JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3269 - 3279
https://publishoa.com
ISSN: 1309-3452

A CONTRIBUTION OF (t_1, t_2, t_3)- NEUTROSOPHIC MULTIFUZZY SUBRING

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Abstract: we have deliberated the notion (t_1, t_2, t_3)- Neutrosophic multifuzzy (Normal)subring and ideal and proved some theorems thar are related to the notion. Utilized the concept of level sets and homomorphic property we have explored some theorems.

Keywords: Neutrosophic fuzzy set (NFS), Neutrosophic multi fuzzy set (NMFS), t-Neutrosophic multifuzzy subring (t-NMFSR), (t_1, t_2, t_3)- Neutrosophic multifuzzy subring ((t_1, t_2, t_3)- NMFSR), (t_1, t_2, t_3)- Neutrosophic multifuzzy left(right) ideals ((t_1, t_2, t_3)- NMFL(R)I), (t_1, t_2, t_3) - Neutrosophic multifuzzy normal subring((t_1, t_2, t_3) -NMFNSR).

1. Introduction

The defined thought of fuzzy set was enlightened by L.A.Zadeh[12]. Smarandache[15] initiated Neutrosophic set to build the thought of Atanassov's[1] intuitionistic fuzzy sets which is, the part of philosophy. Gradually, some developments of this term were developed. It has been expanded by researchers in fields such as medical diagnosis, decision making, etc. In view of fuzzy set hypothesis, Multifuzzy set was initiated by Sabu and Ramakrishnan [8,9]. The unified notions of Multifuzzy set and Groups called as multifuzzy groups was examined by Muthuraj and Balamurugan [5,6]. Also, he has discussed its Level Subgroups. The combined concepts Intuitionistic Fuzzy sets and Fuzzy Multisets together reached as Intuitionistic Fuzzy multisets by Shinoj [11]. To elaborate the neutrosophic set theory, the conception neutrosophic multiset was originated by Deli [13] for modelling vagueness and uncertainty. The thought of t-Intuitionistic fuzzy groups along with homomorphic property had explored by Sharma [2,10]. It's generalized notion was examined by B.Anitha.[3] et.al. The scope of this work is utilizing the notion of Neutrosophic set and multifuzzy set in conjucation with rings we have tendency to characterized here an idea of (t_1, t_2, t_3)- Neutrosophic multifuzzy subrings along with some properties and create sense of certain outcomes connected with them.

2. Preliminaries

Definition.2.1[13] A NMFS д on Ҳ be defined as follows: д = {< x, ($ԋ^1_д$(x), $ԋ^2_д$(x), ..., $ԋ^n_д$(x)),($И^1_д$(x), $И^2_д$ (x), ..., $И^n_д$(x)), ($ғ^1_д$(x), $ғ^2_д$(x), ..., $ғ^n_д$(x)) >: x ∈ Ҳ }, where, $ԋ^i_д$(x), $И^i_д$(x), $ғ^i_д$(x): Ҳ → [0, 1] Э , 0 ≤ sup $ԋ^i_д$(x)+ sup $И^i_д$(x) + sup $ғ^i_д$(x) ≤ 3 (i = 1, 2, ..., n) and for any x truth membership $ԋ^1_д$(x) ≥ $ԋ^2_д$(x) ≥ ... ≥ $ԋ^n_д$(x) as decreasing order but no restrictions for indeterminacy and falsity membership . Furthermore, n is called the dimension of д, denoted d (д).

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3269 - 3279
https://publishoa.com
ISSN: 1309-3452

Definition.2.2[10] Let д be an Intuitionistic fuzzy set of a ring R. Let t∈ [0,1] then д′ of R is called t-intuitionistic fuzzy set of R with respect to intuitionistic fuzzy set д and is defined as $д' = (\mu_{д'}, И_{д'})$ with $\mu_{д'}(x) = \min\{\mu_д(x), t\}$ and $И_{д'}(x) = \max\{И_д(x), 1-t\}$ ∀ x∈ R.

Definition.2.3[14] Let X, Y be two non-empty sets and $F: X \to Y$ be a function.

(i) If $B = \langle y, \mu_B(y), И_B(y), f_B(y)\rangle / y \in Y$ is a NFS in Y, then the pre-image of B under F, denoted by $F^{-1}(B)$, is the NFS in X defined by $F^{-1}(B) = \{\langle x, F^{-1}(\mu_B)(x), F^{-1}(И_B)(x), F^{-1}(f_B)(x)\rangle : x \in X\}$ where $F^{-1}(\mu_B)(x) = (\mu_B)(F(x))$.

(ii) If $д$ is a Neutrosophic set in X, then the image of $д$ under F, denoted by $F(д)$, is the NFS in Y defined by $F(д) = \{(y), F(\mu_д)(y), F(И_д)(y), F(f_д)(y) : y \in Y\}$, where

$$F(\mu_д)(y) = \begin{cases} \sup_{x \in F^{-1}(y)} (\mu_д)(x), & \text{if } x \in F^{-1}(y) \\ 0, & \text{otherwise} \end{cases}$$

$$F(И_д)(y) = \begin{cases} \inf_{x \in F^{-1}(y)} (И_д)(x), & \text{if } x \in F^{-1}(y) \\ 1, & \text{otherwise} \end{cases}$$

$$F(f_д)(y) = \begin{cases} \inf_{x \in F^{-1}(y)} (f_д)(x), & \text{if } x \in F^{-1}(y) \\ 1, & \text{otherwise} \end{cases}$$

Where $F(f_д)(y) = (1 - F(1 - (f_д)))(y)$

3. (t_1, t_2, t_3)-Neutrosophic Multifuzzy subring

Definition.3.1 Let д be a NMFS of a ring R. Let t∈ [0,1] then д′ of R is called t-NMFS with respect to NMFS д and is defined as $д' = (\mu_{д'}, И_{д'}, f_{д'})$ where $\mu_{д'} = (\mu^1_{д'}, \mu^2_{д'} \dots \mu^n_{д'})$, $И_{д'} = (И^1_{д'}, И^2_{д'} \dots И^n_{д'})$, $f_{д'} = (f^1_{д'}, f^2_{д'} \dots f^n_{д'})$ with $\mu_{д'}(x) = \min\{\mu_д(x), t\}$, $И_{д'}(x) = \max\{И_д(x), 1-t\}$ and $f_{д'}(x) = \max\{f_д(x), 1-t\}$ ∀ x∈ R ,i=1,2,…,n.

Definition.3.2 Let д be a NMFS of a ring R. Let $t_1, t_2, t_3 \in [0,1]$ and $t_2 \le 1-t_1$, $t_3 \le 1-t_1$. Then NMFS д′ of R is called (t_1, t_2, t_3) -NMFS of R with respect to NMFS д and is defined as $д' = (\mu_{д'}, И_{д'}, f_{д'})$ .where $\mu_{д'} = (\mu^1_{д'}, \mu^2_{д'} \dots \mu^n_{д'})$, $И_{д'} = (И^1_{д'}, И^2_{д'} \dots И^n_{д'})$, $f_{д'} = (f^1_{д'}, f^2_{д'} \dots f^n_{д'})$ with $\mu^i_{д'}(x) = \min\{\mu^i_д(x), t_1\}$, $И^i_{д'}(x) = \min\{И^i_д(x), t_2\}$, $f^i_{д'}(x) = \min\{f^i_д(x), t_3\}$ ∀ x∈ R ,i=1,2,…,n.

Note.3.3 When $t_2 = 1-t_1$, $t_3 = 1-t_1$. Then (t_1, t_2, t_3)-NMFS coincide with t_1-NMFS. Thus, every t-NMFS is (t,1-t,1-t)-NMFS.

Definition.3.4 Let д be a NMFS of a ring R. Let $t_1, t_2, t_3 \in [0,1]$ and $t_2 \le 1-t_1$, $t_3 \le 1-t_1$. Then д is called (t_1, t_2, t_3) -NMFSR of R if д′ satisfies the following condition:

(i) $\mu_{д'}(x-y) \ge \min(\mu_{д'}(x), \mu_{д'}(y))$ (ii) $\mu_{д'}(xy) \ge \min(\mu_{д'}(x), \mu_{д'}(y))$

$И_{д'}(x-y) \le \max(И_{д'}(x), И_{д'}(y))$ $И_{д'}(xy) \le \max(И_{д'}(x), И_{д'}(y))$

$f_{д'}(x-y) \le \max(f_{д'}(x), f_{д'}(y))$ $f_{д'}(xy) \le \max(f_{д'}(x), f_{д'}(y))$

With $\mu^i_{д'}(x), И^i_{д'}(x), f^i_{д'}(x): X \to [0, 1] \ni 0 \le \sup \mu^i_{д'}(x) + \sup И^i_{д'}(x) + \sup f^i_{д'}(x) \le 3$ where truth membership function $\mu^1_{д'}(x) \ge \mu^2_{д'}(x) \ge \dots \ge \mu^n_{д'}(x)$ as decreasing order but no restrictions for indeterminacy and falsity membership function.

Remark.3.5 If д is (t_1, t_2, t_3)-NMFSR of R for all $t_1, t_2, t_3 \in [0,1]$ with $t_2 \le 1-t_1$, $t_3 \le 1-t_1$. Then д is t-NMFSR of R for all t∈ [0,1](assume $t = t_1$, $t_2 = 1-t_1$, $t_3 = 1-t_1$). However if д is t-NMFSR of R for some t∈ [0,1] then it is not necessary that д is (t_1, t_2, t_3)-NMFSR of R when $t = t_1$, $t_2 < 1-t_1$, $t_3 < 1-t_1$ as the accompanying example will show our case.

Example. 3.6 Consider the ring $(Z_4,+,\cdot)$. Define NMFS д of Z_4 by д= {(<0(0.7,0.6,0.4) (0.3,0.4,0.5) (0.2,0.4,0.6)>, <1(0.4,0.3,0.2) (0.5,0.6,0.6) (0.3,0.5,0.6)>, <2(0.6,0.4,0.4) (0.4,0.5,0.8) (0.4,0.5,0.6)>, <3(0.4,0.3,0.2) (0.5,0.6,0.6) (0.4,0.5,0.6)>}.

If we take $t_1 = 0.2$ then $\text{ɥ}_{д'}(x)= (0.2,0.2,0.2)$; $И_{д'}(x) = (0.8,0.8,0.8) = ғ_{д'}(x)$; $\forall\ x\in Z_4$.Hence д is 0.2-NMFSR of Z_4.Suppose $t_1,t_2,t_3 = 0.2, 0.6, 0.7$ then $\text{ɥ}_{д'}(x)= (0.2,0.2,0.2)\ \forall\ x\in Z_4$

$И_{д'}(x) = \begin{cases} (0.6,0.6,0.6) & if\ x = 0,1,3 \\ (0.6,0.6,0.8) & if\ x = 2 \end{cases}$; $ғ_{д'}(x) = (0.7,0.7,0.7)\forall\ x\in Z_4$.Hence д is not (0.2,0.6,0.7)-NMFSR of Z_4 as $И_{д'}(3-1)$ =0.6,0.6,0.8; $\max\{И_{д'}(3),И_{д'}(1)\}$=(0.6,0.6,0.6); $И_{д'}(3-1) \nleq \max\{И_{д'}(3),И_{д'}(1)\}$

Hence д is 0.2-NMFSR of Z_4 but not (0.2,0.6,0.7)-NMFSR of Z_4

Proposition.3.7 If д is NMFSR of R then д is also(t_1, t_2, t_3) -NMFSR of R with $t_2 \le 1-t_1$, $t_3 \le 1-t_1$ where $t_1, t_2, t_3 \in [0,1]$.

Proof Let д be NMFSR of R $\forall$ i and $x, y \in$ R.

$(i)\text{ɥ}^i_{д'}(x-y) = \min\{\text{ɥ}^i_д(x-y),t_1\} \ge \min\{\min\{\text{ɥ}^i_д(x),\text{ɥ}^i_д(y)\},t_1\} = \min\{\min\{\text{ɥ}^i_д(x),t_1\},\min\{\text{ɥ}^i_д(y),t_1\}\}$ $=\min\{\text{ɥ}^i_{д'}(x),\text{ɥ}^i_{д'}(y)\}$

$И^i_{д'}(x-y) = \max\{И^i_д(x-y),t_2\} \le \max\{\max\{И^i_д(x),И^i_д(y)\},t_2\} = \min\{\max\{И^i_д(x),t_2\},\max\{И^i_д(y),t_2\}\}$ $= \max\{И^i_{д'}(x),И^i_{д'}(y)\}$

Similarly, $ғ^i_{д'}(x-y) \le \max\{ғ^i_{д'}(x),ғ^i_{д'}(y)\}$

$(ii)\text{ɥ}^i_{д'}(x\,y) = \min\{\text{ɥ}^i_д(xy),t_1\} \ge \min\{\min\{\text{ɥ}^i_д(x),\text{ɥ}^i_д(y)\},t_1\}= \min\{\min\{\text{ɥ}^i_д(x),t_1\},\min\{\text{ɥ}^i_д(y),t_1\}\}$ $= \min\{\text{ɥ}^i_{д'}(x),\text{ɥ}^i_{д'}(y)\}$

$И^i_{д'}(x\,y) = \max\{И^i_д(xy),t_2\} \le \max\{\max\{И^i_д(x),И^i_д(y)\},t_2\} = \min\{\max\{И^i_д(x),t_2\},\max\{И^i_д(y),t_2\}\}$ $= \max\{И^i_{д'}(x),И^i_{д'}(y)\}$

Similarly, $ғ^i_{д'}(x\,y) \le \max\{ғ^i_{д'}(x),ғ^i_{д'}(y)\}$.From this д is ($t_1, t_2, t_3$)-NMFSR of R.

Remark.3.8 If д is (t_1, t_2, t_3) -NMFSR of R then it isn't really a fact that д is NMFSR of R as is obvious from the accompanying case.

Example.3.9 Consider the ring $(Z_4,+,\cdot)$. Define NMFS д of Z_4 by д= {(<0(0.9,0.8,0.7) (0.3,0.5,0.6) (0.1,0.2,0.3)>, <1(0.8,0.5,0.4) (0.3,0.6,0.7) (0.2,0.3,0.3)>, <2(0.9,0.5,0.4) (0.3,0.5,0.6) (0.3,0.4,0.5)>, <3(0.9,0.5,0.3) (0.3,0.6,0.7) (0.3,0.3,0.4)>}

It is clear that д is not a NMFSR of R as of $ғ_{д'}(3-1)$ =0.3,0.4,0.5; $\max\{ғ_{д'}(3),ғ_{д'}(1)\}$=(0.3,0.3,0.4); $ғ_{д'}(3-1) \nleq \max\{ғ_{д'}(3),ғ_{д'}(1)\}$.Suppose we take $t_1,t_2,t_3 = 0.2, 0.7, 0.5$ then $\text{ɥ}_{д'}(x)=$ (0.2,0.2,0.2); $И_{д'}(x) = (0.7,0.7,0.7)$ $ғ_{д'}(x) = (0.5,0.5,0.5)\forall\ x\in Z_4$. It is clear that д′ is NMFSR of Z_4 and thus д is (0.2,0.7,0.5)-NMFSR of Z_4.

Definition.3.10 Let д′= {x, $\text{ɥ}_{д'}$, $И_{д'}$, $ғ_{д'}$: x∈X} and Ɓ′ = {x, $\text{ɥ}_{Ɓ'}$, $И_{Ɓ'}$, $ғ_{Ɓ'}$: x∈X} be any two (t_1, t_2, t_3) -NMFS having the same cardinality n of X. Then

(i) д′ ⊆ Ɓ′ iff $\text{ɥ}_{д'}(x)\le\text{ɥ}_{Ɓ'}(x)$, $И_{д'}(x) \le И_{Ɓ'}(x)$and $ғ_{д'}(x) \le ғ_{Ɓ'}(x)$)$\forall\ x\in X$

(ii) д′ = Ɓ′ iff $\text{ɥ}_{д'}(x)=\text{ɥ}_{Ɓ'}(x)$, $И_{д'}(x) = И_{Ɓ'}(x)$and $ғ_{д'}(x) = ғ_{Ɓ'}(x)$)$\forall\ x\in X$

(iii)д′ ∩ Ɓ′ =($\text{ɥ}_{д\cap Ɓ}{}'$, $И_{д\cap Ɓ}{}'$, $ғ_{д\cap Ɓ}{}'$)

where $(\text{ɥ}_{д\cap Ɓ}{}')$ (x) =min $\{\text{ɥ}_{д'}(x),\text{ɥ}_{Ɓ'}(x)\}$= min $\{\text{ɥ}^i_{д'}(x),\text{ɥ}^i_{Ɓ'}(x)\}$; $И_{д\cap Ɓ}{}'(x)$=max $\{И_{д'}(x),И_{Ɓ'}(x)\}$=max$\{И^i_{д'}(x),И^i_{Ɓ'}(x)\}$$ғ_{д\cap Ɓ}{}'(x)$=max $\{ғ_{д'}(x),ғ_{Ɓ'}(x)\}$=max$\{ғ^i_{д'}(x),ғ^i_{Ɓ'}(x)\}$ $\forall$ x $\in X$ and i=1, 2..., n.

(iv)д′ ∪ Ɓ′ =($\text{ɥ}_{д\cup Ɓ}{}'$, $И_{д\cup Ɓ}{}'$, $ғ_{д\cup Ɓ}{}'$)

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3269 - 3279
https://publishoa.com
ISSN: 1309-3452

where $(ɥ_{д\cup Ɓ}{}')(x)$ =min $\{ɥ_{д'}(x), ɥ_{Ɓ'}(x)\}$= min $\{ɥ^i_{д'}(x), ɥ^i_{Ɓ'}(x)\}$; $И_{д\cup Ɓ}{}'(x)$=max $\{И_{д'}(x), И_{Ɓ'}(x)\}$=max$\{И^i_{д'}(x), И^i_{Ɓ'}(x)\}$$ғ_{д\cup Ɓ}{}'(x)$=max $\{ғ_{д'}(x), ғ_{Ɓ'}(x)\}$=max$\{ғ^i_{д'}(x), ғ^i_{Ɓ'}(x)\}$ $\forall$ x $\in X$ and i=1, 2..., n.

Result.3.11 Let $д'=\{x, ɥ_{д'}, И_{д'}, ғ_{д'}: x\in X\}$ and $Ɓ'=\{x, ɥ_{Ɓ'}, И_{Ɓ'}, ғ_{Ɓ'}: x\in X\}$ be any two (t_1, t_2, t_3) -NMFS of a ring R. Then $(д\cap Ɓ)'=д'\cap Ɓ'$.

Proof Let $x\in$ R. Then$ɥ^i_{(д\cap Ɓ)'}(x)$= $\min\{ɥ^i_{д\cap Ɓ}(x), t_1\}$= min $\{\min\{ɥ^i_д(x), ɥ^i_Ɓ(x)\}, t_1\}$ =min $\{\min\{ɥ^i_д(x), t_1\}, \min\{ɥ^i_Ɓ(x), t_1\}\}$= $\min\{ɥ^i_{д'}(x), ɥ^i_{Ɓ'}(x)\}$=$ɥ^i_{д'\cap Ɓ'}(x)$.Similarly, we can show that $И^i_{(д\cap Ɓ)'}(x)$= $И^i_{д'\cap Ɓ'}(x)$, $ғ^i_{(д\cap Ɓ)'}(x)$= $ғ^i_{д'\cap Ɓ'}(x)$.Thus $(д\cap Ɓ)'=д'\cap Ɓ'$.

Proposition.3.12 The intersection of two (t_1, t_2, t_3)-NMFSR of R is also (t_1, t_2, t_3)-NMFSR of R.

Proof Let д and Ɓ be two (t_1, t_2, t_3)- NMFSR of R and $x, y\in$ R. Then for all i

(i) $ɥ^i_{(д\cap Ɓ)'}(x-y) = \min\{ɥ^i_{д\cap Ɓ}(x-y), t_1\}$=min $\{\min\{ɥ^i_д(x-y), ɥ^i_Ɓ(x-y)\}, t_1\}$

= $\min\{\min\{ɥ^i_д(x-y), t_1\}, \min\{ɥ^i_Ɓ(x-y), t_1\}\}$= $\min\{ɥ^i_{д'}(x-y), ɥ^i_{Ɓ'}(x-y)\}$

$\geq\min\{\min\{ɥ^i_{д'}(x), ɥ^i_{д'}(y)\}\min\{ɥ^i_{Ɓ'}(x), ɥ^i_{Ɓ'}(y)\}\}$ = $\min\{ɥ^i_{д'\cap Ɓ'}(x), ɥ^i_{д'\cap Ɓ'}(y)\}$ =min $\{ɥ^i_{(д\cap Ɓ)'}(x), ɥ^i_{(д\cap Ɓ)'}(y)\}$

$И^i_{(д\cap Ɓ)'}(x-y) = \max\{И^i_{д\cap Ɓ}(x-y), t_2\}$ =max $\{\max\{И^i_д(x-y), И^i_Ɓ(x-y)\}, t_2\}$

= $\max\{\max\{И^i_д(x-y), t_2\}, \max\{И^i_Ɓ(x-y), t_2\}\}$= $\max\{И^i_{д'}(x-y), И^i_{Ɓ'}(x-y)\}$ $\leq$max $\{\max\{И^i_{д'}(x), И^i_{д'}(y)\}\max\{И^i_{Ɓ'}(x), И^i_{Ɓ'}(y)\}\}$= $\max\{И^i_{д'\cap Ɓ'}(x), И^i_{д'\cap Ɓ'}(y)\}$ = max $\{И^i_{(д\cap Ɓ)'}(x), И^i_{(д\cap Ɓ)'}(y)\}$

(ii) $ɥ^i_{(д\cap Ɓ)'}(xy) = \min\{ɥ^i_{д\cap Ɓ}(xy), t_1\}$ =min $\{\min\{ɥ^i_д(xy), ɥ^i_Ɓ(xy)\}, t_1\}$

= $\min\{\min\{ɥ^i_д(xy), t_1\}, \min\{ɥ^i_Ɓ(xy), t_1\}\}$ = $\min\{ɥ^i_{д'}(xy), ɥ^i_{Ɓ'}(xy)\}$

$\geq\min\{\min\{ɥ^i_{д'}(x), ɥ^i_{д'}(y)\}\min\{ɥ^i_{Ɓ'}(x), ɥ^i_{Ɓ'}(y)\}\}$= $\min\{ɥ^i_{д'\cap Ɓ'}(x), ɥ^i_{д'\cap Ɓ'}(y)\}$=min $\{ɥ^i_{(д\cap Ɓ)'}(x), ɥ^i_{(д\cap Ɓ)'}(y)\}$

$И^i_{(д\cap Ɓ)'}(xy) = \max\{И^i_{д\cap Ɓ}(xy), t_2\}$ =max $\{\max\{И^i_д(xy), И^i_Ɓ(xy)\}, t_2\}$

=max$\{\max\{И^i_д(xy), t_2\}, \max\{И^i_Ɓ(xy), t_2\}\}$= $\max\{И^i_{д'}(xy), И^i_{Ɓ'}(xy)\}$ $\leq$max $\{\max\{И^i_{д'}(x), И^i_{д'}(y)\}\max\{И^i_{Ɓ'}(x), И^i_{Ɓ'}(y)\}\}$=$\max\{И^i_{д'\cap Ɓ'}(x), И^i_{д'\cap Ɓ'}(y)\}$=max$\{И^i_{(д\cap Ɓ)'}(x), И^i_{(д\cap Ɓ)'}(y)\}$

In similar way we can easily show that,

$ғ^i_{(д\cap Ɓ)'}(x-y) \leq \max\{ғ^i_{(д\cap Ɓ)'}(x), ғ^i_{(д\cap Ɓ)'}(y)\}$

$ғ^i_{(д\cap Ɓ)'}(xy) \leq \max\{ғ^i_{(д\cap Ɓ)'}(x), ғ^i_{(д\cap Ɓ)'}(y)\}$.Hence $д\cap Ɓ$ is a (t_1, t_2, t_3)-NMFSR of R $\forall$ $x, y\in$ R ,.

Definition.3.13 Let д be a NMFS of R. Let $t_1, t_2, t_3 \in [0,1]$ and $t_2\leq 1-t_1$, $t_3\leq 1-t_1$.Then д is called $(i)(t_1, t_2, t_3)$ -NMFLI of R if:

1. $ɥ_{д'}(x-y) \geq \min(ɥ_{д'}(x), ɥ_{д'}(y))$
   $И_{д'}(x-y) \leq \max(И_{д'}(x), И_{д'}(y))$
   $ғ_{д'}(x-y) \leq \max(ғ_{д'}(x), ғ_{д'}(y))$
2. $ɥ_{д'}(xy) \geq ɥ_{д'}(y)$
   $И_{д'}(xy) \leq И_{д'}(y)$
   $ғ_{д'}(xy) \leq ғ_{д'}(y)$

$(ii)(t_1, t_2, t_3)$ -NMFRI of R if:

1. $ɥ_{д'}(x-y) \geq \min(ɥ_{д'}(x), ɥ_{д'}(y))$
   $И_{д'}(x-y) \leq \max(И_{д'}(x), И_{д'}(y))$
   $ғ_{д'}(x-y) \leq \max(ғ_{д'}(x), ғ_{д'}(y))$
2. $ɥ_{д'}(xy) \geq ɥ_{д'}(x)$
   $И_{д'}(xy) \leq И_{д'}(x)$
   $ғ_{д'}(xy) \leq ғ_{д'}(x)$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3269 - 3279
https://publishoa.com
ISSN: 1309-3452

$(iii)(t_1, t_2, t_3)$ -NMFI of R if:

1. $μ_{д'}(x - y) \geq \min\left(μ_{д'}(x), μ_{д'}(y)\right)$
   $И_{д'}(x - y) \leq \max\left(И_{д'}(x), И_{д'}(y)\right)$
   $F_{д'}(x - y) \leq \max\left(F_{д'}(x), F_{д'}(y)\right)$
2. $μ_{д'}(xy) \geq \max\left(μ_{д'}(x), μ_{д'}(y)\right)$
   $И_{д'}(xy) \leq \min\left(И_{д'}(x), И_{д'}(y)\right)$
   $F_{д'}(xy) \leq \min\left(F_{д'}(x), F_{д'}(y)\right)$

$\forall\ x, y \in R.$

Proposition.3.14 If д is NMFL(R)I of a ring R, then д is also (t_1, t_2, t_3)-NMFL(R)I of R.

Proof Let x, $y \in R$.д is NMFSR. (By proposition 3.7)

It is enough to show

$$μ_{д'}(xy) \geq μ_{д'}(y); И_{д'}(xy) \leq И_{д'}(y);\ F_{д'}(xy) \leq F_{д'}(y)$$

Then for all i

(i) $μ^i_{д'}(xy) = \min\{μ^i_д(xy), t_1\} \geq \min\{μ^i_д(y), t_1\} = μ^i_{д'}(y)$

(ii) $И^i_{д'}(xy) = \max\{И^i_д(xy), t_2\} \leq \max\{И^i_д(y), t_2\} = И^i_{д'}(y)$

Similarly, we can show easily $F_{д'}(xy) \leq F_{д'}(y) \forall\ x, y \in R.$

Thus д is also (t_1, t_2, t_3)-NMFLI of R. Similarly, we can show easily for NMFRI.

Proposition.3.15 If д is NMFI of a ring R, then д is also (t_1, t_2, t_3)-NMFI of R.

Proof Follows from the above proposition.

4. Homomorphism (hom...) of (t_1, t_2, t_3)-NMFSR:

Definition.4.1 If $д = (μ^i_д, И^i_д, F^i_д)$ is a(t_1, t_2, t_3)- NMFS in R, then $Ƒ(д') = B'$, is defined by

$$Ƒ(μ^i_{д'})(y) = \begin{cases} \sup\limits_{x \in Ƒ^{-1}(y)} (μ^i_{д'})(x), & \text{if } x \in Ƒ^{-1}(y) \\ 0, & \text{otherwise} \end{cases}$$

$$Ƒ(И^i_{д'})(y) = \begin{cases} \sup\limits_{x \in Ƒ^{-1}(y)} (И^i_{д'})(x), & \text{if } x \in Ƒ^{-1}(y) \\ 1, & \text{otherwise} \end{cases}$$

$$Ƒ(F^i_{д'})(y) = \begin{cases} \inf\limits_{x \in Ƒ^{-1}(y)} (F^i_{д'})(x), & \text{if } x \in Ƒ^{-1}(y) \\ 1, & \text{otherwise} \end{cases}$$

Where Ƒ is ring homomorphism of R onto R_1. Also $Ƒ^{-1}(B') = \{\langle x, Ƒ^{-1}(μ^i_{B'})(x), Ƒ^{-1}(И^i_{B'})(x), Ƒ^{-1}(F^i_{B'})(x)\rangle : x \in д\}$ where $Ƒ^{-1}(B')(x) = (B')(Ƒ(x))$.

Theorem.4.2 Let Ƒ be a hom... of ring R onto R_1. If $B \in (t_1, t_2, t_3)$-NMFSR of R_1 then $Ƒ^{-1}(B) \in (t_1, t_2, t_3)$- NMFSR of R.

Proof: Let $x, y \in R$. Let $B \in (t_1, t_2, t_3)$- NMFSR of R_1 Then for all i

(i) $Ƒ^{-1}\left((μ^i_{B'})(x - y)\right) = μ^i_{B'}(Ƒ(x - y)) = μ^i_{B'}(Ƒ(x) - Ƒ(y)) \geq \min(μ^i_{B'}(Ƒ(x)), μ^i_{B'}(Ƒ(y)))$

$= \min(Ƒ^{-1}(μ^i_{B'})(x), Ƒ^{-1}(μ^i_{B'})(y))$

$Ƒ^{-1}\left((И^i_{B'})(x - y)\right) = И^i_{B'}(Ƒ(x - y)) = И^i_{B'}(Ƒ(x) - Ƒ(y)) \leq \max(И^i_{B'}(Ƒ(x)), И^i_{B'}(Ƒ(y)))$

$= \max(Ƒ^{-1}(И^i_{B'})(x), Ƒ^{-1}(И^i_{B'})(y))$

(ii) $Ƒ^{-1}\left((μ^i_{B'})(xy)\right) = μ^i_{B'}(Ƒ(xy)) = μ^i_{B'}(Ƒ(x)Ƒ(y)) \geq \min(μ^i_{B'}(Ƒ(x)), μ^i_{B'}(Ƒ(y)))$

$= \min(Ƒ^{-1}(μ^i_{B'})(x), Ƒ^{-1}(μ^i_{B'})(y))$

$Ƒ^{-1}\left((И^i_{B'})(xy)\right) = И^i_{B'}(Ƒ(xy)) = И^i_{B'}(Ƒ(x)Ƒ(y)) \leq \max(И^i_{B'}(Ƒ(x)), И^i_{B'}(Ƒ(y)))$

$= \max(Ƒ^{-1}(И^i_{B'})(x), Ƒ^{-1}(И^i_{B'})(y))$

Similarly, we can easily show that $Ƒ^{-1}(F^i_{B'})(xy) \leq \max\left(Ƒ^{-1}(F^i_{B'})(x), Ƒ^{-1}(F^i_{B'})(y)\right)$

Hence $Ғ^{-1}(Ɓ) \in (t_1, t_2, t_3)$- NMFSR of R.

Definition.4.3 Let д be a NMFS of a ring R. Let $t_1, t_2, t_3 \in [0,1]$ and $t_2 \leq 1-t_1$, $t_3 \leq 1-t_1$.If a (t_1, t_2, t_3) -NMFSR д of R is called (t_1, t_2, t_3) -NMFNSR of R if it satisfies the following condition: $ӌ_{д'}(xy) = ӌ_{д'}(xy)$; $И_{д'}(xy) = И_{д'}(xy)$, $ғ_{д'}(xy) = ғ_{д'}(xy)$ $\forall$ x,y $\in$ R.

Theorem.4.4 Let R and R_1 be any two rings and $Ғ$ be a hom... of R onto R_1. If $Ɓ \in (t_1, t_2, t_3)$- NMFNSR of R_1 then $Ғ^{-1}(Ɓ) \in (t_1, t_2, t_3)$- NMFNSR of R.

Proof Let x, y $\in R$. Let $Ɓ \in (t_1, t_2, t_3)$- NMFSR of R_1. By theorem 4.2 $Ғ^{-1}(Ɓ) \in (t_1, t_2, t_3)$- NMFSR of R Then for all i $Ғ^{-1}\left((ӌ^i_{Ɓ'})(xy)\right) = ӌ^i_{Ɓ'}(Ғ(xy)) = ӌ^i_{Ɓ'}(Ғ(x)Ғ(y)) = ӌ^i_{Ɓ'}(Ғ(y)Ғ(x)) = ӌ^i_{Ɓ'}(Ғ(yx))$

$=Ғ^{-1}\left((ӌ^i_{Ɓ'})(yx)\right)$

$Ғ^{-1}\left((И^i_{Ɓ'})(xy)\right) = И^i_{Ɓ'}(Ғ(xy)) = И^i_{Ɓ'}(Ғ(x)Ғ(y)) = И^i_{Ɓ'}(Ғ(y)Ғ(x)) = И^i_{Ɓ'}(Ғ(yx))$

$=Ғ^{-1}\left((И^i_{Ɓ'})(yx)\right)$ .Similarly, we can easily show that $Ғ^{-1}\left((ғ^i_{Ɓ'})(xy)\right) = Ғ^{-1}\left((ғ^i_{Ɓ'})(yx)\right)$

Hence $Ғ^{-1}(Ɓ) \in (t_1, t_2, t_3)$- NMFNSR of R.

Theorem.4.5 Let R and R_1 be any two rings and $Ғ$ be a hom... of R onto R_1. If $Ɓ \in (t_1, t_2, t_3)$-NMFI of R_1 then $Ғ^{-1}(Ɓ) \in (t_1, t_2, t_3)$- NMFL(R)I of R.

Proof Let x, y $\in R$. Let $Ɓ \in (t_1, t_2, t_3)$- NMFLI of R_1 Then for all i, By theorem 4.2 $Ғ^{-1}(Ɓ) \in (t_1, t_2, t_3)$- NMFSR of R.It is enough to show $Ғ^{-1}\left((ӌ^i_{Ɓ'})(xy)\right) \geq Ғ^{-1}\left((ӌ^i_{Ɓ'})(y)\right)$; $Ғ^{-1}\left((И^i_{Ɓ'})(xy)\right) \leq Ғ^{-1}\left((И^i_{Ɓ'})(y)\right)$ $Ғ^{-1}\left((ғ^i_{Ɓ'})(xy)\right) \leq Ғ^{-1}\left((ғ^i_{Ɓ'})(y)\right)$

$Ғ^{-1}\left((ӌ^i_{Ɓ'})(xy)\right) = ӌ^i_{Ɓ'}(Ғ(xy)) = ӌ^i_{Ɓ'}(Ғ(x)Ғ(y)) \geq ӌ^i_{Ɓ'}(Ғ(y)) = Ғ^{-1}\left((ӌ^i_{Ɓ'})(y)\right)$

$Ғ^{-1}\left((И^i_{Ɓ'})(xy)\right) = И^i_{Ɓ'}(Ғ(xy)) = И^i_{Ɓ'}(Ғ(x)Ғ(y)) \leq И^i_{Ɓ'}(Ғ(y)) = Ғ^{-1}\left((И^i_{Ɓ'})(y)\right)$

Similarly, we can easily show that $Ғ^{-1}\left((ғ^i_{Ɓ'})(xy)\right) \leq Ғ^{-1}\left((ғ^i_{Ɓ'})(y)\right)$

Hence $Ғ^{-1}(Ɓ) \in (t_1, t_2, t_3)$- NMFLI of R.In the same way we prove that $Ғ^{-1}(Ɓ) \in (t_1, t_2, t_3)$- NMFRI of R.

Theorem.4.6 Let $Ғ$ be a hom... of ring R onto R_1. If $Ɓ \in (t_1, t_2, t_3)$-NMFI of R_1 then $Ғ^{-1}(Ɓ) \in (t_1, t_2, t_3)$- NMFI of R.

Proof Follows from the above theorem.

Theorem.4.7 Let $Ғ$ be a hom... of ring R onto R_1. If $д \in (t_1, t_2, t_3)$- NMFSR of R then $Ғ(д) \in (t_1, t_2, t_3)$- NMFSR of R_1.

Proof Let $y_1, y_2 \in R_1$ then there exist $x_1, x_2 \in R$ such that $Ғ(x_1)=y_1$, $Ғ(x_2)=y_2$

If $д \in (t_1, t_2, t_3)$- NMFSR of R. Then

(i) $Ғ\left((ӌ^i_{д'})(y_1 - y_2)\right) = \min\left\{Ғ\left((ӌ^i_д)(y_1 - y_2), t_1\right)\right\}$

$=\min\left\{Ғ\left((ӌ^i_д)(Ғ(x_1) - Ғ(x_2)), t_1\right)\right\} = \min\left\{Ғ\left((ӌ^i_д)Ғ(x_1 - x_2), t_1\right)\right\} \geq \min\left\{\left((ӌ^i_д)(x_1 - x_2), t_1\right)\right\}$

$=(ӌ^i_{д'})(x_1 - x_2) \geq \min(ӌ^i_{д'}(x_1), ӌ^i_{д'}(x_2) = \min\left(\sup_{x \in Ғ^{-1}(y)}(ӌ^i_{д'}(x_1)), \sup_{x \in Ғ^{-1}(y)}(ӌ^i_д(x_2))\right)$

$= \min\left(Ғ(ӌ^i_{д'}(y_1)), Ғ(ӌ^i_{д'}(y_2))\right)$

$Ғ\left((И^i_{д'})(y_1 - y_2)\right) = \max\left\{Ғ\left((И^i_д)(y_1 - y_2), t_2\right)\right\} = \max\left\{Ғ\left((И^i_д)(Ғ(x_1) - Ғ(x_2)), t_2\right)\right\}$

$= \max\left\{Ғ\left((И^i_д)Ғ(x_1 - x_2), t_2\right)\right\} \leq \max\left\{\left((И^i_д)(x_1 - x_2), t_2\right)\right\} = (И^i_{д'})(x_1 - x_2) \leq \max(И^i_{д'}(x_1), И^i_{д'}(x_2)$

$) = \max\left(\inf_{x \in Ғ^{-1}(y)}(И^i_{д'}(x_1)), \inf_{x \in Ғ^{-1}(y)}(И^i_д(x_2))\right) = \max\left(Ғ(И^i_{д'}(y_1)), Ғ(И^i_{д'}(y_2))\right)$

(ii) $Ғ\left((ӌ^i_{д'})(y_1 y_2)\right) = \min\left\{Ғ\left((ӌ^i_д)(y_1 y_2), t_1\right)\right\} = \min\left\{Ғ\left((ӌ^i_д)(Ғ(x_1)Ғ(x_2)), t_1\right)\right\}$

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Volume 13, No. 3, 2022, p. 3269 - 3279
https://publishoa.com
ISSN: 1309-3452

$= \min\{Ғ((\mu^i_{д})Ғ(x_1x_2),t_1)\} \geq \min\{((\mu^i_{д})(x_1x_2),t_1)\} = (\mu^i_{д'})(x_1x_2) \geq \min(\mu^i_{д'}(x_1), \mu^i_{д'}(x_2)$

$= \min\left(\sup_{x\in Ғ^{-1}(y)}(\mu^i_{д'}(x_1)), \sup_{x\in Ғ^{-1}(y)}(\mu^i_{д}(x_2))\right) = \min\left(Ғ(\mu^i_{д'}(y_1)), Ғ(\mu^i_{д'}(y_2))\right)$

$Ғ((\nu^i_{д'})(y_1-y_2)) = \max\{Ғ((\nu^i_{д})(y_1y_2),t_2)\} = \max\{Ғ((\nu^i_{д})(Ғ(x_1)Ғ(x_2)),t_2)\}$

$= \max\{Ғ((\nu^i_{д})Ғ(x_1x_2),t_2)\} \leq \max\{((\nu^i_{д})(x_1x_2),t_2)\} = (\nu^i_{д'})(x_1x_2) \leq \max(\nu^i_{д'}(x_1), \nu^i_{д'}(x_2)$

$= \max\left(\inf_{x\in Ғ^{-1}(y)}(\nu^i_{д'}(x_1)), \inf_{x\in Ғ^{-1}(y)}(\nu^i_{д}(x_2))\right) = \max\left(Ғ(\nu^i_{д'}(y_1)), Ғ(\nu^i_{д'}(y_2))\right)$

Similarly, $Ғ((\digamma^i_{д'})(y_1-y_2)) \leq \max\left(Ғ(\digamma^i_{д'}(y_1)), Ғ(\digamma^i_{д'}(y_2))\right)$

$Ғ((\digamma^i_{д'})(y_1-y_2)) \leq \max\left(Ғ(\digamma^i_{д'}(y_1)), Ғ(\digamma^i_{д'}(y_2))\right)$. Hence $Ғ(д) \in (t_1,t_2,t_3)$- NMFSR of R_1.

Theorem.4.8 Let Ғ be a hom... of R onto R_1. If $д \in (t_1,t_2,t_3)$- NMFNSR of R then $Ғ(д) \in (t_1,t_2,t_3)$ - NMFNSR of R_1.

Proof Let $y_1, y_2 \in R_1$ then there exist $x_1, x_2 \in R$ such that $Ғ(x_1)=y_1$, $Ғ(x_2)=y_2$

If $д \in (t_1,t_2,t_3)$- NMFNSR of R. Then by theorem $Ғ(д) \in (t_1,t_2,t_3)$- NMFSR of R_1. Then for all i,

$Ғ((\mu^i_{д'})(y_1y_2)) = Ғ((\mu^i_{д'})(Ғ(x_1)Ғ(x_2))) = Ғ((\mu^i_{д'})(Ғ(x_1x_2))) = \sup_{x_1x_2\in Ғ^{-1}(y_1y_2)}(\mu^i_{д'}(x_1x_2))$

$= \sup_{x\in Ғ^{-1}(y)}(\mu^i_{д'}(x_2x_1)) = Ғ((\mu^i_{д'})(y_2y_1))$

In similar way $Ғ((\nu^i_{д'})(y_1y_2)) = Ғ((\nu^i_{д'})(y_2y_1))$

$Ғ((\digamma^i_{д'})(y_1y_2)) = Ғ((\digamma^i_{д'})(y_2y_1))$. So $Ғ(д) \in (t_1,t_2,t_3)$- NMFNSR of R_1.

Theorem.4.9 Let Ғ be a hom... of ring R onto R_1. If $д \in (t_1,t_2,t_3)$- NMFL(R)I of R then $Ғ(д) \in (t_1,t_2,t_3)$- NMFL(R)I of R_1.

Proof Let $y_1, y_2 \in R_1$ then there exist $x_1, x_2 \in R$ such that $Ғ(x_1)=y_1$, $Ғ(x_2)=y_2$

If $д \in (t_1,t_2,t_3)$- NMFI of R. Then by theorem $Ғ(д) \in (t_1,t_2,t_3)$- NMFSR of R_1.

Then for all i,

(i) $Ғ((\mu^i_{д'})(y_1y_2)) = \min\{Ғ((\mu^i_{д})(Ғ(x_1)Ғ(x_2)),t_1)\} = \min\{Ғ((\mu^i_{д})(Ғ(x_1x_2)),t_1)\}$

$= \min\{((\mu^i_{д})((x_1x_2)),t_1)\} = (\mu^i_{д'})((x_1x_2)) \geq (\mu^i_{д'})((x_2)) = \min\{((\mu^i_{д})((x_2)),t_1)\}$

$= \min\{((\mu^i_{д})((x_2)),t_1)\} = \min\{Ғ((\mu^i_{д'})(y_2),t_1))\} = Ғ((\mu^i_{д'})(y_2))$

(ii) $Ғ((\nu^i_{д'})(y_1y_2)) = \max\{Ғ((\nu^i_{д})(Ғ(x_1)Ғ(x_2)),t_2)\} = \max\{Ғ((\nu^i_{д})(Ғ(x_1x_2)),t_2)\}$

$= \max\{((\nu^i_{д})((x_1x_2)),t_2)\} = (\nu^i_{д'})((x_1x_2)) \leq (\nu^i_{д'})((x_2)) = \max\{((\nu^i_{д})((x_2)),t_2)\}$

$= \max\{((\nu^i_{д})((x_2)),t_2)\} = \max\{Ғ((\nu^i_{д'})(y_2),t_2))\} = Ғ((\nu^i_{д'})(y_2))$

Similarly, $Ғ((\digamma^i_{д'})(y_1y_2)) \leq Ғ((\digamma^i_{д'})(y_2))$

Hence $Ғ(д) \in (t_1,t_2,t_3)$- NMFLI of R_1. In similar way we can show $Ғ(д) \in (t_1,t_2,t_3)$- NMFRI of R_1.

Theorem.4.10 Let Ғ be a hom... of ring R onto R_1. If $д \in (t_1,t_2,t_3)$- NMFI of R then $Ғ(д) \in (t_1,t_2,t_3)$- NMFI of R_1.

Proof. Follows from the above theorem.

Definition.4.11 Let д be (t_1,t_2,t_3)- NMFS of R with respect to NMFS д. Let $\alpha_i, \beta_i, \gamma_i \in [0,1]$. With $0 \leq \alpha_i + \beta_i + \gamma_i \leq 3$. Then the set $д_{(\alpha,\beta,\gamma)'}$ is called a level set of д, where for any $x \in д_{(\alpha,\beta,\gamma)'}$ the following inequalities hold $\mu^i_{д'}(x) \geq \alpha_i$; $\nu^i_{д'}(x) \leq \beta_i$; $\digamma^i_{д'}(x) \leq \gamma_i$; and $\alpha_i \leq t_1; \beta_i \leq t_2; \gamma_i \leq t_3$.

Definition.4.12 Let д and Ɓ be two (t_1,t_2,t_3)- NMFS in R. Then $\forall$ x, $y \in R$, $д' \circ Ɓ'$ is defined as,

$$(Д' \circ Ɓ')(x) = \begin{cases} \underbrace{sup}_{x=yz} \; min\left(ɥ^i_{Д'}(y), ɥ^i_{Ɓ'}(z)\right) \\ \underbrace{inf}_{x=yz} \; max(И^i_{Д'}(y), И^i_{Ɓ'}(z)) & \text{if } x = yz \\ \underbrace{inf}_{x=yz} \; max(ғ^i_{Д'}(y), ғ^i_{Ɓ'}(z)) \\ (0,1,1) & \text{if } x \neq yz \end{cases}$$

Theorem.4.13 If $Д'$ and $Ɓ'$ is any two (t_1,t_2,t_3)- NMFSR of a ring R. Then $Д' \circ Ɓ'$ is a (t_1,t_2,t_3)- NMFSR of R $\Leftrightarrow Д' \circ Ɓ' = Ɓ' \circ Д'$.

Proof Suppose $Д' \circ Ɓ'$ is a (t_1,t_2,t_3)- NMFSR of R. $\Leftrightarrow$ Each $(Д' \circ Ɓ')_{\alpha,\beta,\gamma}$ are subrings of R, for all $\alpha_i,\beta_i,\gamma_i \in[0,1]$,i={1,2,..n}.Now, $Д'_{\alpha,\beta,\gamma} \circ Ɓ'_{\alpha,\beta,\gamma}$ is a subring of R. Since $Д'$ and $Ɓ'$ are (t_1,t_2,t_3)- NMFSR of R, each $Д'_{\alpha,\beta,\gamma}$ and $Ɓ'_{\alpha,\beta,\gamma}$ are subrings of R. $\Leftrightarrow Д'_{\alpha,\beta,\gamma} \circ Ɓ'_{\alpha,\beta,\gamma} = Ɓ'_{(\alpha,\beta,\gamma)} \circ Д'_{(\alpha,\beta,\gamma)}$. $\because H_1$ and H_2 are two subrings of R then $H_1 H_2$ is a subring of R. $\Leftrightarrow H_1 H_2 = H_2 H_1 \Leftrightarrow (Д' \circ Ɓ')_{\alpha,\beta,\gamma} = (Ɓ' \circ Д')_{\alpha,\beta,\gamma}$ for all $\alpha_i,\beta_i,\gamma_i \in[0,1]$,i={1,2,..n}.

$\Leftrightarrow Д' \circ Ɓ' = Ɓ' \circ Д'$.

Theorem.4.14 If $Д'$ is any (t_1,t_2,t_3)- NMFSR of a ring R then $Д' \circ Д' = Д'$

Proof If for all $\alpha_i,\beta_i,\gamma_i \in[0,1]$, i= {1, 2...n}. $Д'_{\alpha,\beta,\gamma} \circ Д'_{\alpha,\beta,\gamma} = Д'_{\alpha,\beta,\gamma}$ .Since $Д'$ is (t_1,t_2,t_3)- NMFSR of R, each $Д'_{\alpha,\beta,\gamma}$ is a subring of R.

$\Rightarrow (Д' \circ Д')_{\alpha,\beta,\gamma} = Д'$ $\because$H is a subring of R $\Rightarrow$ H H = H

$\Rightarrow Д' \circ Д' = Д'$.

Theorem.4.15 Let $Д'$, $Ɓ'$ be two (t_1,t_2,t_3)- NMFS in R. If $Д'$ and $Ɓ'$ be (t_1,t_2,t_3)- NMFI of R then $Д' \circ Ɓ' \subset Д' \cap Ɓ'$.

Proof Let $x \in R$. Suppose $Д' \circ Ɓ' = (0,1,1)$ then there is nothing to prove.

Suppose $Д' \circ Ɓ' \neq (0,1,1)$

Then,

$$(Д' \circ Ɓ')(x) = \begin{cases} \underbrace{sup}_{x=yz} \; \min\left(ɥ^i_{Д'}(y), ɥ^i_{Ɓ'}(z)\right) \\ \underbrace{inf}_{x=yz} \; \max(И^i_{Д'}(y), И^i_{Ɓ'}(z)) & \text{if } x= yz \\ \underbrace{inf}_{x=yz} \; \max(ғ^i_{Д'}(y), ғ^i_{Ɓ'}(z)) \end{cases}$$

Since $Д'$, $Ɓ'$ are(t_1,t_2,t_3)- NMFI of R then

(i) $ɥ^i_{Д'}(y) \leq ɥ^i_{Д'}(yz) = ɥ^i_{Д'}(x)$ (ii) $ɥ^i_{Ɓ'}(z) \leq ɥ^i_{Ɓ'}(yz) = ɥ^i_{Ɓ'}(x)$

$И^i_{Д'}(y) \geq И^i_{Д'}(yz) = И_{Д'}(x)$ $И^i_{Ɓ'}(z) \geq И^i_{Ɓ'}(yz) = И^i_{Ɓ'}(x)$

$ғ^i_{Д'}(y) \geq ғ^i_{Д'}(yz) = ғ^i_{Д'}(x)$ $ғ^i_{Ɓ'}(z) \geq ғ^i_{Ɓ'}(yz) = ғ^i_{Ɓ'}(x)$

Thus,

$$ɥ^i_{(Д' \circ Ɓ')}(x) = \underbrace{sup}_{x=yz} \{min(ɥ^i_{Д'}(y), ɥ^i_{Ɓ'}(z)\} \leq min(ɥ^i_{Д'}(x), ɥ^i_{Ɓ'}(x)) = ɥ^i_{Д' \cap Ɓ'}(x)$$

$$И^i_{(Д' \circ Ɓ')}(x) = \underbrace{inf}_{x=yz} \{max(И^i_{Д}(y), И^i_{Ɓ}(z)\} \geq T_c(И^i_{Д}(y), И^i_{Ɓ}(z)) = И^i_{Д' \cap Ɓ'}(x)$$

Similarly, $ғ^i_{(Д' \circ Ɓ')}(x) \geq ғ^i_{Д' \cap Ɓ'}(x)$. Hence $Д \circ Ɓ \subset Д' \cap Ɓ'$.

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Volume 13, No. 3, 2022, p. 3269 - 3279
https://publishoa.com
ISSN: 1309-3452

Theorem.4.16 If д is (t_1, t_2, t_3)- NMFS of R with respect to NMFS д . Then д is(t_1, t_2, t_3)- NMFSR of R iff $д'_{\alpha,\beta,\gamma}$ is a subring of R where for all $\alpha_i, \beta_i, \gamma_i \in[0,1]$ and $\alpha_i \leq t_1; \beta_i \leq t_2; \gamma_i \leq t_3$.

Proof Assume that д is(t_1, t_2, t_3)- NMFSR of R.

Let x, y ∈ $д'_{\alpha,\beta,\gamma}$.Then for all i , $\mu^i_{д'}(x) \geq \alpha_i$; $И^i_{д'}(x) \leq \beta_i$; $\digamma^i_{д'}(x) \leq \gamma_i$;

$\mu^i_{д'}(x-y), \mu^i_{д'}(xy) \geq \min\{\mu^i_{д'}(x), \mu^i_{д'}(y)\} \geq \min\{\alpha_i, \alpha_i\} \geq \alpha_i$

$И^i_{д'}(x-y), И^i_{д'}(xy) \leq \max\{И^i_{д'}(x), И^i_{д'}(y)\} \leq \max\{\beta_i, \beta_i\} \leq \beta_i$

Similarly, $\digamma^i_{д'}(x-y), \digamma^i_{д'}(xy) \leq \gamma_i$.

Thus $\mu_{д'}(x-y), \mu_{д'}(xy) \geq \alpha$, $И_{д'}(x-y), И_{д'}(xy) \leq \beta$, $\digamma_{д'}(x-y), \digamma_{д'}(xy) \leq \gamma$.

Which implies x- y, x y ∈ $д'_{\alpha,\beta,\gamma}$.Hence $д'_{\alpha,\beta,\gamma}$is a subring of R.

Conversely, let д is(t_1, t_2, t_3)- NMFS of R. Each $д_{(\alpha,\beta,\gamma)'}$ is subring of R.

Let x, y ∈ R, for all i, $\alpha_i = \min\{\mu^i_{д'}(x), \mu^i_{д'}(y)\}$

$\beta_i = \max\{И^i_{д'}(x), И^i_{д'}(y)\}$, $\gamma_i = \max\{\digamma^i_{д'}(x), \digamma^i_{д'}(y)\}$. Then for all i , $\mu^i_{д'}(x) \geq \alpha_i$; $И^i_{д'}(x) \leq \beta_i$; $\digamma^i_{д'}(x) \leq \gamma_i$

Which implies $\mu_{д'}(x) \geq \alpha$; $И_{д'}(x) \leq \beta$; $\digamma_{д'}(x) \leq \gamma$; .

Thus x, y ∈ $д'_{\alpha,\beta,\gamma}$since $д'_{\alpha,\beta,\gamma}$ is a subring of R such that $\alpha_i \leq t_1; \beta_i \leq t_2; \gamma_i \leq t_3$.

So, $\mu^i_{д'}(x-y), \mu^i_{д'}(xy) \geq \alpha_i = \min\{\mu^i_{д'}(x), \mu^i_{д'}(y)\}$

$И^i_{д'}(x-y), И^i_{д'}(xy) \leq \beta_i = \max\{И^i_{д'}(x), И^i_{д'}(y)\}$; $\digamma^i_{д'}(x-y), \digamma^i_{д'}(xy) \leq \gamma_i = \max\{\digamma^i_{д'}(x), \digamma^i_{д'}(y)\}$.

$\mu_{д'}(x-y), \mu_{д'}(xy) \geq \min\{\mu_{д'}(x), \mu_{д'}(y)\}$, $И_{д'}(x-y), И_{д'}(xy) \leq \max\{И_{д'}(x), И_{д'}(y)\}$, $\digamma_{д'}(x-y), \digamma_{д'}(xy) \leq \max\{\digamma_{д'}(x), \digamma_{д'}(y)\}$.Hence д is$(t_1, t_2, t_3)$- NMFSR of R.

Example.4.17 Consider the ring $(Z_5, +, \cdot)$. Define NMFS д of Z_5 by д= {(<0(0.9,0.8) (0.3,0.5) (0.2,0.4)>, <1(0.8,0.4) (0.4,0.6) (0.2,0.5)>, <2(0.9,0.5) (0.3,0.5) (0.3,0.6)>, <3(0.5,0.3) (0.4,0.6) (0.3,0.6)><4(0.7,0.5) (0.4,0.6) (0.4,0.7)>} Suppose we take $t_1, t_2, t_3 = 0.2, 0.5, 0.7$

Then, α_i=0.2,0.1; β_i=0.8,0.9; γ_i=0.7,0.8 for i= 1,2.then $\mu_д(x)$= (0.3,0.3); $\digamma_д(x) = (0.7, 0.7) \forall$ x $\in Z_5$.$И_{д'}(x) = \begin{cases} (0.5,0.5) & if\ x = 0,2 \\ (0.5,0.6) & if\ x = 1,3,4 \end{cases}$ It is easy to verify that $д'_{\alpha,\beta,\gamma}$ is a subring of R but д not a NMFSR of R as of $\mu_д$(4-1) =0.5,0.3; $\min\{\mu_д(4), \mu_д(1)\}$=(0.7,0.4); $\digamma_{д'}$(3-1) $\ngeq \min\{\mu_д(4), \mu_д(1)\}$. Hence д not a NMFSR of R

Theorem.4.18 If $\digamma: R \to R_1$is a surjective ring hom.. The homomorphic image of a level set which is a subring of (t_1, t_2, t_3)-NMFSR of R is again a level set which is a subring of (t_1, t_2, t_3)-NMFSR of R_1.

Proof Let д be (t_1, t_2, t_3)-NMFSR of R. Let $x_1, x_2 \in R$. Then $\digamma$(д) is a (t_1, t_2, t_3)-NMFSR of R_1 and $\digamma(x_1) = y_1$, $\digamma(x_2) = y_2$.

Let $д'_{\alpha,\beta,\gamma}$ be a level set of д .Then clearly $д'_{\alpha,\beta,\gamma}$ is a subring of R. suppose $x_1, x_2 \in д'_{\alpha,\beta,\gamma}$.

Then, $\digamma\left((\mu^i_{д'})(\digamma(x_1) - \digamma(x_2))\right) = \digamma\left((\mu^i_д)\digamma(x_1 - x_2)\right)$

$$= \sup_{x_1 - x_2 \in \digamma^{-1}(y_1 - y_2)} \left(\mu^i_{д'}(x_1 - x_2)\right)$$

$$\geq \sup_{x \in \digamma^{-1}(y)} \{\min(\mu^i_{д'}(x_1), \mu^i_{д'}(x_2)\}$$

$$= \min\left(\sup_{x_1 \in \digamma^{-1}(y_1)} (\mu^i_{д'}(x_1)), \sup_{x_2 \in \digamma^{-1}(y_2)} (\mu^i_д(x_2))\right)$$

$$= \min\{\alpha_i, \alpha_i\}$$

$$= \alpha_i$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3269 - 3279
https://publishoa.com
ISSN: 1309-3452

Similarly, $Ƒ\left(\left(\mu^{i}_{Д'}\right)\left(Ƒ(x_1)-Ƒ(x_2)\right)\right) \geq \alpha_i$

$$Ƒ\left(\left(И^{i}_{Д'}\right)\left(Ƒ(x_1)-Ƒ(x_2)\right)\right) = Ƒ\left(\left(И^{i}_{Д}\right)Ƒ(x_1-x_2)\right)$$
$$= \inf_{x_1-x_2 \in Ƒ^{-1}(y_1-y_2)}\left(И^{i}_{Д'}(x_1-x_2)\right)$$
$$\geq \inf_{x \in Ƒ^{-1}(y)}\left\{\max\left(И^{i}_{Д'}(x_1),\, И^{i}_{Д'}(x_2)\right)\right\}$$

$$= \max\left(\inf_{x_1 \in Ƒ^{-1}(y_1)}\left(И^{i}_{Д'}(x_1)\right),\, \inf_{x_2 \in Ƒ^{-1}(y_2)}\left(И^{i}_{Д}(x_2)\right)\right)$$
$$= \max\{\beta_i, \beta_i\}$$
$$= \beta_i$$

Similarly, $Ƒ\left(\left(И^{i}_{Д'}\right)\left(Ƒ(x_1)-Ƒ(x_2)\right)\right) \leq \beta_i$

In the same way we can show that $Ƒ\left(\left(ғ^{i}_{Д'}\right)\left(Ƒ(x_1)-Ƒ(x_2)\right)\right) \leq \gamma_i Ƒ\left(\left(ғ^{i}_{Д'}\right)\left(Ƒ(x_1)-Ƒ(x_2)\right)\right) \leq \gamma_i$

$\therefore Ƒ(x_1)-Ƒ(x_2),\, Ƒ(x_1)Ƒ(x_2) \in Ƒ\left(Д'_{\alpha,\beta,\gamma}\right)$

Hence $Ƒ\left(Д'_{\alpha,\beta,\gamma}\right)$ is a subring in R_1.

Theorem.4.19 If $Ƒ: R \to R_1$ is a surjective ring hom... The homomorphic pre image of a level set which is a subring of (t_1, t_2, t_3)-NMFSR of R_1 is again a level set which is a subring of (t_1, t_2, t_3)-NMFSR of R.

Proof Let $Ɓ \in (t_1, t_2, t_3)$- NMFSR of R_1 .Then clearly $Ƒ^{-1}(Ɓ)$ is (t_1, t_2, t_3)- NMFSR of R. Let $Ƒ(x_1)$, $Ƒ(x_2) \in Д'_{\alpha,\beta,\gamma}$. Then for all i, $\mu^{i}_{Ɓ'}\left(Ƒ(x_1)\right) \geq \alpha_i$, $И^{i}_{Ɓ'}\left(Ƒ(x_1)\right) \leq \beta_i$, $ғ^{i}_{Ɓ'}\left(Ƒ(x_1)\right) \leq \gamma_i \Rightarrow \mu_{Ɓ'}\left(Ƒ(x_1)\right) \geq \alpha$, $И_{Ɓ'}\left(Ƒ(x_1)\right) \leq \beta$, $ғ_{Ɓ'}\left(Ƒ(x_1)\right) \leq \gamma$

$Ƒ^{-1}\left(\left(\mu^{i}_{Ɓ'}\right)(x_1-x_2)\right) = \mu^{i}_{Ɓ'}\left(Ƒ(x_1-x_2)\right) = \mu^{i}_{Ɓ'}\left(Ƒ(x_1)-(x_2)\right) \geq \min\{\mu^{i}_{Ɓ'}\left(Ƒ(x_1)\right), \mu^{i}_{Ɓ'}\left(Ƒ(x_2)\right)\} = \alpha_i$

Similarly, $Ƒ^{-1}\left(\left(\mu^{i}_{Ɓ'}\right)(x_1x_2)\right) \geq \alpha_i$

$Ƒ^{-1}\left(\left(И^{i}_{Ɓ'}\right)(x_1-x_2)\right) = И^{i}_{Ɓ'}\left(Ƒ(x_1-x_2)\right) = И^{i}_{Ɓ'}\left(Ƒ(x_1)-Ƒ(x_2)\right) \leq \max\{И^{i}_{Ɓ'}\left(Ƒ(x_1)\right), И^{i}_{Ɓ'}\left(Ƒ(x_2)\right)\} = \beta_i$

Similarly, $Ƒ^{-1}\left(\left(И^{i}_{Ɓ'}\right)(x_1x_2)\right) \leq \beta_i$.

In similar way we can show that, $Ƒ^{-1}\left(\left(ғ^{i}_{Ɓ'}\right)(x_1-x_2)\right), Ƒ^{-1}\left(\left(ғ^{i}_{Ɓ'}\right)(x_1x_2)\right) \leq \gamma_i$

Hence $Ƒ^{-1}\left(Ɓ'_{\alpha,\beta,\gamma}\right)$ is a subring in R.

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