JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 1, 2022, p. 490-497
https://publishoa.com
ISSN: 1309-3452

Realization of Relative Entropy Evolution in the Sudarshan-Lindblad for two Quantum Systems

Bhaveshkumar B. Prajapati[1] and Dr. Nirbhay kumar Chaubey[2]

[1] Bhaveshkumar B. Prajapati [https://orcid.org/0000-0002-8015-7934] ,IT Department, L.D. College of Engineering, GujaratTechnological University, Gujarat, India. E-mail: b.b.prajapati@gmail.com (Corresponding Author)

[2] Dr. Nirbhay kumar Chaubey [https://orcid.org/0000-0001-6575-7723] , Senior Member of IEEE, Dean of Computer Science, Ganpat University, Gujarat, India E-mail: nirbhay@ieee.org

Abstract:

The idea of this paper is to investigate and realizing the quantum relative entropy normally used in quantum information using physical systems for two quantum systems. We have described these respectively by Hamiltonian and Lindblad operators (H_k, L_k), $k = 1,2,\cdots$,and we calculate the rate at which the relative entropy distance between the states evolving according to their two systems evolve. A consequence of the Baker–Campbell–Haussdorff formula in lie algebra theory, the computation uses the differentiation formula for the exponential map. The asymptotic formula for this relative entropy rate is obtained in terms of the scattering matrix corresponding to the two Hamiltonian.

Keywords: Schrödinger equation, Perturbation theory, Ito calculus, quantum relative entropy, Sudarshan- Lindblad equation, Lie algebra theory

1. Introduction:

The field of quantum information can be processed with a high level of emerging technologies with covering of a wider application of the various areas. It is using a large ensemble of ultra-cold and trapped atoms, to develop with high precision in spectrography and metrology to build technological enhancement. We have to consider quantum information processing through photons pair, which includes quantum computation, metrology, commutation and a quantum sensing. Photon pair techniques are the best method in the quantum communication to established secured and fast process because photons pairs are the trustworthy carriers modulation for quantum information application. The most important part of the photon pair of quantum information leads to quantum communication and computation in which all processing tasks are performed in atomic scale with high accuracy. The emerging developments with a full-scale of quantum communication and information is still untouched and still challenging. So optical or photon pair systems are the most promising approaches for implementation quantum computing and quantum channels. The basic understanding of classical and quantum theory are representations, In classical information, the states denoted by ZERO and ONE, it means a single bit. But in quantum theory, the information is carried by qubits and it follows the quantum superposition. Here, two hypotheses are applicable and validate everywhere in the quantum system, first quantum superposition and other quantum entanglement.

In Quantum superposition, If we have a classical bit, Ket $\{|0\rangle$ and $|1\rangle\}$ would be the only two stage. But as per the law of quantum mechanics allow a qubit, which is to be a combination of $|0\rangle$ *and* $|1\rangle$, called a *superposition states.* Quantum mechanics defines coherent states of the 3D (three dimensional) - quantum harmonic oscillator, and proposed by Roy Glauber for the first time to describe the quantum wave function in the form of States. As per the Dirac notation of the qubit, it should be laid on 2D (two dimensional) of complex Hilbert space H in the quantum dynamics systems.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 1, 2022, p. 490-497
https://publishoa.com
ISSN: 1309-3452

In Quantum entanglement, which is composed of two or more parts of the quantum system and also constitutes between quantum system and classical are correlated. It will support a basic understanding between quantum-classical boundaries. Besides these fundamental aspects, quantum entanglement is a very important tool for developing the area in quantum information science [1-3]. In short, entangled states are connected with various applications in quantum cryptography, quantum teleportation, and quantum sensing. These days, entanglement is the best process to recognised as the best approach in the field of quantum communication and quantum computation tasks [4-6].

The Advantage of entanglement, is provided with a method for preparing an entangled state of the atomic scale. The method involves respective optical trap of each atom loaded more than a dipole-dipole interaction length separates the atoms of the atomic ensemble[7-9]. We have focused primarily so far, the electromagnetic quantum field has only been studied from a spatial perspective. In any case, finding the time evolution of quantum fields is important for all physical applications, and for quantum information processing in particular to achieve and describe two equivalent ways. In either case, we can either take into account the time dependence when we describe the quantum state (the Schrödinger picture) or we can describe the time dependence in terms of the operators (the Heisenberg picture).

Based on this ideas the quantum systems are defined through postulates. So one of the quantum postulates are explaining the coherent states, which is described by a density operator ρ in a Hilbert space H at given time period T. It is a positive semidefinite and the trace of the density operator (ρ) is unity. When its rank is one, the state of destiny operator is defined as pure states.

So, we can write as $\rho = |\psi >< \psi|$, where$|\psi >$is a unit vector in the given Hilbert space H.

In the Hilbert space H, there is also an orthogonal projection operator P. The probability of this event P occurring when the system is in the state ρ is defined as Tr(ρP) [10-14].

A Novelty of quantum information processing has been proposed to be carried out on physical devices. There are characterized by the fact that they should be supported to qubits or other quantum systems. When entanglement exists, the result of an experiment that collapses the quantum state of a first particle can be correlated with the result of an experiment performed on a second particle that is entangled with the first, even if the particles are separated by a macroscopic distance that prevents mutual interaction at the time of measurement [15-17].

One application of the paper's ideas is in systems at the nanoscale, such as transmitting information between atoms or from one gene to another in microorganisms. It is also possible to determine the information in atoms is transferred through quantum electromagnetic (EM) field and received by quantum receiver within another atom in the system, which is embedded [18-21]. Recently, a multi-particle entanglement measure based on Hilbert space geometry has been proposed. In pure states, this geometrical measurement of entangled states is to be determined by optimal overlap between two possible entangled states and another the unentangled state, which is mathematically computed. This measurement of two distinct upper bounds of multipartite and bi-particles of the entangled states has to be applied in the pure and mixed states [22-25].

The main contribution of this paper is to explore the relationship between the discrepancy of measured value and the relative entropy for two quantum systems. But the condition of the pure states in the some bi-partite and multi-particle for lower bound has to be saturated, and thus their relative entropy of entanglement should be calculated mathematically, based on their known geometrical measured parameters of the entangled states. Moreover, an upper bounds of the quantum relative entropy has to be established for certain mixed states of the entanglement. Numbers strongly suggest that these upper bounds are the relative entropy of entanglement, that is, they are the relative entropy of entanglement [16, 20, 23-32]. The results, although not general enough to solve the problem of calculating the relative entropy of entanglement for arbitrary multipartite states, nevertheless provide some insight into the problem. The study may provide some insight into future analytic progress related to the relative entropy of entanglement, as well as serve as a testbed for future work. As follows is the paper's structure: In Sec. II we are describing Mathematical Descriptions of the Generalized Sudarshan-Lindblad. In Sec. III description of the two entanglement state: methodology for evolution of Quantum relative entropy of entanglement. In Sec. IV we are concluding remarks.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 1, 2022, p. 490-497
https://publishoa.com
ISSN: 1309-3452

2. Mathematical Descriptions of the Generalized Sudarshan-Lindblad:

Let $\mathfrak{h}$ us consider the system Hilbert space and $\mathcal{H} = L^2(\mathbb{R}_+)$ the noise Hilbert space. $\Gamma_s(\mathcal{H}) = \Gamma_s(L^2(\mathbb{R}_+))$ is the noise Boson Fock space. According to the Hudson-Parthasarathy noisy Time dependent Schrödinger equation, the evolution operator U(t) of the system plus bath in the system plus noise Hilbert space is $\mathfrak{h} \otimes \Gamma_s(\mathcal{H})$:

$$dU(t) = (-iH(t)dt + L_1(t)dA(t) - L_2(t)dA^*(t) + S(t)d\Lambda(t))U(t)$$

where, $L_1(t), L_2(t), S(t), H(t)$ are system operators, that is, in $L(\mathfrak{h})$ and $A(t), A^*(t), \Lambda(t)$ act in the Boson Fock space $\Gamma_s(\mathcal{H}_{t]})$. More precisely, $A(t), A^*(t)$ and $\Lambda(t)$ act in $\Gamma_s(\mathcal{H}_{t]})$ and act as identity operators in $\Gamma_s(\mathcal{H}_{(t)})$. Here, we identity the Hilbert space $\Gamma_s(\mathcal{H}) = \Gamma_s(\mathcal{H}_{t]} \oplus \mathcal{H}_{(t)}$ with $\Gamma_s(\mathcal{H}_{t]}) \otimes \Gamma_s(\mathcal{H}_{(t}$ via the canonical isomorphism defined using exponential vectors: $e(u) \approx e(u\chi_{[}0,t]) \otimes e(u\chi_{(t,\infty)})$. Note that for any Borel subset B of $\mathbb{R}_+\$, \$(u\chi_B)(t)$ equals u(t) for $t \in B$ and zero for $t \notin B$. The quantum Ito formula based on the differentiation of the creation, annihilation and conservation process, which are simplified in the infinite dimension of the Boson Fock space. Using these calculus, the HP equation noisy Schrödinger equation should be described below:

$$dA(t)dA^*(t) = dt, dA^*(t)dA(t) = 0, d\Lambda(t)dA^*(t) = dA^*(t), d\Lambda d\Lambda = d\Lambda$$

$$dA(t)d\Lambda(t) = dA(t), d\Lambda(t)dA(t) = 0, dA^*(t)d\Lambda(t) = 0$$

Let $X \in L(\mathfrak{h})$ be Hermitian. $X(t) = U^*(t)XU(t) = U^*(t)(X \otimes I)U(t)$.

By the quantum Ito formula, we have

$$d(U^*(t)U(t)) = dU^*dU + U^*dU + dU^*dU$$

$$= U^*(i(H^* - H)dt + (L_1^* - L_2)dA^* + (L_1 - L_2^*)dA + (S^* + S)d\Lambda)U$$
$$+U^*(L_1 * dA^* - L_2^*dA + S^*d\Lambda)(L_1dA - L_2dA^* + Sd\Lambda)U$$

$$= U^*[(i(H^* - H) - L_2^*L_2)dt + (S^* + S + S^*S)d\Lambda + (L_1^* - L_2 - S^*L_2)dA^* + (L_1 - L_2^* - L_2^*S)dA]U$$

For U(t) to be unitary for all t, we require that $d(U^*U) = 0$ and this happens iff $i(H^* - H) + L_2^*L_2 = 0, S + S^* + S^*S = 0, L_1 = L_2^*(1 + S)$, we assume that these conditions are satisfied. Another application of the quantum Ito formula yields the Heisenberg equation for X(t):

$$dX(t) = dU^*.X.U + U^*XdU + dU^*XdU$$

$$= U^*(i(H^*X - XH - L_2^*XL_2)dt + (L_1^*X - XL_2 - S^*XL_2)dA^* + (XL_1 - L_2^*X - L_2^*XS)dA + (XS^* + SX + S^*XS)d\Lambda)U \tag{1}$$

We get for $\rho_s(0) \in \mathcal{L}(\mathfrak{h})$ being the initial system state and $u \in \mathcal{H}$,

$$Tr(\rho_s(0) \otimes |e(u) >< e(u)|dX(t)) = Tr_1(\rho_s(0)Tr_2(|e(u) >< e(u)|dX(t)))$$

Now from (1),

$$Tr(\rho_s(0) \otimes |e(u) >< e(u)|dX(t)) = i.Tr[U(t)\rho_s(0) \otimes |e(u) >< e(u)|U(t)^*)(H^*X - XH - L_2^*XL_2]dt$$

$$+Tr(U(t)(\rho_s(0) \otimes |e(u) >< e(u)|)U(t)^*(L_1^*X - XL_2 - S^*XL_2))\overline{u}(t)dt$$

$$+Tr(U(t)(\rho_s(0) \otimes |e(u) >< e(u)|)U(t)^*(XL_1 - L_2^*X - L_2^*XS))u(t)dt$$

$$+Tr(U(t)(\rho_s(0) \otimes |e(u) >< e(u)|)U(t)^*(XS^* + SX + S^*XS))|u(t)|^2dt$$

It follows that if we define the system state at time t as

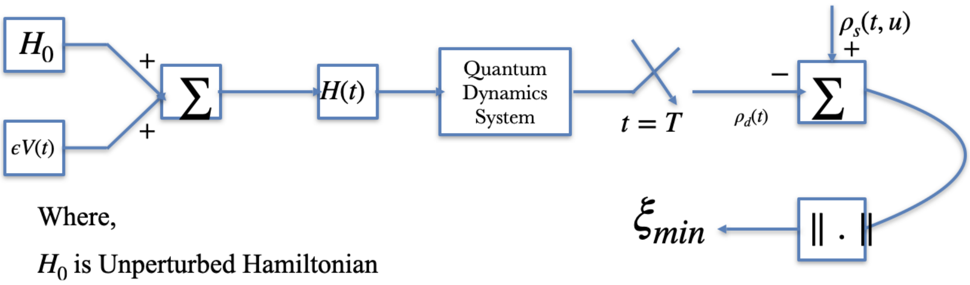
JOURNAL OF ALGEBRAIC STATISTICS

Volume 13, No. 1, 2022, p. 490-497

htt

ISS

Model of optimum value for of the Density operators:



Where,

H_0 is Unperturbed Hamiltonian

$H_1 = \epsilon V(t)$ is Perturbed Hamiltonian

So Total Hamiltonian; $H(t) = H_0 + \epsilon V(t)$

$\rho_s(t,u)$ is a desired operators

$\rho_d(t)$ is a Derived operators

$\| . \|$ is a Frobenius Norm and

$\xi_{min} = ||\rho_s(t,u) - \rho_d(t)||$ is an optimum values

$$\rho_s(t) = \rho_s(t,u) = Tr_2(U(t)(\rho_s(0) \otimes |e(u) >< e(u)|)U(t)^*)$$

then $\rho_s(t)$ satisfies the generalized Sudarshan-Lindblad equation

$$\rho_s'(t) = -i(H(t)\rho_s(t) - \rho_s(t)H(t)^* - L_2(t)\rho_s(t)L_2(t)^*) + \overline{u}(t)(\rho_s(t)L_1(t)^* - L_2(t)\rho_s(t) - L_2(t)\rho_s(t)S(t)^*)$$

$$+u(t)(L_1(t)\rho_s(t) - \rho_s(t)L_2(t)^* - S(t)\rho_s(t)L_2(t)^*) + |u(t)|^2(S(t)^*\rho_s(t) + \rho_s(t)S(t)^* + S(t)\rho_s(t)S(t)^*)$$

Given a state evolution $\rho_d(t), 0 \le t \le T$, assuming L_1, L_2, H, S to be constant operators in $\mathfrak{h}$ (i.e. independent of time). The optimum function $u(t), 0 \le t \le T\$(\$u \in L^2(\mathbb{R}_+))$ so that $\int_0^T \| \rho_s(t,u) - \rho_d(t) \|^2 \, dt$ is a minimum.

Assuming the function $u(t), 0 \le t \le T$ and H given, determine the optimum operators L_1, L_2, S, so that $\int_0^T \| \rho_s(t,u) - \rho_d(t) \|^2 \, dt$ is a minimum in given block diagram.

3. Methodology for Evolution of Quantum relative entropy:

The evolution has to be appeared in the form of Von Neumann entropy for an entangled states ρ, which is defined as $S(\rho) -= Tr(\rho.log(\rho))$. When the measurement has to be carried out, the evolution of the entangled state may get disturbed by BATH states due to Heisenberg uncertainty principles. So in order to avoid this, we can approach the non-demolition measurement, which can be made on the system with BATH states. Suppose we send a random classical alphabet x with probability p(x) and the resulting state is ρ_x [31-37]. Then the conditional entropy of the output given the input is given by $S(Y|X) = \sum_x p(x)S(\rho_x)$and the entropy of output is given by

$$S(Y) = S(\sum_x p(x)\rho_x).$$

A quantum relative entropy is evolving in between two quantum systems $\rho_1(t)$ and $\rho_2(t)$ are density matrices satisfying the Sudarshan-Lindblad equation.

$$\rho_1'(t) = -\iota[H_1, \rho_1(t)] - \frac{1}{2}\theta_1(\rho_1(t))$$

$$\rho_2'(t) = -\iota[H_2, \rho_2(t)] - \frac{1}{2}\theta_2(\rho_2(t))$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 1, 2022, p. 490-497
https://publishoa.com
ISSN: 1309-3452

Where, H_1 and H_2 are time dependent Hamiltonian, which are hermitian operators and θ_1 and $\theta_2$are the Noise operators, deigned through observable.

$$\theta_1(X) = \sum_{k=1}^{p} (L_k^* L_k X + X L_k^* L_k - 2L_k X L_k^*)$$

$$\theta_2(X) = \sum_{k=1}^{p} (M_k^* M_k X + X M_k^* M_k - 2M_k^* M_k)$$

Assume $H_2 - H_1$ and $M_k - L_k$ upto $O(\varepsilon)$, then calculate upto $O(\varepsilon^2)$.

$$\frac{d}{dt} T_r(\rho_1 log\rho_1) = T_r(\frac{d\rho_1}{dt}) + T_r(\rho_1 \frac{d}{dt} log\rho_1)$$

so by the Baker–Campbell–Haussdorff formula

$$\rho_1 = Z_1, \rho_2 = e^{Z_1}, \rho_1' = e^{Z_1} \frac{I - e^{-adZ_1}}{adZ_1}$$

Let,

$$Z_1' = \rho_1^{-1} \sum_{r=1}^{\infty} c_r (adZ_1)^r (\rho_1^{-1} \rho_1')$$

$$T_r(\rho_1 \frac{d}{dt} log\rho_1) = T_r(\rho_1 Z_1) = \sum_{r=1}^{\infty} c_r T_r(\rho_1 (adlog\rho_1)^r (\rho_1^{-1} \rho_1'))$$

(since $T_r(\rho_1') = 0$). If X is a self-adjoint (like Z_1), then $adX$is a skew-Hermitian.

Since,

$T_r((adX)(y)Z) = T_r([X,Y]Z)$ $= T_r((XY - YX)Z) = -T_r(Y(XZ - ZX)) = -T_r(Y(adX)(Z))$

Let, $(adX)^* = adX$, Hence all the eigenvalues or adX are pure imaginary and the above series for $\frac{adZ}{1-e^{-adZ}}$ does not converge.

Instead, we try

$$\frac{adZ_1}{1 - e^{-adZ_1}} = (ad_1) \sum_{m=0}^{\infty} e^{-madZ_1}$$

$$= adZ_1 \sum_{m=0}^{\infty} Ad(e^{-mZ_1}) = (adZ_1)(I + \sum_{m=1}^{\infty} Ad(\rho_1^{-m}))$$

so, $$T_r(\rho_1 \frac{d}{dt} log\rho_1) = T_r(\rho_1 Z_1') = T_r(\rho_1 \frac{adZ_1}{1-e^{-adZ_1}} (\rho_1^{-1} \rho_1')$$

$$= T_r(\rho_1 (adZ_1))(\rho_1^{-1} \rho_1' + \sum_{m=1}^{\infty} \rho_1^{-m-1} \rho_1' \rho_1^m))$$

$$= T_r(\rho_1 (\rho_1 \rho^{-1} aZ_1(\rho_1') + \sum_{m=1}^{\infty} e_1^{-m-1} [Z_1, \rho_1'] \rho_1^m))$$

$$= T_r([Z_1, \rho_1']) + \sum_{m=1}^{\infty} T_r(\rho_1^{-m} [Z_1, \rho_1'] \rho_1^m)$$

so,

$$\frac{d}{dt} T_r(\rho_1 log\rho_1) = T_r[\rho_1' log\rho_1]$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 1, 2022, p. 490-497
https://publishoa.com
ISSN: 1309-3452

and

$$\frac{d}{dt}T_r(\rho_1 log\rho_2) = T_r(\rho_1' log\rho_2) + T_(\rho_1 Z_2)$$

Now,

$$Z_2 = log\rho_2,$$

$$Z_2' = \frac{adZ_2}{1-e^{-adZ_2}}(\rho_1^{-1}\rho_2')$$

$$T_r(\rho_1 Z_2') = T_r\{\rho_1 Z_2 \sum_{m=0}^{\infty} e^{m.ad.Z_2}(\rho_1^{-1}\rho_2')\} \quad = T_r\{\rho_1(adZ_2(\rho_1^{-1}\rho_2' + \sum_{m=1}^{\infty}\rho_2^{-m-1}\rho_2'\rho_2^m))\}$$

$$= T_r\{\rho_1\rho_2^{-1}[Z_2,\rho_2']\} + \sum_{m=1}^{\infty} T_r(\rho_1\rho_2^{-m-1}[Z_2,\rho_2']\rho_2^m)$$

$$= T_r\{\rho_1\rho_2^{-1}[Z_2,\rho_2']\} + \sum_{m=1}^{\infty} T_r(\rho_2^m\rho_1\rho_2^{-m-1}[Z_2 - \rho_2'])$$

$$\sum_{m=0}^{\infty} T_r(\rho_2^m\rho_1\rho_2^{Z_2,\rho_2'})$$

so,

$$\frac{d}{dt}S(\rho_1,\rho_2) = \frac{d}{dt}T_r(\rho_1 log\rho_1 - \rho_1 log\rho_2)$$

$$T_r(\rho_1' log\rho_1) - T_r(\rho_1' log\rho_2) - T_r(\rho_1 Z_2')$$

$$T_r(T_1(\rho_1)log\rho_1)) - T_r(T_1(\rho_1)log\rho_2)$$

$$\sum_{m=0}^{\infty} T_r(\rho_2^m\rho_1\rho_2^{-m-1}[log\rho_2, T_2(\rho_2)])$$

When,

$T_k(\rho) = -\iota[H_k,\rho] - \frac{1}{2}\theta_k(\rho)$, $k = 1,2$ .... Special case $\theta_1 = \theta_2 = 0$ (No noise),

Then

$$T_k(\rho) = -\iota[H_k,\rho]$$

$$T_r(T_1(\rho_1)log\rho_1) = -\iota T_r\{[H_1,\rho_1]log\rho_1\} = -\iota T_r\{H_1\rho_1 log\rho_1 - \rho_1 H_1 log\rho_1\}$$

$$= -\iota T_r(\{H_1[\rho_1, log\rho_1]\}) = 0$$

$$T_r(T_1(\rho_1)log\rho_2) = -\iota T_r\{[H_1,\rho_1]log\rho_2\} = -\iota T_R\{H_1(\rho_1 log\rho_2 - log\rho_2\rho_1)\}$$

$$T_r(\rho_2^m\rho_1^m\rho_2^{-m-1}[log\rho_2, T_2(\rho_2)]) = -\iota T_r(\rho_2^m\rho_1^m\rho_2^{-m-1}[log\rho_2, H_2\rho_2 - \rho_2 H_2])$$

$$= -\iota T_r(\rho_2^m\rho_1^m\rho_2^{-m-1}((log\rho_2)(H_2\rho_2 - \rho_2 H_2) - (H_2\rho_2 - \rho_2 H_2)log\rho_2))$$

$$= \iota T_r(\{\rho_2^{m+1}\rho_1\rho_2^{-m-1}log\rho_2 - \rho_2^m\rho_1\rho_2^{-m}log\rho_2 - log\rho_2\rho_1^{m+1}\rho_2^{-m-1} + (log\rho_2)\rho_2^m\rho_1\rho_2^{-m}\}H_2)$$

$$= -\iota T_r\{\{[\rho_2^{m+1}\rho_1\rho_2^{-m-1}, log(\rho_2)] + [log(\rho_2), \rho_2^m rho_1\rho_2^{-m}]\}H_2\}$$

$$= -\iota T_r\{\{\rho_2^{m+1}[\rho_1, log\rho_2]\rho_2^{-m-1} + \rho_2^m[log\rho_2,\rho_1]\rho_2^{-m}\}\}$$

In this case, we thus find

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 1, 2022, p. 490-497
https://publishoa.com
ISSN: 1309-3452

$$\frac{d}{dt}S(\rho_1,\rho_2) = \quad -\iota T_r\{H_1[\rho_1, log\rho_2]\} + \iota\sum_{m=0}^{\infty} T_r\{H_2(\rho_2^{m+1}[rho_1, log(\rho_2)]\rho_2^{-m-1} - \rho_2^{m}[\rho_1 log\rho_2]\rho_2^{-m})\}$$

Now,

$$\sum_{m=0}^{\infty}(\rho_2^{m+1}[\rho_1, log\rho_2]\rho_2^{-m-1} - \rho_2^{m}[\rho_1, log\rho_2]\rho_2^{-m}) = -[\rho_1, log\rho_2]$$

Provided

$$\lim_{N\to\infty}\rho_2^N[\rho_1, log\rho_2]\rho_2^{-N} = 0$$

In this case we find

$$\frac{d}{dt}S(\rho_1,\rho_2) = \iota T_r\{(H_1 - H_2)[\rho_1, log\rho_2]\}$$

The aim of this paper is to solve the relative entropy of the quantum systems without noise. Our research is presently describe the quantum average measure value to extra the information with effecting of the AWGN and stochastic noise and we will evaluate the performance of our algorithm in the presence of BATH states, that is, compute the noise to signal ratio of the given estimate of $\delta\theta$, that is, $E(||\delta(\theta) - \delta(\theta)||^2)$.

4. Conclusions:

The quantum information processing has wider application and it is a generalized form of non-demolition Von-Neumann entropy for the entangled quantum states. So, the concluding remarks of the geometrical measurement for entangled operators have calculated with lower bound of the pure states. It has to satisfy the condition of quantum postulate for arbitrary bi-particle and multipartite. For mixed states, we have explained the mathematical methodology for the relative entropy without effecting of the BATH space of the entangled states. The BATH states will be finally optimized by using HP equations to the specific example of the trapped atom model. And we will realize and simulate through MATLAB for evaluating the performance criteria to show the NSR (Noise to signal ratio) plot to get the estimate value of observable form the noisy angular momentum. Future research will need to address many other important issues to understand the quantum noise analysis and more discuss about proposed filter bank, like Belavkin filter.

References:

1. Schr¨odinger E.: An Undulatory Theory of the Mechanics of Atoms and Molecules. Phys. Rev., 28, (1926).
2. Von Neumann J.: Mathematical Foundations of Quantum Me- chanics, Princeton Univ. Press, Princeton, NJ, (1955).
3. Dirac P.A.M.: The principles of quantum mechanics, Oxford university press, 4th edition, New York, (1958).
4. Helstrom C.W.: Detection Theory and Quantum Mechanics. Information and Control, 10, 254-291(1967).
5. Helstrom C.W.: Detection Theory and Quantum Mechanics (II). Information and Control, 13, 156-171(1968).
6. Peyton B., Sard E., Lange R., Arams F.R.: On Optimal Quan- tum Receivers for Digital Signal Detection. Proceedings of the IEEE, (1970).
7. Holevo A.S.: Statistical Decision Theory for Quantum Systems. Journal Of Multivariate Analysis, 3, 337-394(1973).
8. Yuen H.P., Kennedy R.S., Lax M.: Optimum Testing of Multiple Hypotheses in Quantum Detection Theory. IEEE Transaction on Information Theory, 21(2), (1975).
9. Kennedy R.S.: A near-optimum receiver for the binary coherent state quantum channel. Research Lab. Electron MIT Cambridge Tech. Rep., 108, (1973).
10. Dolinar S.J.: An optimum receiver for the binary coherent state quantum channel. Research Lab. Electron MIT Cam- bridge Tech. Rep., 111, (1973).
11. Helstrom C.W.: Bayes-Cost Reduction Algorithm in Quantum Hypothesis Testing. IEEE Transaction on Information Theory 28(2), 359-366 (1982).
12. Ekert A.K.: Quantum cryptography based on Bells theorem. Physical review letters, 67(6), 661 (1991).

13. Atmanspacher H., Kurths J., Scheingraber H., Wackerbauer R., Witt A.: Complexity and meaning in nonlinear dynamical systems, Open Systems and Information Dynamics, 1(2), 269- 289 (1992).
14. Christopher A. F., Carlton M. C.: Mathematical techniques for quantum communication theory. Open Systems and Information Dynamics, 3(3), 345-356(1995).
15. Bennett C.H., Brassard G., Popescu S., Schumacher B., Smolin J.A. and Wootters W.K.: Purification of noisy entanglement and faithful teleportation via noisy channels. Physical review letters, 76(5), 722, (1996).
16. Bennett C.H., DiVincenzo D.P., Smolin J.A. and Wootters W.K.: Mixed-state entanglement and quantum error correction. Physical Review A, 54(5), 3824 (1996).
17. Mensky M., Audretsch J.: Continuous QND measurements: no quantum noise. Journal Applied Physics B, 64(2), 129-136, (1997).
18. Karlsson A., Bjrk G.: Quantum correlations in dual quantum measurements. Journal Applied Physics B, 64(2), 235-241, (1997).
19. Nielsen, M.A., Chuang, I.L.: Quantum Computation and Quan- tum Information, pp. 171286. Cambridge University Press, Cambridge (2001).
20. Kato K., Hirota O.: Square-Root Measurement for Quantum Symmetric Mixed State Signals. IEEE Transaction on Information Theory, 49(12), (2003).
21. Srikanth R.: A Computational Model for Quantum Measurement. Quantum Information Processing, 2(3), 153-199, (2003).
22. Long G.L. and Xiao L.: Parallel quantum computing in a single ensemble quantum computer. Physical Review A, 69(5), 052303 (2004).
23. Brickmont J., Kupiainen A.: Towards a Derivation of Fouri- ers Law for Coupled Anharmonic Oscillators. Communication in Mathematical Physics 274, 555-626, (2007).
24. Dowker F., Tabatabai Y.G.: Dynamical wavefunction collapse models in quantum measure theory. J. Phy. A Mathematical and Theoritical, 41(20), (2008).
25. Hayashi M., Universal Coding for Classical-Quantum Channel, Communication in Mathematical Physics 289, 1087-1098, (2009).
26. Griffiths D. J.: Introduction to quantum mechanics, Pearson Hall, 2nd edition, USA, (2005).
27. Feng G., Xu G. and Long G.: Experimental realization of nona- diabatic holonomic quantum computation. Physical review letters, 110(19), 190501 (2013).
28. Parthasarathy K.R.: ”Coding theorems of classical and quantum Information theory”, Hindustan Book Agency; 2013.
29. Wang D., Hoen R.D., Ye L., Kais S.: Generalized Remote Preparation of Arbitrary m-qubit Entagled States via Genuine Entanglements, Entropy (17), 1755-1774, (2015).
30. Gautam K., Chauhan G., Rawat T.K., Parthasarathy H. and Sharma N.: Realization of quantum gates based on three-dimensional harmonic oscillator in a time-varying electromag- netic field. Quantum Information Processing, 14(9), 3279-3302 (2015).
31. Gautam K., Rawat T.K., Parthasarathy H. and Sharma N.: Realization of commonly used quantum gates using perturbed har- monic oscillator. Quantum Information Processing, 14(9), 3257- 3277 (2015).
32. Li, X., Ghose S.: Optimal joint remote state preparation of equa- torial states. Quantum Information Processing, 14(12), 4585- 4592 (2015).
33. Wang D., Huang A.J., Sun W.Y., Shi J.D., Ye L.: “Practicle single-photon-assisted remote state preparation with non-maximally entenglement”, Quantum Information Processing, (15), 3367-3381, (2016).
34. Li T. and Long G.L.: Hyperparallel optical quantum computation assisted by atomic ensembles embedded in double-sided optical cavities. Physical Review A, 94(2), 022343 (2016).
35. Gautam K., Rawat T.K., Parthasarathy H., Sharma N. and Upadhyaya V.: Realization of the three-qubit quantum controlled gate based on matching Hermitian generators. Quantum Information Processing, 16(5), 113 (2017).
36. Sharma N., Rawat T.K., Parthasarathy H. and Gautam K.: "Performance Analysis of Quantum Unitary Gates in Presence of Noise in the Field of Quantum Communication", IEEE International conference on Power Electronics (IICPE, Thapar University), 2016.
37. Sharma N., Gautam K. and Parthasarathy H.: “Estimating Hamiltonian fluctuations from quantum time averages”, in IET Quantum Communication, 2020.