JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3600-3609
https://publishoa.com
ISSN: 1309-3452

Transcendental representation of Diophantine Equation $x^n + y^n = z^n$ to Generate At most All Pythagorean and Reciprocal Pythagorean Triples

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Received: 2022 March 25; Revised: 2022 April 28; Accepted: 2022 May 15

Abstract:
This paper revisits one of the Diophantine Equations $x^n + y^n = z^n$ And its transcendental representation $\left(\frac{z}{y}\right)^{\frac{n}{2}} = 1 + \frac{2}{x^2-1}$. By substituting **n=2**, the quadratic Diophantine equation satisfies Pythagorean Theorem. This paper introduced to a generation of all primitive and Nonprimitive Pythagorean triples for each positive integer 'x '. By substituting **n = -2** in the above Diophantine equation it satisfies the Reciprocal Pythagorean Theorem $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$. Also verified each Pythagorean Triple (a, b, c) is generates Reciprocal Pythagorean Triple (ac, bc, ab) .
Apply this corollary to generate Set of Reciprocal Pythagorean Triples RPT=.$\left\{(a\ , b, c): \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}\right\}$. Also, verified each p=(a, b, c) ∈ RPT, $h = \frac{ab}{c}$ **(Also, c =** $\frac{ab}{h}$**);** **h =** $\sqrt{c_a c_b}$; $a^2 = c_a . c$; $b^2 = c_b . c$; **c = 2R; r =** $\frac{a+b-c}{2}$;
$sin_{RPT}(\theta_1)= \frac{h}{a}$, $sin_{RPT}(\theta_2)= \frac{h}{b}$, $cos_{RPT}(\theta_1)= \frac{c_a}{a}$, $cos_{RPT}(\theta_2)= \frac{c_b}{b}$, $Tan_{RPT}(\theta_1)= \frac{h}{c_a}$, $Tan_{RPT}(\theta_2)= \frac{h}{c_b}$
with $\theta_1 + \theta_2 = \frac{\pi}{2}$**.**

Introduction

The solutions to the quadratic Diophantine equation P= =.$\{(x\ , y, z): x^2 + y^2 = z^2\}$ are given by the Pythagorean theorem. While several methods are exploded to generate Pythagorean triples with repetition, two of them are represented below. **Also, we are proposed one of the methods to generate the Pythagorean triples without repetition for each positive integer.**

Corollary 1: G = $\{(4m\ , 4m^2 - 1\ , 4m^2 + 1) : for\ some\ integer\ m\}$ is a subset of P.

Proof: Consider Reciprocal Summation of two consecutive odd number, which is equal to $\frac{a}{b}$ and c = b + 2 can become Pythagorean Triple (a , b, c) , which is $\frac{1}{2m-1} + \frac{1}{2m+1} = \frac{4m}{4m^2-1}$ implies the result $(4m\ , 4m^2 - 1\ , 4m^2 + 1)$ becomes to Pythagorean primitive Triple for m = 1,2 ,3,4,……,because of above triple can Satisfies statement of Pythagorean Theorem $(4m)^2 + (4m^2 - 1)^2 = (4m^2 + 1)^2$.

It proves that **S =** $\{(4m\ , 4m^2 - 1\ , 4m^2 + 1) : for\ some\ integer\ m\}$ is a subset of P.

Corollary 2: K = $\{\ (2n + 1, 2n(n + 1), 2n(n + 1) + 1))\ : for\ some\ integer\ n\}$ is a subset of P.

Proof: Consider Reciprocal Summation of two consecutive even numbers which is equal to $\frac{a}{b}$ and c = b + 1 can become Pythagorean Triple (a , b, c) , which is $\frac{1}{2n} + \frac{1}{2(n+1)} = \frac{2n+1}{2n(n+1)}$ implies the result

$(2n + 1, 2n(n + 1), 2n(n + 1) + 1))$ is becomes to Pythagorean Triple for n = 1, 2,3,4,….

It proves that **K =** $\{\ (2n + 1, 2n(n + 1), 2n(n + 1) + 1))\ : for\ some\ integer\ n\}$ is a subset of P.

Limitations on Existing Results to generate Pythagorean Triples:

1) Euclid's Method generates Pythagorean Triples only for odd integers.

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Volume 13, No. 2, 2022, p. 3600-3609
https://publishoa.com
ISSN: 1309-3452

2) No particular Method to generate all Pythagorean Primitive and Nonprimitive triples systematically for each even integer.

3) No particular Method to generate all Pythagorean Triples systematically.

Transcendental representation of Diophantine Equation $x^n + y^n = z^n$

From Reference [1], Reference [2] , Consider the Diophantine equation $x^n + y^n = z^n$.
$\left(x^{\frac{n}{2}}\right)^2 + \left(y^{\frac{n}{2}}\right)^2 = \left(z^{\frac{n}{2}}\right)^2$ implies **1**= $\left(\frac{z^{\frac{n}{2}}+y^{\frac{n}{2}}}{x^{\frac{n}{2}}}\right)\left(\frac{z^{\frac{n}{2}}-y^{\frac{n}{2}}}{x^{\frac{n}{2}}}\right)$. It states two factors are reciprocals to each other.
Choose two positive odd integers a and b with sufficiently **a must be multiple of b.**
Then $\frac{z^{\frac{n}{2}}+y^{\frac{n}{2}}}{x^{\frac{n}{2}}} = \frac{a}{b}$, $\frac{z^{\frac{n}{2}}-y^{\frac{n}{2}}}{x^{\frac{n}{2}}} = \frac{b}{a}$;
Clearly, $z^{\frac{n}{2}} = \frac{1}{2}\, x^{\frac{n}{2}} \left(\frac{a}{b}+\frac{b}{a}\right)$, $y^{\frac{n}{2}} = \frac{1}{2}\, x^{\frac{n}{2}} \left(\frac{a}{b}-\frac{b}{a}\right)$.
Let **x** = $\frac{a}{b}$ and simplify the above two equations, Obtain the Transcendental representation of above Diophantine equation is $\left(\frac{z}{y}\right)^{\frac{n}{2}} = \frac{a^2+b^2}{a^2-b^2} = 1 + \frac{2}{x^2-1}$.

Transcendental Representation of Pythagorean Theorem

Let **n = 2**, obtain Transcendental Representation of Pythagorean theorem $(x^2 + y^2 = z^2)$ is $\frac{z}{y} = 1 + \frac{2}{x^2-1}$ for 'x 'is an odd number. From Reference [1].[2] we can go to define subset A of P as follows
Case 1.1:

A = $\left\{(x, y, z): \frac{z}{y} = 1 + \frac{2}{x^2-1}\ if\ x\ is\ odd\ prime\ number\ or\ its\ powers\right\}$

TABLE 1.1: Choosing of a, b values for x is an odd prime number and its powers.

a	**b**	**X** = $\frac{a}{b}$	$\frac{z}{y} = 1 + \frac{2}{x^2-1}$	**y**	**z**	**Primitive triple**
3	1	3	$\frac{5}{4}$	4	5	(3,4,5)
5	1	5	$\frac{13}{12}$	12	13	(5,12,13)
9	3	3	$\frac{5}{4}$	4	5	(3,4,5)
15	3	5	$\frac{13}{12}$	12	13	(5,12,13)
21	3	7	$\frac{25}{24}$	24	25	(7,24,25)
15	5	3	$\frac{5}{4}$	4	5	(3,4,5)
27	3	9	$\frac{41}{40}$	40	41	(9,40,41)

Case 1.2: If x is a odd composite positive number or its power
Generalized Transcendental representation of the Pythagorean Theorem is $\frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{(2p-1)^2}\right)^2 - 1}$, p = 1,2,3,.. A number of prime factors of x. From Reference [1], [2] Define Nonempty Subset 'B' for Set of Pythagorean Triples is

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https://publishoa.com
ISSN: 1309-3452

B= $\left\{(x,y,z): \frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{(2p-1)^2}\right)^2 - 1} \, if\ x\ is\ odd\ composite\ \ OR\ its\ powers, for\ some\ p = 1,2,3..\right\}$

TABLE 1.2: Choose x is is odd composite number or its power

Odd composite number 'x'	Representation of Product of primes	Number of product of Primes	$\frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{(2p-1)^2}\right)^2 - 1}$
15	15= 3 x 5	2	(15,112,113),(15,8,17)
21	21= 3 x 7	2	(21,220,221),(21,20,29)
33	33= 3 x 11	2	(33,544,545),(33,56,65)
315	315= 3^2x 5 x 7	3	(315,49612,49613), [(35,612,613) for p=2], (315,1972,1997),(315,988,1037)
105	105= 3 x 5 x 7	3	(105,5512,5513),(105,608,617),(105,208,233)
55	55= 5 x 11	2	(55,1512,1513),(55,48,73)

Case 1.3: If $x = \langle 2^n \rangle, n = 1,2,3, \ldots ..$

Let $x = \langle 2^n \rangle$ then Transcendental representation $\frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{2p^2}\right)^2 - 1}$.

We can generate all Pythagorean triples up to x >$2p^2$ for some p = 1,2,…

If **(x,** $2p^2$) $\neq$ **2** and $\frac{x}{2p^2}$ = t(say) implies (t ,y, z) is becomes primitive triple.

It shows that **C**= $\left\{(x,y,z): \frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{2}\right)^2 - 1} \, if\ x\ is\ geometric\ power\ of\ 2\right\}$ is a Nonempty subset of P.

We can obtain Pythagorean triples by choosing x is geometric power of 2 . i.e x = 2,4,8,16,…

Table 1.3: Choosing of $x = \langle 2^n \rangle, n = 1,2,3, \ldots ..$

x is power of 2	Corresponding Primitive Pythagorean triples (for p = 1)
4,8,16,32,……..	(4,3,5),(8,15,17),(16,63,65),(32,255,257),…….

Case 1.4: If x is an even positive composite number or its powers

If x is an even composite number that is **x=** $2^p {p_1}^{a_1} {p_2}^{a_2} {p_3}^{a_3} \ldots \ldots {p_n}^{a_n}$ here $p_1, p_2, \ldots p_n$ *are odd primes and* $a_1, a_2, \ldots a_n$ are their respective powers.

From Reference [3] ,[4] we can to to define a Non empty subset of P

D= $\left\{(x,y,z): \frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{2p^2}\right)^2 - 1} \, if x\ is\ even\ composite\ and\ its\ power\ , for\ some\ p = 1,2,3 \ldots\right\}$

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https://publishoa.com
ISSN: 1309-3452

TABLE 1.4: x is a even composite positive number and x>$2p^2$.

x	$\frac{x}{2p^2}$ **Improper (x,** $2p^2) \neq 2$ **Choose the value of p**	$\frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{2p^2}\right)^2 - 1}$ **For primitive**	$\frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{2p^2}\right)^2 - 1}$ **For Non primitive**
6	1	(3,4,5)	(6,8,10)
10	1	(5,12,13)	(10,24,25)
12	1,2	(12,35,37), (12,5,13)	-
14	1	(7,24,25)	(14,48,50)

Case 1.5: $S_1 = \left\{\left(x, \left(\frac{x}{2}\right)^2 - 1, \left(\frac{x}{2}\right)^2 + 1\right) : x \text{ is even number}\right\}$ is Subset of P.
Proof: From Reference [1] ,[2] above statement is follows. For generalization, replace x is an even number by 2 m for some m= 2,3,4,…., we obtain Pythagorean Triple $S_1 = \{(2m, (m)^2 - 1, (m)^2 + 1) : m = 2,3,4, \ldots.\}$

Table 1.5: x is an even positive number

m	**X=2m**	**y=** $(m)^2 - 1$	**z=** $(m)^2 + 1$	**Pythagorean Triple**	$\left(x, \left(\frac{x}{2}\right)^2 - 1, \left(\frac{x}{2}\right)^2 + 1\right)$
2	4	3	5	(4,3,5)	(4,3,5)
3	6	8	10	(6,8,10)	(6,8,10)
4	8	15	17	(8,15,17)	(8,15,17)
5	10	24	26	(10,24,26)	(10,24,26)

Case 1.6: **S**$_2$ = $\left\{\left(x, \frac{x^2-1}{2}, \frac{x^2+1}{2}\right) : x \text{ is odd number}\right\}$ **is a subset of P.**
Proof : From Reference [1], [2] above statement is follows.

For generalization we can replace x by 2 m + 1 for some m= 1,2,3,……

S$_2$ = $\{(2m + 1, 2m^2 + 2m, 2m^2 + 2m + 1) : m = 1,2,3, \ldots.\}$

Table 1.6: x is an odd positive integer

m	**x=** $2m + 1$	**y=** $2m^2 + 2m$	**z=** $m^2 + 2m + 1$	**Pythagorean Triple**	$\left(x, \frac{x^2 - 1}{2}, \frac{x^2 + 1}{2}\right)$
1	3	4	5	(3,4,5)	(3,4,5)
2	5	12	13	(5,12,13)	(5,12,13)
3	7	24	25	(7,24,25)	(7,24,25)
4	9	40	41	(9,40,41)	(9,40,41)
5	11	60	61	(11,60,61)	(11,60,61)

Transcendental Representation of Diophantine Equation $x^n + y^n = z^n$ to Generating Set of Reciprocal Pythagorean Triples

Reciprocal Pythagorean theorem relates the two legs a, b to the altitude h is defined as follows $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{h^2}$ with c = $\frac{ab}{h}$.
Now we can introduce following method to generate Set of Reciprocal Pythagorean Triples.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3600-3609
https://publishoa.com
ISSN: 1309-3452

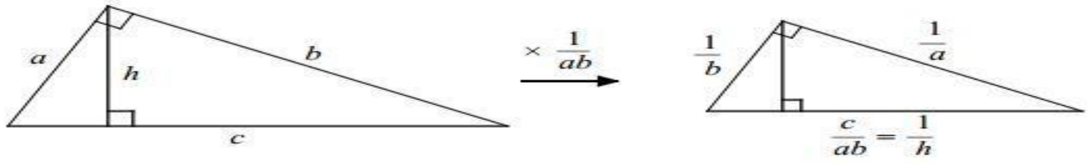

Figure 1: Reciprocal Pythagorean Theorem

From Reference [3] ,(x ,y, z) becomes to Reciprocal triples for some integer p, q where

x= $(p^2+q^2)(p^2-q^2)$ z = $2pq(p^2-q^2)$, y = $2pq(p^2+q^2)$

Also verified each Pythagorean Triple (x, y, z) generates Reciprocal Pythagorean Triple $(xz, yz, xy)'$.

because of $\frac{1}{(xz)^2}+\frac{1}{(yz)^2} = \frac{y^2+x^2}{(xyz)^2} = \frac{z^2}{(xyz)^2} = \frac{1}{(xy)^2}$. It follows that, we can evaluate Reciprocal Pythagorean triples with using of Transcendental representation of Pythagorean Triples from above subsets A, B, C, D of Set of Pythagorean triples. It follows A^1 , B^1 , C^1 , D^1 are subsets of RPT.

Table 1.8: $A^1 = \{(xz, yz, xy) : \frac{z}{y} = 1 + \frac{2}{x^2-1}\ and\ \ x\ is\ an\ odd\ prime\ number\ or\ its\ power\}$

x is odd prime numbers or their powers	Corresponding Primitive Pythagorean triples	Corresponding Reciprocal Pythagorean Triples
(3,5,7,11,13,17,…) and $(3^2, 3^3, 3^4, \dots\dots\ 5^2, 5^3, 5^4, \dots\dots 7^2$	{(3,4,5),(5,12,13),(7,24,25),(11,60,61),(13,84,85),(17,144,145)……and (9,40,41),(27,364,365),(81,3280,3281),……. …....(25,312,313),(125,7812,7813),(625,195312,195313),………..… (49,1200,1201),(343,58824,58825),…………. }	(15, 20, 12), (65, 156, 60), (175, 600, 168), (671, 3660, 660)…….

Table 1.9 : $B^1 = \{(xz, yz, xy) : \frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{(2p-1)^2}\right)^2 - 1}\ and\ \ x\ is\ an\ odd\ composite\ number\ or\ its\ power\}$

x	Number of Prime factors	Pythagorean triples are obtained by replacing p= 1,2,3,..in $\frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{(2p-1)^2}\right)^2 - 1}$	Corresponding Reciprocal Pythagorean Triples
15 = 3x5	2	(15,112,113),(15,8,17)	(1695,12656,1680), (255,136,120)
21=3x7	2	(21,220,221),(21,20,29)	(4641,48620,4620), (609,580,420)
33=3x11	2	(33,544,545),(33,56,65)	(17985,296480,17952), (2145,3640,1848)

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https://publishoa.com
ISSN: 1309-3452

315= 3^2x 5 x 7	3	(315,49612,49613), [(35,612,613) for p=2], (315,1972,1997),(315,988,1037)	(15628095,2461400156, 15627780), (21455,375156,21420), (629055,3938084,621180), (326655,1024556,311220)
105= 3 x 5 x 7	3	(105,5512,5513),(105,608,617), (105,208,233)	(578865,30387656,578760), (64785,375136,63840), (24465,48464,21840)
55= 5 x 11	2	(55,1512,1513),(55,48,73)	(83215,2287656,83160), (4015,3504,2640)

Table 1.10: $C^1 = \{(xz, yz, xy) : \frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{2}\right)^2 - 1}$ $and\ \ x\ is\ only\ power\ of\ 2\}$

x is power of 2	Primitive Pythagorean triples (for p = 1)	Corresponding Reciprocal Pythagorean Triples
4,8,16,32,……..	(4,3,5),(8,15,17),(16,63,65),(32,255,257),…….	(20, 15, 12), (136, 255, 120), (1040, 4095, 1008),…

TABLE 1.11: $D^1 = \{(xz, yz, xy) : \frac{z}{y} = 1 + \frac{2}{\left(\frac{x}{2p^2}\right)^2 - 1}$ $and\ \ x\ is\ an\ even\ composite\ number\ or\ its\ power\ \}$

Even Composite Number	p	primitive Triples	Nonprimitive triple	Corresponding Reciprocal Pythagorean Triples
6	1	(3,4,5)	(6,8,10)	(15,20,12)(60,80,48)
10	1	(5,12,13)	(10,24,25)	(65,156,60),(250,600,240)
12	1,2	(12,35,37) , (12,5,13)	-	(444, 1295, 420), (156,65,60)
14	1	(7,24,25)	(14,48,50)	(175,600,168),(700,2400,720)
18	1	(9,40,41)	(18,80,82)	(369,1640,360), (1476,6560,1440)
20	1,2	(20,99,101) ,(20,21,29)	-	(2020,9999,1980), (580,609,420)
22	1	(11,60,61)	(22,120,122)	(671,3660,660), (2684,14640,2640)
24	1,2,3	(24,143,145),(3,4,5),(24,7,25)	-	(3480,20735,3432),(15,20,12), (600,175,168)

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https://publishoa.com
ISSN: 1309-3452

26	1	(13,84,85)	(26,168,170)	(1105,7140,1092), (4420,28560,4368)
28	1,2	(28,195,197),(28,45,53)	-	(5516,38415,5460), (1484,2385,1260)

Case 1.7: By choosing of 'x ' is an odd integer (with using of Subset S_2).

$Rpt_o = \left\{\left(\frac{x(x^2+1)}{2}, \frac{(x^2-1)(x^2+1)}{4}, \frac{x(x^2-1)}{2}\right) : x \text{ is an odd number greater than } 1\right\}$

For generalization odd numbers is denoted by 2 m + 1 for some m = 1, 2 , 3,…..

It follows that Reciprocal Pythagorean Triples are represented by

$Rpt_o = \left\{\left(\frac{(2m+1)((2m+1)^2+1)}{2}, \frac{((2m+1)^4-1)}{4}, \frac{(2m+1)((2m+1)^2-1)}{2}\right) : m = 1,2,3, \dots\dots\right\}$

Table 1.12: x is a odd positive integer

m	x	$X_1 = \frac{(2m+1)((2m+1)^2+1)}{2}$	$Y_1 = \frac{((2m+1)^4-1)}{4}$	$Z_1 = \frac{(2m+1)((2m+1)^2-1)}{2}$	Reciprocal Pythagorean Triple	$\left(\frac{x(x^2+1)}{2}, \frac{(x^2-1}{}\right.$
1	3	15	20	12	(15,20,12)	(15,20,12)
2	5	65	156	60	(65, 156, 60)	(65, 156, 60)
3	7	175	600	168	(175, 600, 168)	(175, 600, 168)
4	9	369	1640	360	(369, 1640, 360)	(369, 1640, 360)
5	11	671	3660	660	(671, 3660, 660)	(671, 3660, 660)

Case 1.8: By choosing of x is an even integer (With using of Subset S_1).

$Rpt_e = \left\{\left(x\left(\left(\frac{x}{2}\right)^2+1\right)\right), \left(\left(\frac{x}{2}\right)^2-1\right)\left(\left(\frac{x}{2}\right)^2+1\right), x\left(\left(\frac{x}{2}\right)^2-1\right) : x \text{ is an even number greater than } 2\right\}$

For generalization even numbers are denoted by 2 m for some m= 2,3,…..

It follows that Reciprocal Pythagorean Triples are represented by

$Rpt_e = \{(2m(m^2+1)), ((m)^4-1), 2m((m)^2-1) : m = 2,3, \dots\dots\dots\}$

Table 1.13: x is an even positive integer

m	x	$X_1 = (2m(m^2+1)),$	$Y_1 = ((m)^4-1)$	$Z_1 = 2m((m)^2-1)$	Reciprocal Pythagorean Triple	$\left(x\left(\left(\frac{x}{2}\right)^2+1\right)\right), \left(\left(\frac{x}{2}\right)^2-1\right)\left(\left(\frac{x}{2}\right)^2+1\right), x\left(\left(\frac{x}{2}\right)^2-1\right)$
2	4	20	15	12	(20, 15, 12)	(20, 15, 12)
3	6	60	80	48	(60, 80, 48)	(60, 80, 48)
4	8	136	255	120	(136, 255, 120)	(136, 255, 120)
5	10	260	624	240	(260, 624, 240)	(260, 624, 240)
6	12	444	1295	420	(444, 1295, 420)	(444, 1295, 420)

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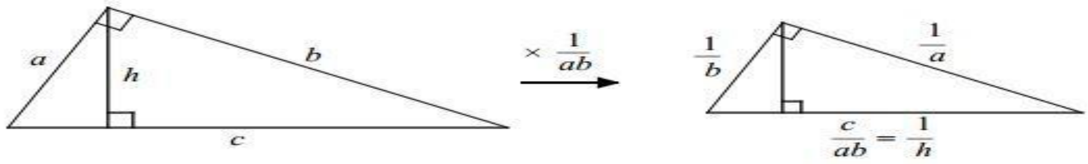
Volume 13, No. 2, 2022, p. 3600-3609
https://publishoa.com
ISSN: 1309-3452

Some Trigonometric relations are satisfied by the elements of Set of Reciprocal Pythagorean Triples



Clearly (a , b, c) is satisfies Pythagorean Theorem $a^2 + b^2 = c^2$.
Area of Δ ABC = $\frac{1}{2} * a * b = \frac{1}{2} * h * c$ follows that $h = \frac{ab}{c}$.
Hence $h^2 = \frac{(ab)^2}{c^2} = \frac{(ab)^2}{a^2+b^2}$, which implies that $\frac{1}{h^2} = \frac{a^2+b^2}{a^2b^2} = \frac{1}{a^2} + \frac{1}{b^2}$, which follows that (a ,b, h) is becomes to Reciprocal Pythagorean Triple.
the elements of RPT are satisfies following conditions.

i) $h = \frac{ab}{c}$ (Also, c = $\frac{ab}{h}$)
ii) h = $\sqrt{c_a c_b}$
iii) $a^2 = c_a . c$
iv) $b^2 = c_b . c$
v) c = 2R
vi) r = $\frac{a+b-c}{2}$

Table 1.14: Above conditions are verified by Some elements of RPT

Reciprocal Pythagorean Triple (a , b ,h)	C= $\frac{ab}{h}$	Pythagorean Triple (a , b , c)	$c_a = \frac{a^2}{c}$	$c_b = \frac{b^2}{c}$	h = $\sqrt{c_a c_b}$	R=$\frac{c}{2}$	r= $\frac{a+b-c}{2}$
(20,15,12)	25	(20,15,25)	16	9	12	12.5	11.5
(60, 80, 48)	100	(60,80,100)	36	64	48	50	46
(65,156,60)	169	(65,156,169)	25	144	60	84.5	80.5
(136, 255, 120)	289	(136,255,289)	64	225	120	144.5	135.5
(260, 624, 240)	676	(260,624,676)	100	576	240	338	322
(444, 1295, 420)	1369	(444,1295,1369)	144	1225	420	684.5	659.5

Also, we can go to verify Trigonometric functions for the elements of RPT **with** $\theta_1 + \theta_2 = \frac{\pi}{2}$ as follows
$sin_{RPT}(\theta_1) = \frac{h}{a}$, $sin_{RPT}(\theta_2) = \frac{h}{b}$, $cos_{RPT}(\theta_1) = \frac{c_a}{a}$, $cos_{RPT}(\theta_2) = \frac{c_b}{b}$, $Tan_{RPT}(\theta_1) = \frac{h}{c_a}$, $Tan_{RPT}(\theta_2) = \frac{h}{c_b}$

Lemma 1: $sin_{RPT}(\theta_1 + \theta_2)$= sin $(\frac{\pi}{2}) = 1$
Proof:$sin_{RPT}(\theta_1 + \theta_2) = sin_{RPT}(\theta_1)cos_{RPT}(\theta_2) + cos_{RPT}(\theta_1)sin_{RPT}(\theta_2)$
$= \frac{h}{a} * \frac{c_b}{b} + \frac{c_a}{a} * \frac{h}{b} = \frac{c_b}{c} + \frac{c_a}{c} = \frac{c}{c} = 1$

Lemma 2: $Cos_{RPT}(\theta_1 + \theta_2)$= cos $(\frac{\pi}{2}) = 0$
Proof:$Cos_{RPT}(\theta_1 + \theta_2) = cos_{RPT}(\theta_1)cos_{RPT}(\theta_2) - sin_{RPT}(\theta_1)sin_{RPT}(\theta_2)$
$= \frac{c_a}{a} * \frac{c_b}{b} - \frac{h}{a} * \frac{h}{b} = \frac{c_a c_b - h^2}{ab} = 0$ (from condition (ii)) .It will clearly indicate that $\theta_1 + \theta_2 = \frac{\pi}{2}$.

Lemma 3: $Sin^2{}_{RPT}(\theta_1) + Sin^2{}_{RPT}(\theta_2) = 1$
Proof: $Sin^2{}_{RPT}(\theta_1) + Sin^2{}_{RPT}(\theta_2) = \left(\frac{h}{a}\right)^2 + \left(\frac{h}{b}\right)^2 = h^2\left(\frac{1}{a^2} + \frac{1}{b^2}\right) = h^2 * \frac{1}{h^2} = 1$

Lemma 4: $Sin^2{}_{RPT}(\theta_1) + Cos^2{}_{RPT}(\theta_1) = 1$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3600-3609
https://publishoa.com
ISSN: 1309-3452

Proof: $Sin^2{}_{RPT}(\theta_1) + Cos^2{}_{RPT}(\theta_1) = \left(\frac{h}{a}\right)^2 + \left(\frac{c_a}{a}\right)^2 = \frac{h^2+(c_a)^2}{a^2} = \frac{c_a c_b+(c_a)^2}{a^2}$
$= \frac{c_a c_b+(c_a)^2}{c_a\, c} = \frac{c_a c_b+(c_a)^2}{c_a\,(c_a+c_b)} = 1$

Lemma 5: $Sin^2{}_{RPT}(\theta_2) + Cos^2{}_{RPT}(\theta_2) = 1$
Proof: $Sin^2{}_{RPT}(\theta_2) + Cos^2{}_{RPT}(\theta_2) = \left(\frac{h}{b}\right)^2 + \left(\frac{c_b}{b}\right)^2 = \frac{h^2+(c_b)^2}{b^2} = \frac{c_a c_b+(c_b)^2}{b^2}$
$= \frac{c_a c_b+(c_b)^2}{c_b\, c} = \frac{c_a c_b+(c_a)^2}{c_a\,(c_a+c_b)} = 1$
Lemma 6: $Sec^2{}_{RPT}(\theta_1) - Tan^2{}_{RPT}(\theta_1) = 1$
Proof: $Sec^2{}_{RPT}(\theta_1) - Tan^2{}_{RPT}(\theta_1) = \left(\frac{a}{c_a}\right)^2 - \left(\frac{h}{c_a}\right)^2 = \frac{a^2 - h^2}{{c_a}^2} = \frac{c_a.c - c_a c_b}{{c_a}^2}$
$= \frac{c_a.(c-c_b)}{{c_a}^2} = \frac{{c_a}^2}{{c_a}^2} = 1$
Lemma 7: $Sec^2{}_{RPT}(\theta_2) - Tan^2{}_{RPT}(\theta_2) = 1$
Proof: $Sec^2{}_{RPT}(\theta_2) - Tan^2{}_{RPT}(\theta_2) = \left(\frac{b}{c_b}\right)^2 - \left(\frac{h}{c_b}\right)^2 = \frac{b^2 - h^2}{{c_b}^2} = \frac{c_b.c - c_a c_b}{{c_b}^2}$
$= \frac{c_b.(c-c_a)}{{c_b}^2} = \frac{{c_b}^2}{{c_b}^2} = 1$
Lemma 8: $Cosec^2{}_{RPT}(\theta_1) - Cot^2{}_{RPT}(\theta_1) = 1$
Proof: consider $Cosec^2{}_{RPT}(\theta_1) - Cot^2{}_{RPT}(\theta_1) = \left(\frac{a}{h}\right)^2 - \left(\frac{c_a}{h}\right)^2 = \frac{a^2 - {c_a}^2}{h^2} = \frac{c_a.c - {c_a}^2}{h^2}$
$= \frac{c_a.(c-c_a)}{c_a c_b} = \frac{c_a c_b}{c_a c_b} = 1$
Lemma 9: $Cosec^2{}_{RPT}(\theta_2) - Cot^2{}_{RPT}(\theta_2) = 1$
Proof: Consider $Cosec^2{}_{RPT}(\theta_2) - Cot^2{}_{RPT}(\theta_2) = \left(\frac{b}{h}\right)^2 - \left(\frac{c_b}{h}\right)^2 = \frac{b^2 - {c_b}^2}{h^2} = \frac{c_b.c - {c_b}^2}{h^2}$
$= \frac{c_b.(c-c_b)}{c_a c_b} = \frac{c_a c_b}{c_a c_b} = 1$

In this way we can verify all the trigonometric Ratios and compound angles for the elements of RPT.

Conclusion :

This paper revisits one of the Diophantine Equations $x^n + y^n = z^n$ And its transcendental representation $\left(\frac{z}{y}\right)^{\frac{n}{2}} = 1 + \frac{2}{x^2-1}$. By substituting **n=2**, the quadratic Diophantine equation satisfies Pythagorean Theorem. This paper introduced to a generation of all primitive and Nonprimitive Pythagorean triples for each positive integer ‘x ‘. By substituting **n = -2** in the above Diophantine equation it satisfies the Reciprocal Pythagorean Theorem $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$. Also verified each Pythagorean Triple (a, b, c) is generates Reciprocal Pythagorean Triple (ac, bc, ab) .
Apply this corollary to generate Set of Reciprocal Pythagorean Triples RPT=.$\left\{(a\,,b,c): \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}\right\}$. Also, verified each p=(a, b, c) ∈ RPT, $h = \frac{ab}{c}$ **(Also, c =** $\frac{ab}{h}$ **); h =** $\sqrt{c_a c_b}$ **;** $a^2 = c_a.c$ **;** $b^2 = c_b.c$; **c = 2R; r =** $\frac{a+b-c}{2}$;
$sin_{RPT}(\theta_1) = \frac{h}{a}$, $sin_{RPT}(\theta_2) = \frac{h}{b}$, $cos_{RPT}(\theta_1) = \frac{c_a}{a}$, $cos_{RPT}(\theta_2) = \frac{c_b}{b}$, $Tan_{RPT}(\theta_1) = \frac{h}{c_a}$, $Tan_{RPT}(\theta_2) = \frac{h}{c_b}$
with $\theta_1 + \theta_2 = \frac{\pi}{2}$**.**

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