical solutions, or assignments that aren't linear). The development of software is therefore vital. For the AP, heuristic methods were more effective at obtaining software development solutions. By utilizing the R package program which is widely used, it can be used both for solving classical and heuristic assignment problems. To solve interval assignment problems without changing them to classical assignments, we present interval versions of the Hungarian method. Numerical examples and the results obtained are discussed in order to form the solution more precisely. A notable difference between our method and other methods is that the optimal solution we obtain is sharper.

References

1. R. L. Daft, New Era of Management (2nd ed.). Thomson South-Western, (2008).
2. P. K. Gupta & D. S. Hira, Operations Research (7th ed.). New Delhi-110055, S. Chand & Company Pvt. Ltd, (2014).
3. V. K. Kapoor, Operations Research Quantitative technique for Management. New Delhi, S. Chand & Sons, (2018).
4. P. A. Ozor, P. O. Chigozirim, A. O. Odekwe, J. I. Ume, G. O. Ozoegwu, Application of the assignment technique to optimization of solid waste management in Enugu region. International Journal of Environment and Waste Management, 19, (2017), no. 2, 52-73.
5. S. Supinl, S. Wahyuni, J. Nahar & Subiyanto, Optimization of personnel Assignment Problem Based on Traveling Time by Using Hungarian Methods: Case study on the Central Post Office Bandung. Paper presented at 4th International Conference on Operational Research (Interi R), (2018).

6. G. A. M. Tettey, A. Stentz & M. B. Dias, The Dynamic Hungarian Algorithm for the Assignment Problem with Changing Costs. Carnegie Mellon University, Research Showcase @ CMU, (2007).
7. P. C. Tulsian & V. Pandey, Quantitative Techniques: Theory and Problems. India, Dorling Kindersley Pvt. Ltd, (2006).