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Fractional Coloring of Some Products of Simple Graphs

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Abstract

Graph coloring is a component of graph labeling in graph theory; it is the assignment of labels generally referred to as "colors" to elements of a graph subject to specified constraints. In this article, we will almost certainly look at fractional colorings of graphs in which the amount of color assigned to a vertex is determined by local characteristics such as its degree or the clique number of its neighborhood. The fractional chromatic number of a graph is inferior to all rational numbers a/b such that there exists a proper a/b-coloring of G. In this article, we studied fractional coloring in graph theory for many types of graphs such as path graph, cycle graph, complete graph and tree related graphs.

Keywords: Graph coloring, clique, fractional coloring, fractional chromatic number

2010 Mathematics Subject Classification: 05C15, 05C72

1 Introduction

Graphsareoneofthemostcommonr epresentationsofbothnaturalandman-made

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structures.Inphysical,biological,social,an dinformationsystems,graphscan beused to model a wide range of relationships and processes.Graphs can be used to illustratea wide range of real-world issues.The fields of graph theory,computer engineering, and operations research experienced exponential growth in the late twentieth century and early twenty-first century.Graphs are used in computer science to describe communicationnetworks,dataorganizatio n,computingdevices,computationflow,an d soon. Adirectedgraph,forexample,candescribea website'slinkstructure,with vertices representing web pages and directed edges representing links from one pagetoanother.Travel,biology,computerc hipdesign,andavarietyofotherindustries can all benefit from a similar approach.As a result, developing algorithms to manage graphs is a hot topic in computer science.Graphrewritesystemsarefrequent ly usedtodescribeanddepictgraphtransforma tions.Graphdatabases,whicharede- signed for transaction-safe,persistent storing and querying of graph-structured data,areacomplementtographtransformat ionsystemsthatfocusonrule-basedin- memory graph manipulation.Each edge of a graph can be given a weight, which can be used to extend its structure.Weighted graphs, also known as graphs with weights, are used to illustrate structures in which pairwise links have numerical values.The weights could, for example, indicate the length of each road in a graph representing a road network.

2 LiteratureReview

M. Larsen, J. Propp, explained the fractional chromatic number of Mycielski's Graphs [4].On some properties of linear complexes was discussed by A. A. Zykov[8].C. Brause, B. Randerath, D. Rautenbach, and I. Schiermeyer,analyzed lower bound on theindependencenumberofagraphinter msofdegreesandlocalcliquesizes[9].In[10]showedthecoloringonnodesofanetw ork.ExplainedColoringquasi- linegraphsin [11].Subcubic triangle- free graphs have fractional chromatic number at most $\frac{14}{5}$. Was analyzed in [13].Z. Dvorak, J.S. Sereni, and J. Volec.Deliberates Fractional coloring of triangle-free planar Graphs [14].In bounding the fractional chromatic number of Kδ-freegraphswasexplained[16].[17]J.R.GriggexaminedLowerboundsonthe independence number in terms of the degrees. Asymptotic choice number for triangle free graphs and studied Fractional total coloring's of graphs of high girth [18],[19]. Thelastfractionofafractionalconjecture wasanalyzedin[20],[21].[22]M.Molloy deliberate the list chromatic number of graphs with small clique number [23].

3 Preliminaries

Definition 3.1."LetGbeagraphwith n verticesandH beanothergraphwithroot vertex v .TherootedproductofGandHisdefinedasthe graphwithonecopyofG and n copiesofH identifyingthevertex u_i ofGwiththevertex v inthe i^{th} copyofH for each $1 \le i \le n$."

Definition3.2. Acoloredgraphisagraphinwhicheachvertexisassignedacolor.

Definition3.3.A properly colored graph is a colored graph whose color assignments conformtothecoloringrulesappliedtothegraph.

Definition3.4. Agraphparameter $\chi(G)$,thechromaticnumberofG,asthesmallest positiveinteger n suchthatthereexistsaproper n -coloringofG.

Definition3.5. The fractional chromatic number of a graph, $\chi_F(G)$ is the infimum of all rational numbers a/b such that there exists a proper a/b-coloring of G.

Definition3.6. Aregulargraphisagraphinwhicheveryvertexhasthesamedegree. An n-regular graph is a regular graph in which all vertices have degree n.

Observation3.7. Let K_n beacompletegraphwithnvertices,then $\chi(K_n \odot K_n) = \chi_f(K_n \odot K_n)$.

Observation3.8. Let P_n and K_n be the path graph and cycle graph with n vertices respectively, then $\chi(P_n \odot K_n) = \chi_f(P_n \odot K_n)$.

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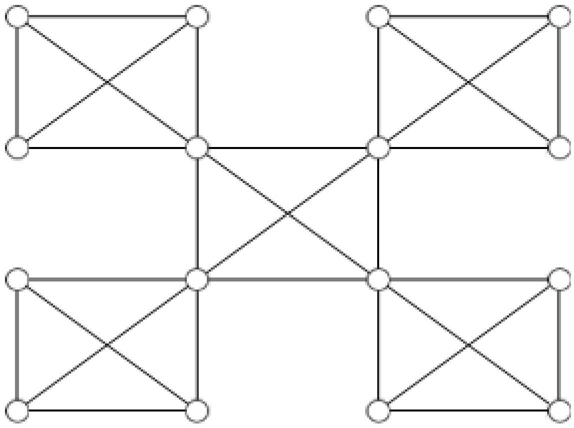

Figure1: $K_n \odot K_n$

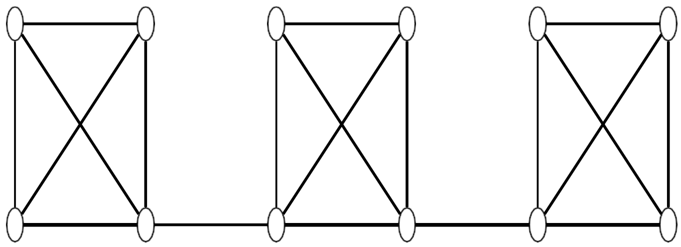

Figure2 : $P_3 \odot K_4$

Observation3.9. Let C_n and K_m be the cycle and path graph with n, m vertices respectively, then $\chi(C_n \odot K_m) = \chi_f(P_n \odot K_m)$.

Theorem 3.10. If $C_{m,m}$be comb graph with n vertices then rooted product of fraction chromatic number is $\chi_f(C_{m,m} \odot C_{m,m}) = \frac{3}{2}$

Proof:

Let G be a comb graph with n vertices and $v_1, v_2, ..., v_n$ be the set of vertices in pathgraphofcaterpillar.Let $u_1, u_2, ..., u_n$ bethesetofverticesintheleavesconnectedbythepathgraph.Therootedproductoftwocombgraph isntimesoffirstgraph connectedbysecondgraph.Thenumberofverticesinrootedproductgraphis n^2. The fraction chromatic number of comb graph is 2.The rooted product of any two trees are always a tree, then the rooted product of two comb graph is tree, so the chromatic number of rooted product of comb graph is less than or equal to two.Hence $\chi_f(C_{m,m} \odot C_{m,m}) = \frac{3}{2}$.

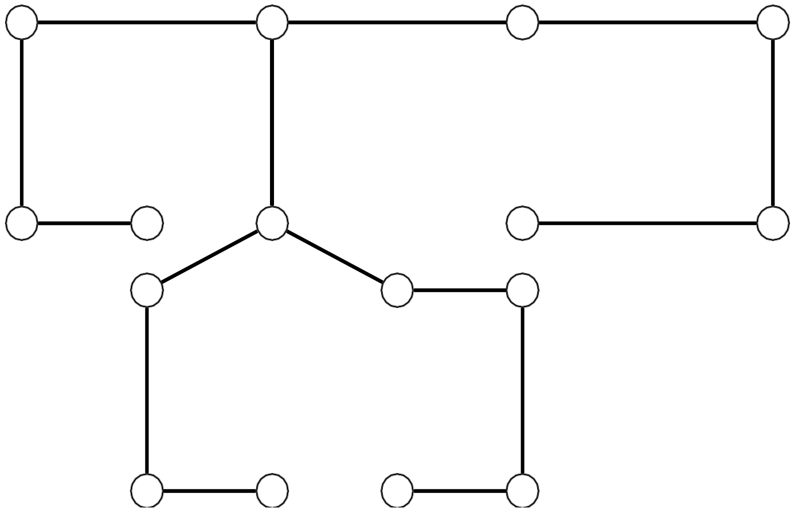

Figure3:$C_{2,2} \odot C_{2,2}$

Theorem3.11. If P_m and P_n be the path graph with m,n vertices respectively then $\chi_f(P_m \odot P_n) = \frac{3}{2}$.

Proof:

Let $v_1, v_2, ..., v_m$ bethesetofverticespathgraphwithmvertices and $u_1, u_2, ..., u_n$ bethesetofverticesinpathgraphwithnvertices.Therootedproductofanytwo graph makes a caterpillar graph.The fractional chromatic number of path graph istwo.In the path graph P_m there are n copies of P_m in the rooted product.The chromatic number of any tree is two.So the fractional chromatic number of caterpillar is lessthanorequaltotwo.Therefore $\chi_f(P_m \odot P_n) = \frac{3}{2}$.

Theorem3.12. If ω_n, ω_m bethetwowheelgraphwithn,mverticesrespectivelythen $\chi_f(\omega_n \odot \omega_m) = \frac{5}{2}$.

Proof:

Let $u_1, u_2, ..., u_m$ besetofverticesofwheelgraphwithmvertices $v_1, v_2, ..., v_n$ be thesetofverticesofsecondwheelgraph.Bythedefinitionofrootedproductntimes of wheel graphs are joined by corresponding vertices of root graph.The chromatic number of wheel graph is three.There are n copies of ω_m are joined by ω_n.The fractional chromatic number is less than three.Hence $\chi_f(\omega_n \odot \omega_m) = \frac{5}{2}$.

Conclusion

In this article we examined fractional chromatic number of rooted product of complete and path graph and cycle graph. We also studied about fractional chromatic number of rooted product of two wheel graphs.

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