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Antimagic Labeling of Fewgraphs

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Abstract

A labeling of a graph is a function that maps vertices, edges or both of a graph to positive integers. Antimagic labeling of a graph $G = (V, E)$ with p vertices and q edges, introduced by Hartsfield and Ringel is an edge labeling which maps E to $\{1,2, \dots, q\}$such that the resulting vertex labels are distinct, where the vertex label is defined to be sum of the labels of the edge incident with the vertex. In this paper we obtain the antimagic labeling for octopus graph , jewel graph and star glued with star graph.

Keywords: Antimagic labeling, Octopus graph , Jewel graph , Star graph.

2010 Mathematics Subject Classification: 05C78

1. Introduction

Labeling is a field of graph theory with diverse applications. Rosa[5] intoduced graph labeling.Hartsfield and Ringel [3] proved that paths , cycles, wheels, and complete graphs admit antimagic labeling. They have also conjectured that all trees except K_2 are antimagic and all connected graphs except K_2 are antimagic. These two conjectures are still open. Vaidya and Vyas[6,7] have proved that graphs obtained by switching of vertex in path, cycle , wheel , helm , fan and some path related graph are antimagic. Baca*et.al*[1]have obtained antimagic labelings of certain families of connected

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graphs and regular graphs.Miller and Baca [4] have obtained antimagic valuation of generalised Petersen graph. A detailed survey of labelings is given by Galian [2]. Antimagic labelings have widepread applications in surveillance security systems, coding theory, completely separating systems and in psychology.

2. Definitions

Definition 1 An Octopus graph O_n , $(n \geq 2)$ is obtained by the one point union of a Fan graph F_n(n ≥ 2)with a Star graph $K_{1,n}$ where n is any positive integer. Octopus graph $O_n$is shown in Figure 1.

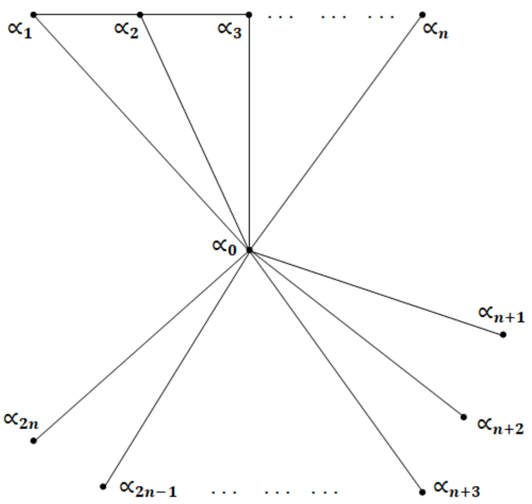

Figure 1: Octopus Graph O_n

Definition 2 The Jewel graph J_n is a graph with vertex set $V(J_n) = \{\alpha_0, \beta_0, \gamma_0, \alpha_i: 1 \leq$ $i \leq n+1$and edge set $EJn=\{\alpha0\alpha1$, $\alpha i\beta 0$, $\alpha_i\gamma_0 : 0 \leq i \leq n+1\}$.

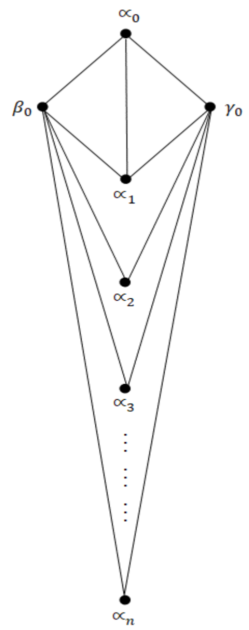

Figure 2: Jewel graph $\boldsymbol{J_n}$

Definition 3 The Star glued with star graph $S_{m,n}$is the graph obtained by attaching a star $K_{1,m}$ to every pendant vertex of the star graph $K_{1,n}$.

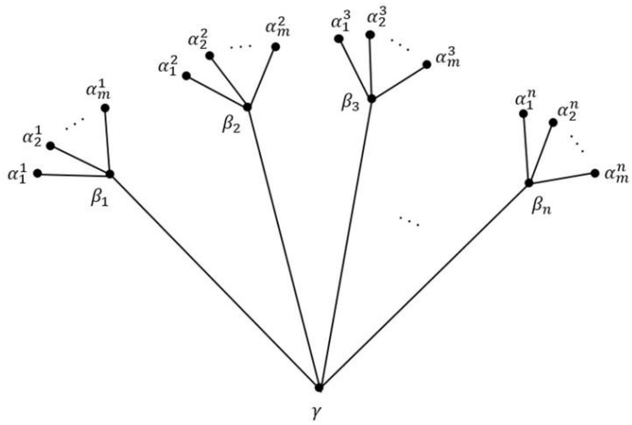

Figure 3: Star Glued with Star graph $\boldsymbol{S_{m,n}}$

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3. Main Results

In this section we prove three theorems.

Theorem 1

The Octopus graph O_n admits antimagic labeling.

Proof:

Let O_n , $(n \geq 2)$ be a octopus graph. Let us denote the apex vertex of O_n as u_0. Let us denote the remaining vertices of degree 2 adjacent to u_0 as $u_1, u_2, \ldots, u_n$ and the pendant vertices adjacent to u_0 as $u_{n+1}, u_{n+2}, \ldots, u_{2n}$ in the anticlockwise direction. Thus the vertex set is $V(O_n) = \{u_0, u_i : 1 \leq i \leq 2n\}$ and the edge set is $E(O_n) = \{ u_0 u_i : 1 \leq i \leq 2n, u_i u_{i+1} : 1 \leq i \leq n-1\}$. Also $|V(O_n)| = 2n+1$, $|E(O_n)| = 3n-1$. Now we define the edge labeling as $\zeta : E \longrightarrow \{1, 2, \ldots, 3n-1\}$

$\zeta(u_0 u_i) = i,$

$\zeta(u_i u_{i+1}) = 2n + i,$

The induced vertex labels are $\xi : V \longrightarrow \mathbb{Z}^+$ are

$\xi(u_0) = n(n+1)$

$g(u_1) = n+2$

$\xi(u_{i+1}) = 4n + 3i + 2$,

$\xi(u_n) = 4n - 1$

$\xi(u_i) = i, i = n+1, n+2, \ldots, 2n$

The vertex labels are distinct. Hence the octopus graph is antimagic.

Theorem 2

The jewel graph J_n is antimagic.

Proof:

Consider the Jewel graph J_n with vertex set $V(J_n) = \{\alpha_0, \beta_0, \gamma_0, \alpha_i : 1 \leq i \leq n+1\}$ and edge set $E(J_n) = \{\alpha_0 \alpha_1, \alpha_i \beta_0, \alpha_i \gamma_0 : 0 \leq i \leq n+1\}$. The total number of vertices are given by $|V(J_n)| = n+4$. The total number of edges are given by $|E(J_n)| = 2n+5$. Now we define the edge labeling as $\zeta : E \longrightarrow \{1, 2, \ldots, 2n+5\}$ as

$\zeta(\alpha_i \beta_0) = 2i + 1, \quad 0 \leq i \leq n+1$

$\zeta(\alpha_i \gamma_0) = 2i + 2, \quad 0 \leq i \leq n+1$

$\zeta(\alpha_0 \alpha_1) = 2n + 5$

$i = 1, 2, \ldots, 2n$

$i = 1, 2, \ldots, n-1$

The induced vertex labels are $\xi : V \longrightarrow \mathbb{Z}^+$ are

$\xi(\alpha_0) = 2n + 8$

$\xi(\beta_0) = (n+2)^2$

$\xi(\gamma_0) = (n+2)(n+3)$

$i = 1, 2, \ldots n-2$

$\xi(\alpha_1) = 2n + 12$

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$\xi(\alpha_i) = 4i + 7$,

The vertex labels are distinct. Hence the Jewel graph is antimagic.

Theorem 3
The Star glued with star graph is antimagic.
Proof :
Consider the Star glued with star graph $S_{m,n}$, the apex vertex is denoted as γ, the vertices adjacent to the apex vertex are denoted as $\beta_1, \beta_2, \beta_3, \ldots. \beta_n$ in the anticlockwise direction. The vertices non adjacent to the apex vertices are denoted as $\alpha_j{}^i, 1 \le i \le n, 1 \le j \le m$. Thus the vertex set $V(S_{m,n}) = \{\alpha_j{}^i, \beta_i, \gamma: 1 \le i \le n, 1 \le j \le m$. The edges incideent with the apex vertex are denoted as $y_1, y_2, y_3, \ldots. y_n$ and the edges not incident with the apex are denoted as edge $x_j{}^i, 1 \le i \le n, 1 \le j \le m$. Thus the edge set $E(S_{m,n}) = \{x_j{}^i : 1 \le i \le n, 1 \le j \le m, y_i : 1 \le i \le n\}$. The total number of vertices are given by $|V(S_{m,n})| = mn + n + 1$. The total number of edges are given by $|E(S_{m,n})| = mn + n$. Now we define the edge labeling as $\zeta: E \longrightarrow \{1, 2, \ldots, mn + n\}$ as

$$\zeta(x_i{}^j) = (j-1)n + i, 1 \le i \le n, 1 \le j \le m$$

$$\zeta(y_i) = mn + n - i + 1, 1 \le i \le n$$

The induced vertex labels are $\xi: V \longrightarrow \mathbb{Z}^+$ are

$\xi(\alpha_i{}^j) = (j-1)n + i$, $1 \le i \le n$, $1 \le j \le m$

$\xi(\beta_i) = \frac{m}{2}[2i + (m-1)n] + mn + n - i + 1$, $1 \le i \le n$

$\xi(\gamma) = \dfrac{n}{2}[2mn + n + 1]$

The vertex labels are distinct. Hence the Star glued with star graph is antimagic.

4. Conclusion

In this paper we have proved that Octopus graph O_n, Jewel graph J_n and Star glued with Star graph $S_{m,n}$ admits antimagic labeling.

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