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AN M/G/1 RENEGING AND BALKING QUEUE OFFERING A VARIETY OF SERVICES AND GETAWAYS

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ABSTRACT

We take a look at an M/G/1 queueing machine for important and optionally available excursion. The arrival follows Poisson process. In this version the server offers 3 kinds of carrier. One is important, the alternative are optionally available carrier. Also, after ES or OS, when there aren't any customers with inside the machine, the server takes - k levels of EV. After the EV, if there are not any consumer with inside the queue, the server can also additionally both wait idle for consumer or can also additionally take k+1th phase of optionally available excursion. Next we bear in mind balking to arise whilst the server is busy or excursion durations and reneging to arise whilst the server is on excursion durations. Both carrier and excursion time primarily based on GD. For this version, the SVT was applied to get the PGF of range of customer with inside the queue. Extensive numerical analyses are achieved to reveal the impact of machine parameters on overall performance measurements.

Mathematics Subject Classification: 60K25, 90B22, 68M20

Key words: M/G/1 line, balking, reneging, suppleymentary variable technique (SVT), general distribution(GD), essential service (ES), essential vacation (EV), optional services (OS), optional vacation (OV), probability generating function (PGF), laplace stieljes transform (LST).

1. Introduction

In the M/G/1 queueing system, the excursion idea has been studied by many authors, including Medhi[8], Kalyanaraman[12, 17], Madan[7], Sathya[16] and Manoharan[13, 14] et al studied the queueing model with optionally available 2nd service. The M/G/1 queue with different leave policies was studied by Choudhury[9], Doshi[6], Godhandaraman[15] and Pavai Madheswari[19]. Cooper [4], Gross and Harris [5] introduced basic queueing theory concept. Cox [1] analyzed non –markovian models and converted them to markovian models by introducing supplementary variable. Kumar [10], Jeeva [11], Maragathasundari[18,20], Subba rao[3] and Haight[2] have analyzed different models of balking and reneging queues. Here this version we remember M/G/1 Reneging and balking queue with 2 types of OS and OV. The relaxation of the paper is prepared as follows. The mathematical description and evaluation of this version is given in phase 2. In phase 3, we derive a few operational homes of the version analyzed in phase 2. Section 4 offers with a few unique instances and phase 5 gives a few numerical outcomes concerning the version analyzed at some stage in this paper. The remaining phase offers a conclusion.

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2. The Mathematical Model and Analysis

In a solitary server queueing framework, the patron arrival follows a poisson process with state dependent parameter λ. The ready room is of of unending potential and provider field is FIFO. The server gives the 3 kinds of administrations, the primary provider is vital and every other provider is optionally available. Let $ß_i^*(s) = E(e^{-sß_i})$ is (LST), first, second and third moments b_{i1}, b_{i2} and b_{i3} respectively. After the ES, the patron may also choose a varieties of optionally available provider with chance $(ÞÞ_1$ or $ÞÞ_2), (Þ_1 + Þ_2) = 1$ or he may also go away the machine with chance $(1 - Þ)$. Let $ß_i(\varkappa), (i = 1,2,3)$ be the distribution and density function of ES and OS. The service time of the ES and OS follows GD.

Balking: The task is with inside the machine however doesn't be part of the execution queue. Let us count on that b is chance of becoming a member of the machine or balks with chance (1-b).

Reneging: The arriving patron with inside the queue and after waiting the some of times leaves the queue without being served. It is assumed to follows ED with parameter $\curlyvee$ and a patron can renege at some stage of time$(t, t + dt]$.

Whenever the system will become empty, the server takes k phases of EV. The EV follows GD with distribution function $V_i(x)$ whose LST is$V_i^*(s), 1 = 1,2, \dots, k$. After the EV, , if there aren't any consumer with inside the queue, the server can also additionally both wait idle for consumer with opportunity Θ_0 or can also additionally take any other one holiday, we name this segment as k+1th phase, which follows GD with distribution function $V_{k+1}(x)$ whose LST is $V_{k+1}^*(s)$ with probability Θ_1. i.e if there are consumer with inside the queue, the server begins administration for the client in the top of the line, in any other case the server waits best for brand new arrival or take simplest single segment holiday. The arriving patron input right into a queue of endless potential, if the carrier isn't on the spot because of server is occupied or go to holiday.

Now altered get-away length is $\mathcal{V} = \begin{cases} \sum_{i=1}^{K} \mathcal{V}_i \\ \sum_{i=1}^{K+1} \mathcal{V}_i \text{ with probability } \Theta_1 \end{cases}$

Assume that $ß_i(0) = 0, ß_i(\infty) = 1, (i = 1 \text{ to } 3), \mathcal{V}_i(0) = 0, \mathcal{V}_i(\infty) = 1, i = 1,2,.,K + 1,$ are continuous at $x = 0$. the elapsed EST (OST) of the purchaser with carrier time ŧ is meant through $ß_i(ŧ), i = 1,2,3$. Elapsed excursion time ŧ is meant through of $\mathcal{V}_i(ŧ), i = 1,2, \dots, K$ and elapsed excursion time of elective section is $\mathcal{V}_{K+1}(ŧ)$.

Let $\mathfrak{Y}(ŧ)$ be the conditions of the server at time ŧ and is described as

$$\mathfrak{Y}(ŧ) = \begin{cases} 0, \text{ if the server is idle} \\ 1, \text{if the server is occupied providing the ES} \\ 2, \text{if the server is occupied providing kind 1 OS} \\ 3, \text{if the server is occupied providing kind 2 OS} \\ 4, \text{if the server is on } 1^{st} \text{ normal excursion} \\ 5, \text{if the server is on } 2^{nd} \text{ discretionary get} - \text{away} \end{cases}$$

Let the rv's $\aleph(ŧ)$ is defined as,

$$\aleph(ŧ) = \begin{cases} 0, \text{if } \mathfrak{Y}(ŧ) = 0 \\ ß_i(ŧ), \text{if } \mathfrak{Y}(ŧ) = 1 \text{ to } 3, i = 1 \text{ to } 3 \\ \mathcal{V}_i(ŧ), \text{if } \mathfrak{Y}(ŧ) = 4 \\ \mathcal{V}_{k+1}(ŧ), \text{if } \mathfrak{Y}(ŧ) = 5 \end{cases}$$

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And Let the rv's $N(t)$ is the quantity of clients in the queue at t, we characterize the limiting probabilities as follows

$$Q(t) = \mathcal{Pr}\{N(t) = 0, \aleph(t) = 0\}$$

$$P_{i,\mathfrak{N}}(x)dx = \mathcal{Pr}\{N(t) = \mathfrak{N}, \aleph(t) = \beta_i(t), x < \beta_i(t) \le x + dx\}, \mathfrak{N} \ge 0, x > 0, i = 1,2,3$$

$$Q_{i,\mathfrak{N}}(x)dx = \mathcal{Pr}\{N(t) = \mathfrak{N}, \aleph(t) = V_i(t), x < V_i(t) \le x + dx\}, \mathfrak{N} \ge 0, x > 0, i = 1,..,K$$

$$R_{K+1,\mathfrak{N}}(x)dx = \mathcal{Pr}\{N(t) = \mathfrak{N}, \aleph(t) = V_{K+1}(t), x < V_{k+1}(t) \le x + dx\}, \mathfrak{N} \ge 0, x > 0$$

Where $\{N(t), \mathfrak{Y}(t), t \ge 0\}$ is a bivariate markov process with state space. In steady state, the equivalent probabilities are $Q = \lim_{t\to\infty} Q(t), P_{1,\mathfrak{N}}(x) = \lim_{t\to\infty} P_{1,\mathfrak{N}}(t,x)$, $P_{2,\mathfrak{N}}(x) = \lim_{t\to\infty} P_{2,\mathfrak{N}}(t,x), P_{3,\mathfrak{N}}(x) = \lim_{t\to\infty} P_{3,\mathfrak{N}}(t,x), Q_{i,\mathfrak{N}}(x) = \lim_{t\to\infty} Q_{i,\mathfrak{N}}(t,x)$ and $R_{K+1,\mathfrak{N}}(x) = \lim_{t\to\infty} R_{K+1,\mathfrak{N}}(t,x)$. Let $\mu_i(x), i = 1,2,3$ be the conditional probability of finalization of the ES and OS in the course of the time period $(x, x + dx]$ given that the elapsed carrier times of three kinds of service is x, so that $\mu_i(x) = \frac{d\beta_i(x)}{1-\beta_i(x)}$. The similar quantity for $V_i(x)$ is $V_i(x) = \frac{dV_i(x)}{1-V_i(x)}, i = 1,2,\dots,K+1$.

The version is administered by the specified differential difference Eqns:

For $x > 0$

$$\frac{d}{dx}P_{1,0}(x) + (\lambda + \mu_1(x))P_{1,0}(x) = (1-b)\lambda P_{1,0}(x) \quad (1)$$

$$\frac{d}{dx}P_{1,\mathfrak{N}}(x) + (\lambda + \mu_1(x))P_{1,\mathfrak{N}}(x) = (1-b)\lambda P_{1,n}(x) + b\lambda P_{1,\mathfrak{N}-1}(x) \quad (2)$$

$$\frac{d}{dx}P_{2,0}(x) + (\lambda + \mu_2(x))P_{2,0}(x) = (1-b)\lambda P_{2,0}(x) \quad (3)$$

$$\frac{d}{dx}P_{2,\mathfrak{N}}(x) + (\lambda + \mu_2(x))P_{2,\mathfrak{N}}(x) = (1-b)\lambda P_{2,n}(x) + b\lambda P_{2,\mathfrak{N}-1}(x) \quad (4)$$

$$\frac{d}{dx}P_{3,0}(x) + (\lambda + \mu_3(x))P_{3,0}(x) = (1-b)\lambda P_{3,0}(x) \quad (5)$$

$$\frac{d}{dx}P_{3,\mathfrak{N}}(x) + (\lambda + \mu_3(x))P_{3,\mathfrak{N}}(x) = (1-b)\lambda P_{3,n}(x) + b\lambda P_{3,\mathfrak{N}-1}(x) \quad (6)$$

$$\frac{d}{dx}Q_{i,0}(x) + (\lambda + \eta_i(x) + \gamma)Q_{i,0}(x) = (1-b)\lambda Q_{i,0}(x) + \gamma Q_{i,0}(x) \quad (7)$$

$$\frac{d}{dx}Q_{i,\mathfrak{N}}(x) + (\lambda + \eta_i(x) + \gamma)Q_{i,\mathfrak{N}}(x) = (1-b)\lambda Q_{i,n}(x) + b\lambda Q_{i,\mathfrak{N}-1}(x) + \gamma Q_{i,0}(x) \quad (8)$$

$$\frac{d}{dx}R_{k+1,0}(x) + (\lambda + \gamma_{k+1}(x) + \gamma)R_{K+1,0}(x) = (1-b)\lambda R_{K+1,0}(x) + \gamma R_{K+1,0} \quad (9)$$

$$\frac{d}{dx}R_{k+1,\mathfrak{N}}(x) + (\lambda + \gamma_{k+1}(x) + \gamma)R_{K+1,\mathfrak{N}}(x) = (1-b)\lambda R_{K+1,\mathfrak{N}}(x) + b\lambda R_{K+1,\mathfrak{N}-1}(x) + \gamma R_{K+1,\mathfrak{N}+1}(x) \quad (10)$$

$$\lambda Q = \Theta_0 \int_0^\infty \eta_K(x)Q_{K,0}(x)dx + \int_0^\infty \gamma_{K+1}(x)R_{K+1,0}(x)dx \quad (11)$$

The boundary conditions at $x = 0$ are

$$P_{1,0}(0) = \lambda Q + \Theta_0 \int_0^\infty \eta_K(x)Q_{K,1}(x)dx + \int_0^\infty \gamma_{K+1}(x)R_{K+1,1}(x)dx + (1-Þ)\int_0^\infty P_{1,1}(x)\mu_1(x)dx + \int_0^\infty P_{2,1}(x)\mu_2(x)dx + \int_0^\infty P_{3,1}(x)\mu_3(x)dx \quad (12)$$

$$P_{1,\mathfrak{N}}(0) = \Theta_0 \int_0^\infty \eta_K(x)Q_{K,\mathfrak{N}+1}(x)dx + \int_0^\infty \gamma_{K+1}(x)R_{K+1,\mathfrak{N}+1}(x)dx + (1-Þ)\int_0^\infty P_{1,\mathfrak{N}+1}(x)\mu_1(x)dx + Þ_2\int_0^\infty P_{2,\mathfrak{N}+1}(x)\mu_2(x)dx + \int_0^\infty P_{3,\mathfrak{N}+1}(x)\mu_3(x)dx \quad (13)$$

$$P_{2,\mathfrak{N}}(0) = ÞÞ_1 \int_0^\infty P_{1,\mathfrak{N}}(x)\mu_1(x)dx \quad (14)$$

$$P_{3,\mathfrak{N}}(0) = ÞÞ_2 \int_0^\infty P_{1,\mathfrak{N}}(x)\mu_2(x)dx \quad (15)$$

$$Q_{1,0}(0) = (1-Þ)\int_0^\infty P_{1,0}(x)\mu_1(x)dx + \int_0^\infty P_{2,0}(x)\mu_2(x)dx + \int_0^\infty P_{3,0}(x)\mu_3(x)dx \quad (16)$$

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$$Q_{1,\mathfrak{N}}(0) = 0, \mathfrak{N} \geq 1 \tag{17}$$

$$Q_{i,\mathfrak{N}}(0) = \int_0^\infty Q_{i-1,\mathfrak{N}}(x)\eta_{i-1}(x)dx, \ \ i = 1,2,\dots,K \tag{18}$$

$$R_{K+1,\mathfrak{N}}(0) = \theta_1 \int_0^\infty Q_{K,\mathfrak{N}}(x)\eta_K(x)dx, \ \ \mathfrak{N} = 0,1,2,\dots \tag{19}$$

The normalizing condition is

$$Q + P_1(1) + P_2(1) + P_3(1) + \sum_{i=1}^{K} Q_i(1) + R_{K+1}(1) = 1 \tag{20}$$

From Eqn (1), we have

$$P_{1,0}(x) = P_{1,0}(0)(1 - B_1(x))e^{-b\lambda x} \tag{21}$$

Similarly from Eqn (3), (5), (7) and (9), we get

$$P_{2,0}(x) = P_{2,0}(0)(1 - B_2(x))e^{-b\lambda x} \tag{22}$$

$$P_{3,0}(x) = P_{3,0}(0)(1 - B_3(x))e^{-b\lambda x} \tag{23}$$

$$Q_{K,0}(x) = Q_{K,0}(0)(1 - V_K(x))e^{-b\lambda x} \tag{24}$$

$$R_{K+1,0}(x) = R_{K+1,0}(0)(1 - V_{K+1}(x))e^{-b\lambda x} \tag{25}$$

Multiply Eqn (2) by $z^{\mathfrak{N}}$, take $\sum_{\mathfrak{N}=1}^{\infty}$ and sum the Eqn (1), then

$$P_1(x,z) = P_1(0,z)(1 - B_1(x))e^{-Tx} \tag{26}$$

Similarly from Eqn (4), (6), (8) and (10), we get

$$P_2(x,z) = P_2(0,z)(1 - B_2(x))e^{-Tx} \tag{27}$$

$$P_3(x,z) = P_3(0,z)(1 - B_3(x))e^{-Tx} \tag{28}$$

$$Q_K(x,z) = Q_K(0,z)(1 - V_K(x))e^{-T_1 x} \tag{29}$$

$$R_{K+1}(x,z) = R_{K+1}(0,z)(1 - V_{K+1}(x))e^{-T_1 x} \tag{30}$$

Where $T = b\lambda(1-z)$ and $T_1 = T + \gamma - \frac{\gamma}{z}$

Multiply Eqn (13) by $z^{\mathfrak{N}}$, take $\sum_{\mathfrak{N}=1}^{\infty}$ and sum the Eqn (12) and $\times^{ly}$ by z, we obtain

$$z P_1(0,z) = z\lambda Q + \theta_0\left[\int_0^\infty Q_K(x,z)\eta_K(x)dx - \int_0^\infty Q_{K,0}(x)\eta_K(x)dx\right] + \left[\int_0^\infty R_{K+1}(x,z)\gamma_{K+1}(x)dx - \int_0^\infty R_{K+1,0}(x)\gamma_{K+1}(x)dx\right]$$
$$+(1-P)\left[\int_0^\infty P_1(x,z)\mu_1(x)dx - \int_0^\infty P_{1,0}(x)\mu_1(x)dx\right] +$$
$$\left[\int_0^\infty P_2(x,z)\mu_2(x)dx - \int_0^\infty P_{2,0}(x)\mu_2(x)dx\right] + \left[\int_0^\infty P_3(x,z)\mu_3(x)dx - \int_0^\infty P_{3,0}(x)\mu_3(x)dx\right] \tag{31}$$

From Eqn (21), we have

$$\int_0^\infty P_{1,0}(x)\mu_1(x)dx = P_{1,0}(0)B_1^*(b\lambda) \tag{32}$$

Similarly, from Eqn (22), (23), (24) and (25), we have

$$\int_0^\infty P_{2,0}(x)\mu_2(x)dx = P_{2,0}(0)B_2^*(b\lambda) \tag{33}$$

$$\int_0^\infty P_{3,0}(x)\mu_3(x)dx = P_{3,0}(0)B_3^*(b\lambda) \tag{34}$$

$$\int_0^\infty Q_{i,0}(x)\eta_i(x)dx = Q_{i,0}(0)V_i^*(b\lambda) \tag{35}$$

$$\int_0^\infty R_{K+1,0}(x)\gamma_{K+1}(x)dx = R_{K+1,0}(0)V_{K+1}^*(b\lambda) \tag{36}$$

From Eqn (26), we have

$$\int_0^\infty P_1(x,z)\mu_1(x)dx = P_1(0,z)B_1^*(T) \tag{37}$$

Similarly from Eqn (27), (28), (29) and (30), we have

$$\int_0^\infty P_2(x,z)\mu_2(x)dx = P_2(0,z)B_2^*(T) \tag{38}$$

$$\int_0^\infty P_3(x,z)\mu_3(x)dx = P_3(0,z)B_3^*(T) \tag{39}$$

$$\int_0^\infty Q_i(x,z)\eta_i(x)dx = Q_i(0,z)V_i^*(T_1), i = 1,2,\dots,K \tag{40}$$

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$\int_0^\infty \mathrm{R}_{K+1}(x,z)\gamma_{K+1}(x)dx = \mathrm{R}_{K+1}(0,z)\mathcal{V}_{K+1}^*(\mathrm{T}_1)$ (41)

Using the equations (32) to (41) in (31), we obtain the Eqn

$z\mathrm{P}_1(0,z) = z\lambda Q + \Theta_0\left[\int_0^\infty \mathrm{Q}_i(0,z)\mathcal{V}_i^*(\mathrm{T}_1) - \mathrm{Q}_{i,0}(0)\mathcal{V}_i^*(b\lambda)\right] +$
$[\mathrm{R}_{K+1}(0,z)\mathcal{V}_{K+1}^*(\mathrm{T}_1)[-\mathrm{R}_{K+1,o}(0)\mathcal{V}_{K+1}^*(b\lambda)] + (1-\mathrm{Þ})[\mathrm{P}_1(0,z)\mathrm{ß}_1^*(\mathrm{T}) -$ (42)
$\mathrm{P}_{1,0}(0)\mathrm{ß}_1^*(b\lambda)] + [\mathrm{P}_2(0,z)\mathrm{ß}_2^*(\mathrm{T}) - \mathrm{P}_{2,0}(0)\mathrm{ß}_2^*(b\lambda)] + [\mathrm{P}_3(0,z)\mathrm{ß}_3^*(\mathrm{T}) - \mathrm{P}_{3,0}(0)\mathrm{ß}_3^*(b\lambda)]$

Multiply Eqn (14) by $z^{\mathfrak{N}}$, take $\sum_{\mathfrak{N}=0}^{\infty}$, then

$\mathrm{P}_2(0,z) = \mathrm{Þ}\mathrm{P}_1\mathrm{P}_1(0,z)\mathrm{ß}_1^*(\mathrm{T})$ (43)

By taking $\mathfrak{N}=0$ in Eqn (14), we get

$\mathrm{P}_{2,0}(0) = \mathrm{Þ}\mathrm{P}_1\mathrm{P}_{1,0}(0)\mathrm{ß}_1^*(b\lambda)$ (44) Multiply Eqn (15) by $z^{\mathfrak{N}}$, take $\sum_{\mathfrak{N}=1}^{\infty}$, we have

$\mathrm{P}_3(0,z) = \mathrm{Þ}\mathrm{P}_2\mathrm{P}_1(0,z)\mathrm{ß}_1^*(\mathrm{T})$ (45)

By taking $\mathfrak{N}=0$ in Eqn (15), we get

$\mathrm{P}_{3,0}(0) = \mathrm{Þ}\mathrm{P}_2\mathrm{P}_{1,0}(0)\mathrm{ß}_1^*(b\lambda)$ (46)

Multiply Eqn (17) by $z^{\mathfrak{N}}$, take $\sum_{\mathfrak{N}=1}^{\infty}$ and sum the Eqn (16), we obtain

$\mathrm{Q}_1(0,z) = [(1-\mathrm{Þ}) + \mathrm{Þ}\mathrm{P}_1\mathrm{ß}_2^*(b\lambda) + \mathrm{Þ}\mathrm{P}_2\mathrm{ß}_3^*(b\lambda)]\mathrm{P}_{1,0}(0)\mathrm{ß}_1^*(b\lambda)$ (47)

By taking $z=1$ in Eqn (47), we get

$\mathrm{Q}_1(0,1) = [(1-\mathrm{Þ}) + \mathrm{Þ}\mathrm{P}_1\mathrm{ß}_2^*(b\lambda) + \mathrm{Þ}\mathrm{P}_2\mathrm{ß}_3^*(b\lambda)]\mathrm{P}_{1,0}(0)\mathrm{ß}_1^*(b\lambda)$ (48)

Multiply Eqn (18) by $z^{\mathfrak{N}}$, summing from $\mathfrak{N}=0$ to ∞, we get

$\mathrm{Q}_i(0,z) = [(1-\mathrm{Þ}) + \mathrm{Þ}\mathrm{P}_1\mathrm{ß}_2^*(b\lambda) + \mathrm{Þ}\mathrm{P}_2\mathrm{ß}_3^*(b\lambda)]\mathrm{P}_{1,0}(0)\mathrm{ß}_1^*(b\lambda)\prod_{l=1}^{i-1}\mathcal{V}_l^*(\mathrm{T}_1), i = 2,3,\dots, K$ (49)

By taking $z=1$ in Eqn (49), we get

$\mathrm{Q}_i(0,1) = [(1-\mathrm{Þ}) + \mathrm{Þ}\mathrm{P}_1\mathrm{ß}_2^*(b\lambda) + \mathrm{Þ}\mathrm{P}_2\mathrm{ß}_3^*(b\lambda)]\mathrm{P}_{1,0}(0)\mathrm{ß}_1^*(b\lambda), i = 1,2,\dots, K$ (50)

By taking $n=0$ in Eqn (18), we get

$\mathrm{Q}_{i,0}(0) = [(1-\mathrm{Þ}) + \mathrm{Þ}\mathrm{P}_1\mathrm{ß}_2^*(b\lambda) + \mathrm{Þ}\mathrm{P}_2\mathrm{ß}_3^*(b\lambda)]\mathrm{P}_{1,0}(0)\mathrm{ß}_1^*(b\lambda)\prod_{l=1}^{i-1}\mathcal{V}_l^*(b\lambda)$ (51)

In a similar way from Eqn (19), we get

$\mathrm{R}_{K+1}(0,z) = \Theta_1[(1-\mathrm{Þ}) + \mathrm{Þ}\mathrm{P}_1\mathrm{ß}_2^*(b\lambda) + \mathrm{Þ}\mathrm{P}_2\mathrm{ß}_3^*(b\lambda)]\mathrm{P}_{1,0}(0)\mathrm{ß}_1^*(b\lambda)\prod_{l=1}^{K}\mathcal{V}_l^*(\mathrm{T}_1)$ (52) By taking $z=1$ in Eqn (52), we get

$\mathrm{R}_{K+1}(0,1) = \Theta_1[(1-\mathrm{Þ}) + \mathrm{Þ}\mathrm{P}_1\mathrm{ß}_2^*(b\lambda) + \mathrm{Þ}\mathrm{P}_2\mathrm{ß}_3^*(b\lambda)]\mathrm{P}_{1,0}(0)\mathrm{ß}_1^*(b\lambda)$ (53)

By taking $\mathfrak{N}=0$ in Eqn (19), we get

$\mathrm{R}_{K+1,0}(0) = \Theta_1[(1-\mathrm{Þ}) + \mathrm{Þ}\mathrm{P}_1\mathrm{ß}_2^*(b\lambda) + \mathrm{Þ}\mathrm{P}_2\mathrm{ß}_3^*(b\lambda)]\mathrm{P}_{1,0}(0)\mathrm{ß}_1^*(b\lambda)\prod_{l=1}^{K}\mathcal{V}_l^*(b\lambda)$ (54)

From Eqn (11), we get

$\lambda Q = [\Theta_0 + \Theta_1\mathcal{V}_{K+1}^*(b\lambda)]\prod_{l=1}^{K}\mathcal{V}_l^*(b\lambda)[(1-\mathrm{Þ}) + \mathrm{Þ}\mathrm{P}_1\mathrm{ß}_2^*(b\lambda) + \mathrm{Þ}\mathrm{P}_2\mathrm{ß}_3^*(b\lambda)]\mathrm{P}_{1,0}(0)\mathrm{ß}_1^*(b\lambda)$ (55)

From Eqn (42), we get

$\mathrm{P}_1(0,z) = \frac{[(1-\mathrm{Þ}) + \mathrm{Þ}\mathrm{P}_1\mathrm{ß}_2^*(b\lambda) + \mathrm{Þ}\mathrm{P}_2\mathrm{ß}_3^*(b\lambda)]\mathrm{P}_{1,0}(0)\mathrm{ß}_1^*(b\lambda)}{z - [(1-\mathrm{Þ}) + \mathrm{Þ}\mathrm{P}_1\mathrm{ß}_2^*(\mathrm{T}) + \mathrm{Þ}\mathrm{P}_2\mathrm{ß}_3^*(\mathrm{T})]\mathrm{P}_{1,0}(0)\mathrm{ß}_1^*(\mathrm{T})}\left[(z-1)\mathrm{A}_1 + \Theta_0\prod_{l=1}^{K}\mathcal{V}_l^*(\mathrm{T}_1) + \mathrm{A}_2 - 1\right]$ (56)

Where $\mathrm{A}_1 = (\Theta_o + \Theta_1\mathcal{V}_{K+1}^*(b\lambda))\,\mathcal{V}_l^*(b\lambda)$ and $\mathrm{A}_2 = (\Theta_1\mathcal{V}_{K+1}^*(\mathrm{T}_1))\,\mathcal{V}_l^*(\mathrm{T}_1)$

Now

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$$\left.\begin{aligned} P_1(z) = \int_0^\infty P_1(x,z)dx = \frac{[1-ß_1^*(T)]}{T}, P_2(z) = \int_0^\infty P_2(x,z)dx = \frac{[1-ß_2^*(T)]}{T} \\ P_3(z) = \int_0^\infty P_3(x,z)dx = \frac{[1-ß_3^*(T)]}{T}, Q_i(z) = \int_0^\infty Q_i(x,z)dx = \frac{[1-V_i^*(T_1)]}{T_1} \\ R_{K+1}(z) = \int_0^\infty R_{K+1}(x,z)dx = \frac{[1-V_{K+1}^*(T_1)]}{T_1} \end{aligned}\right\} \quad (57)$$

To track the $P_{1,0}(0)$ we utilize the normalizing condition

$Q + P_1(1) + P_2(1) + P_3(1) + \sum_{i=1}^{K} Q_i(1) + R_{K+1}(1) = 1$

We get

$$P_{1,0}(0) = \frac{\lambda(1+b\lambda(ß_1^{*\prime}(0)+ÞP_1ß_2^{*\prime}(0)+ÞP_2ß_3^{*\prime}(0))}{[(1-Þ)+ÞP_1ß_2^*(b\lambda)+ÞP_2ß_3^*(b\lambda)]P_{1,0}(0)ß_1^*(b\lambda)C} \quad (58)$$

Where

$C = A_1[1+(b-1)\lambda(ß_1^{*\prime}(0)+ÞP_1ß_2^{*\prime}(0)+ÞP_2ß_3^{*\prime}(0))] + \lambda E(V)[1+\gamma(ß_1^{*\prime}(0)+ ÞP_1ß_2^{*\prime}(0)+ÞP_2ß_3^{*\prime}(0))]$

And substituting Eqn (58) in (55), we get

$$Q = \frac{A_1(1+b\lambda(ß_1^{*\prime}(0)+ÞP_1ß_2^{*\prime}(0)+ÞP_2ß_3^{*\prime}(0))}{C} \quad (59)$$

Conditions in (59) together with (43), (45), (47), (49), (52) and (56) gives the PGF of range of clients with inside the line while server is occupied the assistance is inactive and he is at the k+1[th] durations of get-away separately.

3. Performance Measures

Let L_q stand for the mean queue size respectively, then

$L_q = \frac{d}{dz}\mathfrak{P}(z)|_{z=1}[(1-Þ)+ÞP_1ß_2^*(b\lambda)+ÞP_2ß_3^*(b\lambda)]P_{1,0}(0)ß_1^*(b\lambda)$

$L_q = \frac{d}{dz}\frac{N(z)}{D(z)}|_{z=1}[(1-Þ)+ÞP_1ß_2^*(b\lambda)+ÞP_2ß_3^*(b\lambda)]P_{1,0}(0)ß_1^*(b\lambda) \quad (60)$ Where

$N(z) = \lambda cdy + \lambda exa + A_1 xay$ and $D(z) = V_1(z)V_2(z)V_3(z)$

$a = V_2(z) = T, c = (z-1)A_1 + \Theta_0\prod_{l=1}^{k} V_l^*(T_1) + A_2 - 1,$

$d = 1-(1-Þ)ß_1^*(T) - ÞP_1ß_1^*(T)ß_2^*(T) - ÞP_2ß_1^*(T)ß_3^*(T),$

$e = 1 - V_1^*(T_1)V_2^*(T_1), \dots, V_k^*(T_1) + \Theta_1(1-V_{k+1}^*(T_1))$

$V_1(z) = x = z - [(1-Þ)+ÞP_1ß_2^*(T)+ÞP_2ß_3^*(T)]ß_1^*(T), y = V_3(z) = T_1$

Using the L'Hospital rule

$L_q = \frac{5(D'''(z)N''''(z)-N'''(z)D''''(z))}{20(D'''(z))^2}[(1-Þ)+ÞP_1ß_2^*(b\lambda)+ÞP_2ß_3^*(b\lambda)]P_{1,0}(0)ß_1^*(b\lambda)$

$D'''(z) = 6V_1'(z)V_2'(z)V_3'(z), \quad N'''(z) = 6(\lambda(c'd'y'+e'x'a') + A_1x'a'y'$

$D''''(z) = 12(V_1''(z)V_2'(z)V_3'(z) + V_1'(z)V_2''(z)V_3'(z) + V_1'(z)V_2'(z)V_3''(z))$

$N''''(z) = 12(\lambda(c''d'y'+c'd''y'+c'd'y''+e''x'a'+e'x''a'+e'x'a'') + A_1(x''a'y'+x'a''y'+x'a'y''))$

Where $E(V) = \sum_{i=1}^{K} V_i^{*\prime}(0) + \Theta_1 V_{K+1}^{*\prime}(0)$. Then use of the Little's formula, we get L_q, the mean waiting time with inside the queue as $W_q = \frac{L_q}{\lambda}$ respectively.

4. Particular cases

Case 1: Now we take, the help time and excursion time distribution as ED,

$ß_1^{*\prime}(0) = \frac{-1}{\mu_1}, ß_1^{*\prime\prime}(0) = \frac{2}{\mu_1^2}, ß_2^{*\prime}(0) = \frac{-1}{\mu_2}, ß_2^{*\prime\prime}(0) = \frac{2}{\mu_2^2}, ß_3^{*\prime}(0) = \frac{-1}{\mu_3}, ß_3^{*\prime\prime}(0) = \frac{2}{\mu_3^2} \quad V_i^{*\prime}(0) = \frac{-1}{\gamma_i}, V_i^{*\prime\prime}(0) = \frac{2}{\gamma_i^2}$

and $V_{K+1}^{*\prime}(0) = \frac{-1}{\gamma_{K+1}}, V_{k+1}^{*\prime\prime}(0) = \frac{2}{\gamma_{K+1}^2}$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3302 - 3311
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$$Q = \frac{O_1(\mu_1\mu_2\mu_3 - \mathrm{b}\lambda(\mu_2\mu_3 + \text{ÞÞ}_1\mu_1\mu_3 + \text{ÞÞ}_2\mu_1\mu_2)}{O_4},$$

$$L_q = \lambda\left[\frac{\begin{matrix}-2O_2(-\mathrm{b}^2\lambda^2 + \mathrm{b}\lambda\text{ɤ})(\mu_2^2\mu_3^2 + \text{ÞÞ}_1\mu_1\mu_3^2(\mu_1+\mu_2) + \\ \text{ÞÞ}_2\mu_1\mu_2^2(\mu_1+\mu_3) + (A_3)[(A_3)(2O_1\text{ɤ} - \lambda(-\mathrm{b}\lambda+\text{ɤ})^2O_3) \\ +(\mu_2\mu_3 + \text{ÞÞ}_1\mu_1\mu_3 + \text{ÞÞ}_2\mu_1\mu_2)((\mathrm{b}\lambda-\text{ɤ})^3O_3 + 2\text{ɤ}O_2)\end{matrix}}{2O_4(\mu_1\mu_2\mu_3 - \mathrm{b}\lambda(\mu_2\mu_3 + \text{ÞÞ}_1\mu_1\mu_3 + \text{ÞÞ}_2\mu_1\mu_2))(\mathrm{b}\lambda-\text{ɤ})}\right]$$

Where $A_3 = (\mu_1\mu_2\mu_3 - \mathrm{b}\lambda(\mu_2\mu_3 + \text{ÞÞ}_1\mu_1\mu_3 + \text{ÞÞ}_2\mu_1\mu_2))$

$O_1 = (\,\Theta_o + \Theta_1\frac{\mathcal{V}_{K+1}^*}{\mathrm{b}\lambda + \mathcal{V}_{K+1}^*})(\prod_{l=1}^{k}\frac{\mathcal{V}_l}{\mathrm{b}\lambda+\mathcal{V}_l})$, $O_2 = O_1 + (\mathrm{b}\lambda - \text{ɤ})E(\mathcal{V})$, $O_3 = E(\mathcal{V})^2$

$O_4 = O_1(\mu_1\mu_2\mu_3 - (\mathrm{b}-1)\lambda(\mu_2\mu_3 + \text{ÞÞ}_1\mu_1\mu_3 + \text{ÞÞ}_2\mu_1\mu_2)) + \lambda E(\mathcal{V})(\mu_1\mu_2\mu_3 - \text{ɤ}(\mu_2\mu_3 + \text{ÞÞ}_1\mu_1\mu_3 + \text{ÞÞ}_2\mu_1\mu_2))$

Case 2: the help time and excursion time distribution as ED, if there is no reneging and no balking.

$$Q = \frac{O_1(\mu_1\mu_2\mu_3 - \lambda(\mu_2\mu_3 + \text{ÞÞ}_1\mu_1\mu_3 + \text{ÞÞ}_2\mu_1\mu_2)}{O_4},$$

$$L_q = \lambda^2\left[\frac{\begin{matrix}2O_2\left(\mu_2^2\mu_3^2 + \text{ÞÞ}_1\mu_1\mu_3^2(\mu_1+\mu_2) + \text{ÞÞ}_2\mu_1\mu_2^2(\mu_1+\mu_3)\right) + \lambda O_3(\mu_1\mu_2\mu_3 \\ -\lambda(\mu_2\mu_3 + \text{ÞÞ}_1\mu_1\mu_3 + \text{ÞÞ}_2\mu_1\mu_2)[(\mu_2\mu_3 + \text{ÞÞ}_1\mu_1\mu_3 + \text{ÞÞ}_2\mu_1\mu_2) \\ -(\mu_1\mu_2\mu_3 - \mathrm{b}\lambda(\mu_2\mu_3 + \text{ÞÞ}_1\mu_1\mu_3 + \text{ÞÞ}_2\mu_1\mu_2))]\end{matrix}}{2O_4(\mu_1\mu_2\mu_3 - \lambda(\mu_2\mu_3 + \text{ÞÞ}_1\mu_1\mu_3 + \text{ÞÞ}_2\mu_1\mu_2))}\right]$$

$$W_q = \lambda\left[\frac{\begin{matrix}2O_2\left(\mu_2^2\mu_3^2 + \text{ÞÞ}_1\mu_1\mu_3^2(\mu_1+\mu_2) + \text{ÞÞ}_2\mu_1\mu_2^2(\mu_1+\mu_3)\right) + \lambda O_3(\mu_1\mu_2\mu_3 \\ -\lambda(\mu_2\mu_3 + \text{ÞÞ}_1\mu_1\mu_3 + \text{ÞÞ}_2\mu_1\mu_2)[(\mu_2\mu_3 + \text{ÞÞ}_1\mu_1\mu_3 + \text{ÞÞ}_2\mu_1\mu_2) \\ -(\mu_1\mu_2\mu_3 - \mathrm{b}\lambda(\mu_2\mu_3 + \text{ÞÞ}_1\mu_1\mu_3 + \text{ÞÞ}_2\mu_1\mu_2))]\end{matrix}}{2O_4(\mu_1\mu_2\mu_3 - \lambda(\mu_2\mu_3 + \text{ÞÞ}_1\mu_1\mu_3 + \text{ÞÞ}_2\mu_1\mu_2))}\right]$$

Where

$O_1 = (\,\Theta_o + \Theta_1\frac{\mathcal{V}_{K+1}^*}{\mathrm{b}\lambda + \mathcal{V}_{K+1}^*})(\prod_{l=1}^{k}\frac{\mathcal{V}_l}{\mathrm{b}\lambda+\mathcal{V}_l})$, $O_2 = O_1 + (\mathrm{b}\lambda - \text{ɤ})E(\mathcal{V})$, $O_3 = E(\mathcal{V})^2$

$O_4 = \mu_1\mu_2\mu_3(O_1 + \lambda E(\mathcal{V}))$

Case 3: $\mu_1 = \mu_2 = \mu_3 = \mu$

$$Q = \frac{O_1(\mu - \mathrm{b}\lambda(1+\text{Þ}))}{O_4},$$

$$L_q = \lambda\left[\frac{\begin{matrix}-2O_2(-\mathrm{b}^2\lambda^2 + \mathrm{b}\lambda\text{ɤ})(1+2\text{Þ}) + (\mu - \mathrm{b}\lambda(1+\text{Þ})[((\mu - \mathrm{b}\lambda(1+\text{Þ})) \\ (2O_1\text{ɤ} - \lambda(-\mathrm{b}\lambda+\text{ɤ})^2O_3) + (1+\text{Þ})((\mathrm{b}\lambda-\text{ɤ})^3O_3 + 2\text{ɤ}O_2)]\end{matrix}}{2O_4((\mu - \mathrm{b}\lambda(1+\text{Þ}))(\mathrm{b}\lambda-\text{ɤ})}\right]$$

$$W_q = \left[\frac{\begin{matrix}-2O_2(-\mathrm{b}^2\lambda^2 + \mathrm{b}\lambda\text{ɤ})(1+2\text{Þ}) + (\mu - \mathrm{b}\lambda(1+\text{Þ})[((\mu - \mathrm{b}\lambda(1+\text{Þ})) \\ (2O_1\text{ɤ} - \lambda(-\mathrm{b}\lambda+\text{ɤ})^2O_3) + (1+\text{Þ})((\mathrm{b}\lambda-\text{ɤ})^3O_3 + 2\text{ɤ}O_2)]\end{matrix}}{2O_4((\mu - \mathrm{b}\lambda(1+\text{Þ}))(\mathrm{b}\lambda-\text{ɤ})}\right]$$

Where

$O_1 = (\,\Theta_o + \Theta_1\frac{\mathcal{V}_{K+1}^*}{\mathrm{b}\lambda + \mathcal{V}_{K+1}^*})(\prod_{l=1}^{k}\frac{\mathcal{V}_l}{\mathrm{b}\lambda+\mathcal{V}_l})$, $O_2 = O_1 + (\mathrm{b}\lambda - \text{ɤ})E(\mathcal{V})$, $O_3 = E(\mathcal{V})^2$

$O_4 = O_1(\mu - (\mathrm{b}-1)\lambda(1+\text{Þ})) + \lambda E(\mathcal{V})(\mu - \text{ɤ}(1+\text{Þ}))$

Case 4: Now we take, the two kinds of OS, $\text{Þ}_1 = \text{Þ}_2 = 0$, queue without OS.

$$Q = \frac{\mathrm{A}_1(1 + \mathrm{b}\lambda(\text{ß}_1^{*\prime}(0)))}{\mathrm{A}_1\left[1 + (\mathrm{b}-1)\lambda\text{ß}_1^{*\prime}(0)\right] + \lambda E(\mathcal{V})\left[1 + \text{ɤ}\text{ß}_1^{*\prime}(0)\right]}$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3302 - 3311
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$$L_q = \frac{\lambda\left[\begin{array}{c}\text{ß}_1^{*''}(0)[A_1+(b\lambda-\gamma)E(V)](b^2\lambda^2-b\lambda\gamma)+\left[1+b\lambda\text{ß}_1^{*'}(0)\right][[-\text{ß}_1^{*'}(0)(b\lambda-\gamma)^3E(V^2)+ \\ 2\gamma[A_1+(b\lambda-\gamma)E(V)]+(1+b\lambda\text{ß}_1^{*'}(0))(2A_1\gamma-\lambda\left(-b\lambda+\gamma)^2E(V^2)\right)]\end{array}\right]}{2(1+b\lambda\text{ß}_1^{*'}(0)(b\lambda-\gamma)(A_1[1+(b-1)\lambda\text{ß}_1^{*'}(0)+\lambda E(V)[1+\gamma\text{ß}_1^{*'}(0)]}$$

$$W_q = \frac{\left[\begin{array}{c}\text{ß}_1^{*''}(0)[A_1+(b\lambda-\gamma)E(V)](b^2\lambda^2-b\lambda\gamma)+\left[1+b\lambda\text{ß}_1^{*'}(0)\right] \\ [[-\text{ß}_1^{*'}(0)(b\lambda-\gamma)^3E(V^2)+2\gamma[A_1+(b\lambda-\gamma)E(V)]+ \\ (1+b\lambda\text{ß}_1^{*'}(0))(2A_1\gamma-\lambda\left(-b\lambda+\gamma)^2E(V^2)\right)]\end{array}\right]}{2(1+b\lambda\text{ß}_1^{*'}(0)(b\lambda-\gamma)(A_1[1+(b-1)\lambda\text{ß}_1^{*'}(0)+\lambda E(V)[1+\gamma\text{ß}_1^{*'}(0)]}$$

Case 5: If there is no reneging, no balking, no OS.

$$Q = \frac{A_1(1+\lambda(\text{ß}_1^{*'}(0)))}{A_1+\lambda E(V)}$$

$$L_q = \frac{\lambda^2\left[\text{ß}_1^{*''}(0)[A_1+\lambda E(V)]+(1+\lambda\text{ß}_1^{*'}(0))E(V^2)\right]}{2[A_1+\lambda E(V)](1+\lambda\text{ß}_1^{*'}(0))}$$

$$W_q = \frac{\lambda\left[\text{ß}_1^{*''}(0)[A_1+\lambda E(V)]+(1+\lambda\text{ß}_1^{*'}(0))E(V^2)\right]}{2[A_1+\lambda E(V)](1+\lambda\text{ß}_1^{*'}(0))}$$

5. Mathematical outcomes

In this section we present some numerical results for illustration (Case 2) (Case 3). Assuming certain values for the system parameters like $\lambda = 1.50$ to $1.59, \text{Þ} = 0.3, \mu_1 = 2.5, \mu_2 = 2.25, \mu_3 = 2, k = 5$, and$\Theta_1 = 0.7565$.We calculated the values of ρ, Q, L_q and W_q and they are in table 1. We observe from the table that as λ increases, Q decrease and the steady state increase in both L_q and W_q which is expected.

Table -1 Computing measures of L_q and W_q

λ	ρ	Q	L_q	W_q
1.50	0.81	0.000746462	5.17671	3.45114
1.51	0.8154	0.000720463	5.37499	3.55959
1.52	0.8208	0.000694804	5.58337	3.67327
1.53	0.8262	0.000669479	5.80271	3.79262
1.54	0.8316	0.000654544	6.03399	3.91817
1.55	0.837	0.000619807	6.27832	4.05052
1.56	0.8424	0.000595446	6.53695	4.19035
1.57	0.8478	0.000571395	6.81130	4.33841
1.58	0.8532	0.000547647	6.89450	4.36361
1.59	0.8586	0.000524419	7.41404	4.66292

Assuming certain values for the system parameters such as $\gamma = 0.5$ to $0.69, \text{Þ} = 0.1, \lambda = 1.1, \mu = 1.5, k = 5, b = 1$ and $\Theta_1 = 0.59$. We calculated the values of Q, L_q and W_q and they are in table 2. We note from the table that as γ increases, there is a Q increase and steady state decrease in both L_q and W_q which is expected.

Table -2 Computing measures of L_q and W_q

γ	Q	L_q	W_q

0.51	0.00352925	3.92481	3.56801
0.53	0.00361234	3.89894	3.544492
0.55	0.00369944	3.87408	3.52189
0.57	0.00379084	3.85031	3.50029
0.59	0.00388686	3.82766	3.47968
0.61	0.00398789	3.80637	3.46034
0.63	0.00409430	3.78640	3.44218
0.65	0.00420655	3.76801	3.42547
0.67	0.00432512	3.75133	3.41054
0.69	0.00445058	3.73650	3.39682

6. Conclusion

The examination an M/G/1 Reneging and Balking line with two sorts of OS and $k+1^{th}$ phase of discretionary excursions was considered. For this model, get PGF for the quantity of client inside the framework and also get the L_q, W_q in the queue. A wide mathematical work should be done to notice the concept of the operating qualities.

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