

Extremally Disconnectedness via a Soft Minimal Structure

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Abstract: We interpose an alteration of soft extremally disconnected spaces (SES) which is known as soft minimal-extremally disconnected (SmES) and attain numerous properties of SmES.

Keywords: Soft m-structure, Soft extremally disconnected spaces, soft m-extremally disconnected.

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1. INTRODUCTION

In this research paper, we introduced a soft topological space (briefly STS) $(P, \varsigma, \check{T})$ over P with a soft m-structure $m(P, \check{T})$ over P to be SMS if m-closure (briefly m-CL) of soft open (O, \check{T}) is for each SOPS (O, \check{T}) of $(P, \varsigma, \check{T})$. We defined numerous characterizations of soft m-extremally disconnected spaces. we will show that minimal extremal disconnectedness and soft extremal disconnectedness are independent by giving simple examples. Although, if soft minimal structure $m(P, \check{T}) = SO(P, \varsigma, \check{T})$ or $SPO(P, \varsigma, \check{T})$, then the mixed space $(P, \varsigma, m(P, \check{T}))$ is minimal-extremally disconnected for each $(P, \varsigma, \check{T})$. If $m(P, \check{T}) = \alpha(P, \varsigma, \check{T})$, $PO(P, \varsigma, \check{T})$ or $bo(P, \varsigma, \check{T})$, then $(P, \varsigma, m(P, \check{T}))$ is soft minimal-extremally disconnected for each $(P, \varsigma, \check{T})$. Recently papers have studied few innovative classes of soft sets (briefly ST) through soft minimal-structures (briefly SmS).

2. PRELIMINARIES:

Let Z is initial universe set and set \check{T} is parameters, $P(Z)$ denote the power set of Z . All the way through this paper soft set, SOPS soft topological space, soft minimal structure and soft m-extremally disconnected denotes ST, SOPS, STS, SmS and SmES respectively.

Definition 2.1 [17]: Given mapping $S: \check{T} \rightarrow P(Z)$. A pair (S, \check{T}) is said to be ST over Z .

So, (S, \check{T}) is parameterized family over Z . For all $t \in \check{T}$, $S(t)$ is the set of t -approximate members of (S, \check{T}) .

Definition 2.2 [22]: A soft family $\varsigma \in S(P, \check{T})$ is called soft topology over P if:

1. $\check{\emptyset}, \check{P} \in \varsigma$.
2. ς is closed under union of any number of ST and intersection of any two ST.

Structure $(P, \varsigma, \check{T})$ is called a S.T.S. over P .

Definition 2.3 S.T. (M, \check{T}) of a S.T.S $(P, \varsigma, \check{T})$ is called:

[a] If $(M, \check{T}) = \text{Int}(\text{Cl}(M, \check{T}))$ then soft regular open [7];

[b] if $(M, \check{T}) \subseteq \text{Int}(\text{Cl}(\text{Int}(M, \check{T})))$ then soft α -open [5];

[c] if $(M, \check{T}) \subseteq \text{Cl}(\text{Int}(M, \check{T}))$ then soft semiopen [15];

[d] if $(M, \tilde{T}) \subseteq \text{Int}(\text{Cl}(M, \tilde{T}))$ then soft preopen [3];

[e] if $(M, \tilde{T}) \subseteq \text{Int}(\text{Cl}(M, \tilde{T})) \cup \text{Cl}(\text{Int}(M, \tilde{T}))$ then soft b-open [2].

[f] if $(M, \tilde{T}) \subseteq \text{Cl}(\text{Int}(\text{Cl}(M, \tilde{T})))$ then soft β -open [4]

Family of each soft regular open (resp. soft- α -open, soft-pre-open, soft-semi-open, soft- β -open, soft-b-open) sets of P will be denoted by $\text{SRO}(P, \tilde{T})$ (resp. $\text{S}\alpha\text{O}(P, \tilde{T})$, $\text{SPO}(P, \tilde{T})$, $\text{SSO}(P, \tilde{T})$, $\text{S}\beta\text{O}(P, \tilde{T})$, $\text{SbO}(P, \tilde{T})$).

Remark 2.4 [2]: The concepts of softsemi open and soft preopen sets are independent.

Definition 2.5 [21]: $m(P, \tilde{T})$ of $S(P, \tilde{T})$ over P is called a soft minimal structure (SmS) on P if $\phi \in m(P, \tilde{T})$ and $\check{P} \in (P, \tilde{T})$.

Remark 2.6 [21]: Let (P, ζ, \tilde{T}) be STS. Then the families ζ , $\text{SO}(P, \tilde{T})$, $\text{SPO}(P, \tilde{T})$, $\text{S}\alpha\text{O}(P, \tilde{T})$, $\text{S}\beta\text{O}(P, \tilde{T})$, $\text{SbO}(P, \tilde{T})$ are all SmS on P .

Definition 2.7 [21]: A sms $m(P, \tilde{T})$ over P is called to include the property **B** if the union of any family of $ST \in m(P, \tilde{T})$.

A STS (W, ζ, \tilde{T}) with a SMS $m(P, \tilde{T})$ on P is called a soft mixed space and is denoted by $(P, \zeta, m(P, \tilde{T}))$.

Definition 2.8: A $ST(P, \tilde{T})$ of a soft mixed space $(P, \zeta, m(P, \tilde{T}))$ is said to be:

- (1) $m(P, \tilde{T})$ dense if $m\text{Cl}(P, \tilde{T}) = P$.
- (2) soft $m(P, \tilde{T})$ nowhere dense if $\text{Int}(m\text{Cl}(P, \tilde{T})) = \emptyset$.
- (3) soft α - $m(P, \tilde{T})$ open if $(P, \tilde{T}) \subseteq \text{Int}(m\text{Cl}(\text{Int}(P, \tilde{T})))$.
- (4) soft semi- $m(P, \tilde{T})$ open if $(P, \tilde{T}) \subseteq m\text{Cl}(\text{Int}(P, \tilde{T}))$.
- (5) soft pre- $m(P, \tilde{T})$ open if $(P, \tilde{T}) \subseteq \text{Int}(m\text{Cl}(P, \tilde{T}))$.
- (6) soft β - $m(P, \tilde{T})$ open if $(P, \tilde{T}) \subseteq \text{Cl}(\text{Int}(m\text{Cl}(P, \tilde{T})))$.
- (7) soft semi- $m(P, \tilde{T})$ *-open if $(P, \tilde{T}) \subseteq \text{Cl}(m\text{Int}(P, \tilde{T}))$.
- (8) soft strongly- $\beta m(P, \tilde{T})$ open if $(P, \tilde{T}) \subseteq m\text{Cl}(\text{Int}(m\text{Cl}(P, \tilde{T})))$.

3. PROPERTIES of SOFT m-EXTREMALLY DISCONNECTED SPACES

In this section, soft extremally disconnected spaces and soft m-extremally disconnected spaces means SEDS, SmEDS respectively.

Definition 3.1: $(P, \zeta, m(P, \tilde{T}))$ is said to be soft SmEDS (resp. m-hyperconnected) if $m\text{Cl}(P, \tilde{T}) \in \zeta$ (resp. $m\text{Cl}(P, \tilde{T}) = P$) $\forall (P, \tilde{T}) \in \zeta$.

Example 3.2: $P = \{a, b, c\}$, $\tilde{T} = \{t1, t2\}$ & $(A, \tilde{T}) = (t1, \{a\}), (t2, \{a\}), (B, \tilde{T}) = (t1, \{b\}), (t2, \{b\}), (C, \tilde{T}) = (t1, \{a, b\}), (t2, \{a, b\}), (D, \tilde{T}) = (t1, \{c\}), (t2, \{c\})$ be ST.

Let $\zeta = \{\emptyset, (A, \tilde{T}), (B, \tilde{T}), (C, \tilde{T}), P\}$, $m(P, \tilde{T}) = \{\emptyset, (A, \tilde{T}), (B, \tilde{T}), (D, \tilde{T}), \check{P}\}$. Then STS (P, ζ, \tilde{T}) is not SEDS and soft mixed space $(P, \zeta, m(P, \tilde{T}))$ is SmEDS.

Example 3.3: Let $P = \{a, b, c\}$, $\tilde{T} = \{t1, t2\}$ & $(A, \tilde{T}) = (t1, \{a\}), (t2, \{a\})$, $(B, \tilde{T}) = (t1, \{b, c\}), (t2, \{b, c\})$, $(C, \tilde{T}) = (t1, \{b\}), (t2, \{b\}), (D, \tilde{T}) = (t1, \{a, c\}), (t2, \{a, c\})$ be ST. Let $\zeta = \{\emptyset, (A, \tilde{T}), (B, \tilde{T}), P\}$, $m(P, \tilde{T}) = \{\emptyset, (A, \tilde{T}), (C, \tilde{T}), (D, \tilde{T}), \check{P}\}$. Then STS (P, ζ, \tilde{T}) is SEDS and soft mixed space $(P, \zeta, m(P, \tilde{T}))$ is not SmEDS.

Let $\Omega \subseteq P(W)$ is called a generalized soft topology (i.e., GST) [8] on W if $\emptyset \in \Omega$, $G_i \in \Omega$ for $i \in I$ $\Rightarrow \bigcup_{i \in I} G_i \in \Omega$. We say structure (W, Ω, \tilde{T}) a soft generalized topological space (brief SGTS) on W .

For a SGTS (W, Ω, \tilde{T}) , elements of Ω are said to be Ω -open sets and complements are said to be Ω -closed sets. For $(P, \tilde{T}) \subseteq W$, symbolically by $(C_\Omega, (P, \tilde{T}))$ the intersection of each soft Ω -closed sets $\subseteq (P, \tilde{T})$, $m(W, \tilde{T}) = \Omega$, where $m(W, \tilde{T})$ including property B, as special case of Definition 3.1.

We defined the subsequent definition:

Definition3.4: Let (W, Ω, \tilde{T}) be a SGTS and $G \subseteq W$.

(1) G is called soft Ω -dense if $c_\Omega(G) = W$,

(2) (W, Ω, \tilde{T}) is said soft hyperconnected if G is soft Ω -dense \forall soft Ω -open set

$G \neq \emptyset$ of (W, Ω, \tilde{T}) .

Lemma3.5: Let $(W, \Omega, m(W, \tilde{T}))$ is soft mixed space. Then, subsequent properties hold: (1) If W is soft m -hyperconnected, then W is SmEDS.

(2) If $m(W, \tilde{T}) = SO(W, \tilde{T})$ or $SPO(W, \tilde{T})$, then $(W, \zeta, m(W, \tilde{T}))$ is SmEDS.

(3) Let (W, Ω, \tilde{T}) be SEDS. If $m(W, \tilde{T}) = \alpha(W, \tilde{T})$, $PO(W, \tilde{T})$ or $BO(W, \tilde{T})$, then $(W, \zeta, m(W, \tilde{T}))$ is SmEDS.

Proof. (1) obviously. (2) It is known in [3] that $sCl(W, \tilde{T}) = (W, \tilde{T}) \cap (Int(Cl(W, \tilde{T})))$ and $spCl(W, \tilde{T}) = (W, \tilde{T}) \cap (Int(Cl(Int(W, \tilde{T}))) \forall ST(W, \tilde{T})$ of W . Therefore, $sCl(V, \tilde{T})$ and $spCl(V, \tilde{T})$ are soft open for every soft open set (V, \tilde{T}) and consequently $(W, \Omega, m(W, \tilde{T}))$ is SmEDS for $m(W, \tilde{T}) = SO(W, \tilde{T})$ or $SPO(W, \tilde{T})$. (3) Since $\alpha(W, \tilde{T}) = (W, \tilde{T}) \cup Cl(Int(Cl(W, \tilde{T})))$, $pCl(W, \tilde{T}) = (W, \tilde{T}) \cup Cl(Int(W, \tilde{T}))$ and $bCl(W, \tilde{T}) = sCl(W, \tilde{T}) \cap pCl(W, \tilde{T}) \forall$ soft set (W, \tilde{T}) of W . Therefore, $\alpha(V, \tilde{T})$, $pCl(V, \tilde{T})$ and $bCl(V, \tilde{T})$ are soft open $\forall (V, \tilde{T})$ of a SEDS (W, Ω, \tilde{T}) and consequently $(W, \Omega, m(W, \tilde{T}))$ is SmEDS for $m(W, \tilde{T}) = \alpha(W, \tilde{T})$, $PO(W, \tilde{T})$, or $BO(W, \tilde{T})$. The subsequent eg. shows that converse of every statement of Lemma3.5 are not true.

Example 3.6: Let soft mixed space $(W, \Omega, m(W, \tilde{T}))$, where $W = \{a, b, c, d\}$, $\tilde{T} = \{t1, t2\}$ and $\Omega = \{\emptyset, (t1, \{a\}), (t2, \{a\}), (t1, \{a, d\}), (t2, \{a, d\}), (t1, \{b, d\}), (t2, \{b, d\}), (t1, \{c, d\}), (t2, \{c, d\}), (t1, \{a, b, d\}), (t2, \{a, b, d\}), (t1, \{a, c, d\}), (t2, \{a, c, d\}), (t1, \{b, c, d\}), (t2, \{b, c, d\}), \tilde{W}\}$. If $(P, \tilde{T}) = (t1, \{a\}), (t2, \{a\})$, then (P, \tilde{T}) is soft open and $mCl(P, \tilde{T}) = (t1, \{a\}), (t2, \{a\}) \neq W$. Hence $(W, \Omega, m(W, \tilde{T}))$ is soft not m -hyperconnected. Since soft closure- $m(W, \tilde{T})$ of each soft open is open set, W is SmEDS. Moreover, since $(t1, \{b, d\}), (t2, \{b, d\})$ is not soft β -open, $m(W, \tilde{T})$ is not $SPO(W, \tilde{T})$.

Theorem3.7: Let $(W, \Omega, m(W, \tilde{T}))$ is soft mixed space, the subsequent properties are equivalent:

(1) W is SmEDS;

(2) $m-Int(P, \tilde{T})$ is soft closed \forall soft closed set (P, \tilde{T}) of W ;

(3) $m-Cl(Int(P, \tilde{T})) \subseteq Int(mCl(P, \tilde{T})) \forall$ soft set (P, \tilde{T}) of W ;

(4) Every soft semi- $m(P, \tilde{T})$ open set is soft pre- $m(P, \tilde{T})$ open;

(5) $mCl(P, \tilde{T}) \in \Omega \forall$ strongly-soft β - $m(P, \tilde{T})$ open set (P, \tilde{T}) ;

(6) Every strongly-soft β - $m(P, \tilde{T})$ open set is soft pre- $m(P, \tilde{T})$ open;

(7) (P, \tilde{T}) is soft α - $m(P, \tilde{T})$ open \leftrightarrow it is soft-semi- $m(P, \tilde{T})$ -open $\forall (P, \tilde{T}) \subseteq W$.

Proof. (1) \rightarrow (2): Let (P, \tilde{T}) be a soft closed set in W . Then $W - (P, \tilde{T})$ is open. By (1)

$mCl(W - (P, \tilde{T})) = W - mInt(P, \tilde{T})$ is soft open. Thus, $mInt(P, \tilde{T})$ is soft closed.

(2) \rightarrow (3): Let (P, \tilde{T}) be any soft set of W . So, $W - \text{Int}(P, \tilde{T})$ is closed in W and by (2) $m\text{Int}[W - \text{Int}(P, \tilde{T})]$ is soft closed in W . Therefore, $m\text{Cl}(\text{Int}(P, \tilde{T}))$ is soft open in W and consequently $m\text{Cl}(\text{Int}(P, \tilde{T})) \subseteq \text{Int}(m\text{Cl}(P, \tilde{T}))$.

(3) \rightarrow (4): Let (P, \tilde{T}) be soft semi- $m(P, \tilde{T})$ open. By (3), we have $(P, \tilde{T}) \subseteq m\text{Cl}(\text{Int}(P, \tilde{T})) \subseteq \text{Int}(m\text{Cl}(P, \tilde{T}))$. Thus, (P, \tilde{T}) is soft pre- $m(P, \tilde{T})$ open.

(4) \rightarrow (5): Let (P, \tilde{T}) be a strongly-soft β - $m(P, \tilde{T})$ -open set. Then $m\text{Cl}(P, \tilde{T})$ is soft semi- $m(P, \tilde{T})$ open. By (4), $m\text{Cl}(P, \tilde{T})$ is soft pre- $m(P, \tilde{T})$ open. Thus, $m\text{Cl}(P, \tilde{T}) \subseteq \text{Int}(m\text{Cl}(P, \tilde{T}))$ and consequently $m\text{Cl}(P, \tilde{T})$ is open.

(5) \rightarrow (6): Let (P, \tilde{T}) be a strongly-soft β - $m(P, \tilde{T})$ open set. By (5), $m\text{Cl}(P, \tilde{T}) = \text{Int}(m\text{Cl}(P, \tilde{T}))$.

Thus, $(P, \tilde{T}) \subseteq m\text{Cl}(P, \tilde{T}) = \text{Int}(m\text{Cl}(P, \tilde{T}))$ and consequently (P, \tilde{T}) is soft pre- $m(P, \tilde{T})$ -open.

(6) \rightarrow (7): Let (P, \tilde{T}) is soft semi- $m(P, \tilde{T})$ open set. Since a soft semi- $m(P, \tilde{T})$ open set is

soft strongly- β - $m(P, \tilde{T})$ open, then by (6) it is soft pre- $m(P, \tilde{T})$ open. Since (P, \tilde{T}) is soft semi- $m(P, \tilde{T})$ open and soft pre- $m(P, \tilde{T})$ open, it is soft α - $m(P, \tilde{T})$ open.

(7) \rightarrow (1): Let (P, \tilde{T}) is soft open set of W . Then $m\text{Cl}(P, \tilde{T})$ is soft semi- $m(P, \tilde{T})$ open and by (7) $m\text{Cl}(P, \tilde{T})$ is soft α - $m(P, \tilde{T})$ open. Therefore, $m\text{Cl}(P, \tilde{T}) \subseteq \text{Int}(m\text{Cl}(\text{Int}(m\text{Cl}(P, \tilde{T})))) = \text{Int}(m\text{Cl}(P, \tilde{T}))$, thus $m\text{Cl}(P, \tilde{T}) = \text{Int}(m\text{Cl}(P, \tilde{T}))$. Hence $m\text{Cl}(P, \tilde{T})$ is soft open and W is SmEDS.

Corollary 3.8: Let $(W, \Omega, m(W, \tilde{T}))$ is soft mixed space. Then, subsequent properties are equivalent:

- (1) W is SmEDS;
- (2) $m\text{-Cl}(W, \tilde{T}) \in \Omega \forall$ soft α - $m(W, \tilde{T})$ -open set (W, \tilde{T}) of W ;
- (3) $m\text{-Cl}(W, \tilde{T}) \in \Omega \forall$ soft semi- (W, \tilde{T}) -open set (W, \tilde{T}) of W ;
- (4) $m\text{-Cl}(W, \tilde{T}) \in \Omega \forall$ soft pre- $m(W, \tilde{T})$ -open set (W, \tilde{T}) of W .

Proof: Follows by Theorem 3.7.

Theorem 3.9: Let $(W, \Omega, m(W, \tilde{T}))$ is a soft mixed space and $m(W, \tilde{T})$ have property B. Then, the subsequent properties are equivalent:

- (1) W is SmEDS;
- (2) For any $(P, \tilde{T}) \in \Omega$ and $(B, \tilde{T}) \in m(W, \tilde{T})$ such that $(P, \tilde{T}) \cap (B, \tilde{T}) = \emptyset$, there exist disjoint $m(W, \tilde{T})$ -closed set (U, \tilde{T}) and a closed set (V, \tilde{T}) such that $(P, \tilde{T}) \subseteq (U, \tilde{T})$ and $(B, \tilde{T}) \subseteq (V, \tilde{T})$;
- (3) $m\text{Cl}(U, \tilde{T}) \cap \text{Cl}(V, \tilde{T}) = \emptyset \forall (U, \tilde{T}) \in \zeta$ and $(V, \tilde{T}) \in m(W, \tilde{T})$ with $(U, \tilde{T}) \cap (V, \tilde{T}) = \emptyset$;
- (4) $m\text{Cl}[\text{Int}(m\text{Cl}(U, \tilde{T}))] \cap \text{Cl}(V, \tilde{T}) = \emptyset \forall (U, \tilde{T}) \subseteq W$ and $(V, \tilde{T}) \in m(W, \tilde{T})$ with $(U, \tilde{T}) \cap (V, \tilde{T}) = \emptyset$.

Proof. (1) \rightarrow (2): Let W be SmEDS. Let (P, \tilde{T}) and (B, \tilde{T}) be two disjoint soft open and $m(W, \tilde{T})$ -open sets, respectively. Then $m\text{Cl}(P, \tilde{T})$ and $W - m\text{Cl}(P, \tilde{T})$ are disjoint soft $m(P, \tilde{T})$ -closed and soft closed sets $\subseteq (P, \tilde{T})$ and (B, \tilde{T}) , respectively.

(2) \rightarrow (3): Let $(U, \tilde{T}) \in \zeta$ & $(V, \tilde{T}) \in m(W, \tilde{T})$ with $(U, \tilde{T}) \subseteq (V, \tilde{T}) = \emptyset$. By (2), there exist

disjoint a soft $m(P, \tilde{T})$ closed set (F, \tilde{T}) and a soft closed set (G, \tilde{T}) such that $(U, \tilde{T}) \subseteq (F, \tilde{T})$ and $(V, \tilde{T}) \subseteq (G, \tilde{T})$. Therefore, $m\text{Cl}(U, \tilde{T}) \cap \text{Cl}(V, \tilde{T}) \subseteq (F, \tilde{T}) \cap (G, \tilde{T}) = \emptyset$. Thus, $m\text{Cl}(U, \tilde{T}) \cap \text{Cl}(V, \tilde{T}) = \emptyset$.

(3) \rightarrow (4): Let $(U, \tilde{T}) \subseteq W$ and $(V, \tilde{T}) \in m(W, \tilde{T})$ with $(U, \tilde{T}) \cap (V, \tilde{T}) = \emptyset$. Since $\text{Int}(m\text{Cl}(U, \tilde{T})) \in \zeta$ and $\text{Int}(m\text{Cl}(U, \tilde{T})) \cap (V, \tilde{T}) = \emptyset$, by (3) $m\text{Cl}[\text{Int}(m\text{Cl}(U, \tilde{T}))] \cap \text{Cl}(V, \tilde{T}) = \emptyset$.

(4) \rightarrow (1): Let (U, \tilde{T}) is SOPS. Then $[W - mCl(U, \tilde{T})] \cap (U, \tilde{T}) = \emptyset$. Since $m(P, \tilde{T})$ has property B, $W - mCl(U, \tilde{T}) \in m(P, \tilde{T})$ and by (4) $mCl(Int(mCl(U, \tilde{T}))) \cap Cl(W - mCl(U, \tilde{T})) = \emptyset$. Since $(U, \tilde{T}) \in \zeta$, we have $mCl(U, \tilde{T}) \cap [W - Int(mCl(U, \tilde{T}))] = \emptyset$.

Therefore, $mCl(U, \tilde{T}) \subseteq Int(mCl(U, \tilde{T}))$ and $mCl(U, \tilde{T})$ is soft open. This shows that W is SmEDS.

Definition 3.10: A ST (P, \tilde{T}) of a soft mixed space $(W, \Omega, m(W, \tilde{T}))$ is said to be a soft Rm-openset if $(P, \tilde{T}) = Int(mCl(P, \tilde{T}))$. Complement of a soft Rm-open set is called soft Rm-closed.

Theorem 3.11: Let $(W, \Omega, m(W, \tilde{T}))$ be a soft mixed space and $m(W, \tilde{T})$ have property B. Then, subsequent properties are equivalent:

- (1) W is SmEDS;
- (2) Each soft Rm-open set of W is soft $m(W, \tilde{T})$ -closed in W ;
- (3) Each soft Rm-closed set of W is soft $m(W, \tilde{T})$ -open in W .

Proof. (1) \rightarrow (2): Let W be SmEDS. Let (P, \tilde{T}) be a soft Rm-open set of W . Then $(P, \tilde{T}) = Int(mCl(P, \tilde{T}))$. Since (P, \tilde{T}) is SOPS, then $mCl(P, \tilde{T})$ is soft open. Thus, $(P, \tilde{T}) = Int(mCl(P, \tilde{T})) = mCl(P, \tilde{T})$ and consequently (P, \tilde{T}) is soft $m(W, \tilde{T})$ -closed.

(2) \rightarrow (1): Suppose that every soft Rm-open set of W is $m(W, \tilde{T})$ -closed in W . Let SOPS (P, \tilde{T}) of W . Since $Int(mCl(P, \tilde{T}))$ is soft Rm-open, then it is soft $m(W, \tilde{T})$ -closed. Therefore, $mCl(P, \tilde{T}) \subseteq mCl(Int(mCl(P, \tilde{T}))) = Int(mCl(P, \tilde{T}))$ since $(P, \tilde{T}) \subseteq Int(mCl(P, \tilde{T}))$. Thus, $mCl(P, \tilde{T})$ is soft open and consequently W is SmEDS.

(2) \rightarrow (3): It is obvious.

Theorem 3.12: Let $(W, \Omega, m(W, \tilde{T}))$ be a soft mixed space. Then subsequent properties are equivalent:

- (1) W is SmEDS;
- (2) $mCl(P, \tilde{T}) \in \Omega \forall$ soft Rm-open set (P, \tilde{T}) of W .

Proof. (1) \rightarrow (2): Let (P, \tilde{T}) is a soft Rm-open set of W . Then (P, \tilde{T}) is open and $mCl(P, \tilde{T}) \in \Omega$.

(2) \rightarrow (1): Suppose that $mCl(P, \tilde{T}) \in \Omega \forall$ soft Rm-open set (P, \tilde{T}) of W . Let (V, \tilde{T}) is SOPS of W . Then $Int(mCl(V, \tilde{T}))$ is a soft Rm-open set and $mCl(V, \tilde{T}) = mCl(Int(mCl(V, \tilde{T}))) \in \Omega$. Thus $mCl(V, \tilde{T}) \in \Omega$ and as a result W is SmEDS.

Theorem 3.13: Let $(W, \Omega, m(W, \tilde{T}))$ be a soft mixed space and $m(W, \tilde{T})$ have property B.

Then, the subsequent properties are equivalent:

- (1) W is SmEDS;
- (2) If (P, \tilde{T}) is soft semi- $m(W, \tilde{T})$ open, B is soft semi- $m(W, \tilde{T})^*$ -open and $(A, \tilde{T}) \cap (B, \tilde{T}) = \emptyset$, then $mCl(P, \tilde{T}) \cap Cl(B, \tilde{T}) = \emptyset$.

Proof: (1) \rightarrow (2): Let (P, \tilde{T}) is soft semi- $m(W, \tilde{T})$ open, (B, \tilde{T}) soft semi- $m(W, \tilde{T})^*$ -open and $(P, \tilde{T}) \cap (B, \tilde{T}) = \emptyset$. Since $m(P, \tilde{T})$ is property B, $mInt(B, \tilde{T})$ is soft $-m(W, \tilde{T})$ open and $mCl(P, \tilde{T}) \cap mInt(B, \tilde{T}) = \emptyset$. By Corollary 3.8, $mCl(P, \tilde{T})$ is soft open and $mCl(P, \tilde{T}) \cap Cl(mInt(B, \tilde{T})) = \emptyset$. Since (B, \tilde{T}) is soft semi- $m(W, \tilde{T})^*$ -open, $Cl(B, \tilde{T}) = Cl(mInt(B, \tilde{T}))$ and consequently $mCl(P, \tilde{T}) \cap Cl(B, \tilde{T}) = \emptyset$.

(2) \rightarrow (1): Let (P, \tilde{T}) is soft semi- $m(W, \tilde{T})$ open set. Since (P, \tilde{T}) and $W - mCl(P, \tilde{T})$ are disjoint soft semi- $m(W, \tilde{T})$ open and soft semi- $m(W, \tilde{T})^*$ -open, respectively, by (2) we have $mCl(P, \tilde{T}) \cap Cl[W - mCl(P, \tilde{T})] = \emptyset$. This implies that $mCl(P, \tilde{T}) \subseteq Int(mCl(P, \tilde{T}))$. Thus $mCl(P, \tilde{T})$ is soft open. Hence, by Corollary 3.8, W is SmEDS.

Theorem 3.14: Let $(W, \Omega, m(W, \tilde{T}))$ is soft mixed space and $m(W, \tilde{T})$ have property B. Then W is SmEDS if and only if \forall SOPS (G, \tilde{T}) and every soft $m(W, \tilde{T})$ -closed set (F, \tilde{T}) with $(G, \tilde{T}) \subseteq (F, \tilde{T})$, \exists a SOPS (G_1, \tilde{T}) and a soft $-m(P, \tilde{T})$ closed set (F_1, \tilde{T}) such that $(G, \tilde{T}) \subseteq (F_1, \tilde{T}) \subseteq (G_1, \tilde{T}) \subseteq (F, \tilde{T})$.

Proof. Suppose W is SmEDS. Let (G, \tilde{T}) be a SOPS and (F, \tilde{T}) a soft $-m(P, \tilde{T})$ closed set in W such that $(G, \tilde{T}) \subseteq (F, \tilde{T})$. Then $(G, \tilde{T}) \cap (W - (F, \tilde{T})) = \emptyset$. Then by theorem 3.9 $mCl(G, \tilde{T}) \cap Cl(W - (F, \tilde{T})) = \emptyset$, that is, $mCl(G, \tilde{T}) \subseteq W - Cl(W - (F, \tilde{T}))$. Using the fact that $W - Cl(W - F) \subseteq (F, \tilde{T})$ and writing $mCl(G, \tilde{T}) = (F_1, \tilde{T})$, $W - Cl(W - (F, \tilde{T})) = (G_1, \tilde{T})$, we get $(G, \tilde{T}) \subseteq (F_1, \tilde{T}) \subseteq (G_1, \tilde{T}) \subseteq (F, \tilde{T})$. Conversely, let the condition hold. Let (U, \tilde{T}) be a SOPS and (V, \tilde{T}) be a soft $-m(P, \tilde{T})$ open set in W such that $(U, \tilde{T}) \cap (V, \tilde{T}) = \emptyset$. Then, $(U, \tilde{T}) \subseteq W - (V, \tilde{T})$ and $W - (V, \tilde{T})$ is $-m(P, \tilde{T})$ closed, \exists a SOPS (G, \tilde{T}) and a soft $-m(P, \tilde{T})$ closed set (F, \tilde{T}) such that $(U, \tilde{T}) \subseteq (F, \tilde{T}) \subseteq (G, \tilde{T}) \subseteq W - (V, \tilde{T})$. This implies that $mCl(U, \tilde{T}) \cap W - [Int(W - (V, \tilde{T}))] = \emptyset$. But $W - [Int(W - (V, \tilde{T}))] = Cl(V, \tilde{T})$. That is, $mCl(U, \tilde{T}) \cap Cl(V, \tilde{T}) = \emptyset$ and by Theorem 3.9 W is SmEDS.

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