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Peristaltic Flow of Jeffrey Fluid through a slanted cylinder

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Abstract- Scientists in this decade carry out commendable examinations, which they do regardless of the way that they know about the clever meanings that a peristaltic peculiarity has in various natural and physiological frameworks. Because of the renowned uses of the peristaltic occasion in the gastrointestinal plot, fake lung, heart hardware, and blood course in the body, concentrates on this subject actually require consideration. The reason for this exploration is to research the way in which blood travels through a corridor when a dynamic rush of region withdrawal and development moves along the blood vessel wall. This attempt investigates the clever effect of variable consistency on the peristaltic stream of non-Newtonian Jeffrey fluid through a slanted cylinder. The impact of changing the properties of the liquid as well as the properties of the wall is thought about. The use of the MATLAB 2009b program delivers a graphical portrayal of significant boundaries on the physiological stream amounts. The outcomes that were acquired demonstrate that the conveyances of speed improve when higher upsides of the thickness boundary are utilized.

Keywords: Jeffery fluid, variable viscosity, Wave velocity

1. Introduction-

Peristaltic movement in a vessel is limit driven stream whose fundamental object is the vehicle, the breaking down and the blending of the materials held inside. It is delivered by the tightening and unwinding of the vessel walls. It happens in numerous natural frameworks, including the human body. It is a fundamental system by which food is shipped through the intestinal systems including the throat, the stomach and the small digestive tract; in the progression of blood through the veins, the vessels and the supply routes; in the vehicle of lymph in lymphatic vessels; and in the vehicle of pee from kidney to bladder through the ureter. It is additionally taken advantage of in numerous modern applications including biomechanical and biomedical frameworks in the alleged roller siphon. It is utilized to move clean liquid without defilement, transport poisonous liquid in the atomic business and siphon the blood in the heart-lung machine. In these specialized applications, the proficiency by which material vehicle happens assumes a significant part, since disappointment or deficient stockpile of the pertinent substance can bring about devastating results.

The impacts of visco-elasticity on peristaltic stream were concentrated scientifically by Hayat et al. (2008). Numerous hypothetical examinations have been done to concentrate on the component of peristalsis. Reddy et al. (2011) explored the peristaltic movement of Carreau liquids through a slanted divert in presence of attractive field. The irritation technique is utilized to examine the stream. Akbar and Butt (2015) concentrated on heat move under the activity of the peristaltic stream of gooey liquid with Herschel-Bulkley liquid model while thinking about that liquid coursing through a non-uniform slanted channel. Other numerical models have been created to portray the vehicle of physiological liquids through miniature channels by means of peristaltic waves under different circumstances, e.g., by Shit and Ranjit (2016) and Ranjit et al. (2017). The investigation of intensity and mass exchange in association with the peristaltic transport of exaggerated digression liquid through a divert of differing width in presence of attractive field was finished by Sarvana et al. (2016). Bhatt et al. (2017) as of late considered peristaltic transport and intensity move through non-uniform math. Walls are thought to be penetrable and found that temperature diminishes with expansion in

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Darcy number. Driven by the uses of intensity and mass exchange on the peristaltic stream of Newtonian and non-Newtonian liquids, Latha (2018) have broke down the effect of these properties in different mathematical circumstances. Propelled by the peristaltic stream application in demonstrating the chyme development in the gastrointestinal plot, Pandey et al. (2019) have researched the peristaltic stream utilizing different non-Newtonian liquids. Manjunatha et al. (2020) really look at the movement of Jeffery liquid in a non-uniform permeable conductor under warm conductivity and variable thickness to evaluate the impact of intensity and mass exchange. Divya et al. (2021) explored the impact of an attractive field and fluctuated fluid qualities on the peristaltic interaction of a Casson liquid while likewise thinking about the transmission of intensity and mass. With an end goal to research the peristaltic development of a micropolar liquid that has greased up walls, Mahmood et al. (2022) led their examination. They involved exceptionally flimsy layers of Ostwald-de-Waele liquid to get the ideal degree of oil.

2. Mathematical Formulation-

Let the equation of arterial wall be given by

$$\eta = a\left[1 + \varepsilon\cos\frac{2\pi}{\lambda}(Z - ct)\right] \tag{1}$$

Where a is the radius of the original undisturbed artery, ε is the amplitude of the wave, λ is the wave length and c is the wave velocity.

We will use the moving coordinate system (r, z) traveling with the wave so that

$$r = R,\ z = Z - ct \tag{2}$$

The equation of continuity and motion reduce respectively to

$$\frac{\partial}{\partial r}(ru_1) + \frac{\partial}{\partial r}(ru_2) = 0 \tag{3}$$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(r\tau) + \frac{\sin\theta}{F} \tag{4}$$

Where τ for Jeffrey fluid is give by

$$\tau = \frac{\mu(r)}{1+\lambda_1}\frac{du_2}{dr} \tag{5}$$

The variation in viscosity is given by

$$\mu(r) = 1 - \beta r \tag{6}$$

where β is the viscosity coefficient.

Where u_1 and u_2 are the components of velocity for the motion of the blood in relation to the moving coordinate system.

The boundary conditions for solving (3) and (4) are

$$u_1 = \frac{d\eta}{dt},\ u_2 = -c \text{ when } r = \eta \tag{7}$$

Using (5) and (6), Integrating (4), we get

$$\frac{\partial p}{\partial z} - \frac{\sin\theta}{F} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{(1-\beta r)}{1+\lambda_1}\frac{du_2}{dr}\right] \Rightarrow \frac{\partial}{\partial r}\left[r(1-\beta r)\frac{du_2}{dr}\right] = \left(\frac{\partial p}{\partial z} - \frac{\sin\theta}{F}\right)(1+\lambda_1)r$$

$$\Rightarrow \frac{\partial}{\partial r}\left[(r - \beta r^2)\frac{du_2}{dr}\right] = kr \text{ , where } k = \left(\frac{\partial p}{\partial z} - \frac{\sin\theta}{F}\right)(1+\lambda_1)$$

$$\Rightarrow (r - \beta r^2)\frac{d^2u_2}{dr^2} + (1 - 2\beta r)\frac{du_2}{dr} = kr$$

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$$\Rightarrow \frac{d^2u_2}{dr^2} + \left[\frac{1-2\beta r}{r(1-\beta r)}\right]\frac{du_2}{dr} = \frac{kr}{r(1-\beta r)} \Rightarrow \frac{d^2u_2}{dr^2} + \left[\frac{1-\beta r-\beta r}{r(1-\beta r)}\right]\frac{du_2}{dr} = \frac{k}{1-\beta r}$$

$$\Rightarrow \frac{d^2u_2}{dr^2} + \left[\frac{1}{r} - \frac{\beta}{1-\beta r}\right]\frac{du_2}{dr} = \frac{k}{1-\beta r}$$

Let $\frac{du_2}{dr} = M$

$$\frac{dM}{dr^2} + \left[\frac{1}{r} - \frac{\beta}{1-\beta r}\right]\frac{dM}{dr} = \frac{k}{1-\beta r}$$

$$\text{I.F.} = e^{\log r + \log(1-\beta r)} = e^{\log r(1-\beta r)} = r(1-\beta r)$$

$$M.r(1-\beta r) = \int \frac{k}{1-\beta r} r(1-\beta r)dr$$

$$M.r(1-\beta r) = \frac{kr^2}{2} \Rightarrow M = \frac{k}{2}\frac{r}{(1-\beta r)} \Rightarrow \frac{du_2}{dr} = -\frac{k}{2\beta}\frac{1-\beta r-1}{(1-\beta r)}$$

$$\Rightarrow \frac{du_2}{dr} = -\frac{k}{2\beta}\left(1 - \frac{1}{1-\beta r}\right)$$

$$\Rightarrow u_2 = -\frac{k}{2\beta}[r + \log(1-\beta r)] + d$$

$$-c = -\frac{k}{2\beta}[\eta + \log(1-\beta\eta)] + d$$

$$d = -c + \frac{k}{2\beta}[\eta + \log(1-\beta\eta)]$$

$$u_2 = -\frac{k}{2\beta}[r + \log(1-\beta r)] - c + \frac{k}{2\beta}[\eta + \log(1-\beta\eta)]$$

$$u_2 = -c - \frac{k}{2\beta}\left[\eta - r + \log\frac{(1-\beta\eta)}{(1-\beta r)}\right] \tag{8}$$

Now $q = 2\pi\int_0^\eta ru_2\, dr$

$$q = 2\pi\int_0^\eta \left[-cr - \frac{k}{2\beta}\left\{\eta r - r^2 + r\log\frac{(1-\beta\eta)}{(1-\beta r)}\right\}\right]dr$$

$$q = 2\pi\int_0^\eta \left[-cr - \frac{k}{2\beta}\left\{\eta r - r^2 + r\log\frac{(1-\beta\eta)}{(1-\beta r)}\right\}\right]dr$$

$$= 2\pi\left[-\frac{cr^2}{2} - \frac{k}{2\beta}\frac{\eta r^2}{2} + \frac{k}{2\beta}\frac{r^3}{3} - \frac{k}{2\beta}\frac{r^2}{2}\log(1-\beta\eta) + \frac{1}{\beta^2}\frac{k}{2\beta}\left\{\frac{(1-\beta r)^2}{2}\log(1-\beta r) - \frac{(1-\beta r)^2}{4} - (1-\beta r)\log(1-\beta r) - (1-\beta r)\right\}\right]_0^\eta$$

$$= 2\pi\left[-\frac{cr^2}{2} - \frac{k\eta r^2}{4\beta} + \frac{kr^3}{6\beta} - \frac{kr^2}{4\beta}\log(1-\beta\eta) + \frac{k}{2\beta^3}\left\{\frac{(1-\beta r)^2}{2}\log(1-\beta r) - \frac{(1-\beta r)^2}{4} - (1-\beta r)\log(1-\beta r) + (1-\beta r)\right\}\right]_0^\eta$$

$$q = 2\pi\left[-\frac{c\eta^2}{2} - \frac{k\eta^3}{4\beta} + \frac{k\eta^3}{6\beta} - \frac{k\eta^2}{4\beta}\log(1-\beta\eta) + \frac{k}{2\beta^3}\left\{\frac{(1-\beta\eta)^2}{2}\log(1-\beta\eta) - \frac{(1-\beta\eta)^2}{4} - (1-\beta\eta)\log(1-\beta\eta) + (1-\beta\eta)\right\}\right]$$

$$\frac{q}{2\pi} + \frac{c\eta^2}{2} = k\left[-\frac{\eta^3}{12\beta} - \frac{\eta^2}{4\beta}\log(1-\beta\eta) + \frac{1}{2\beta^3}\left\{\frac{(1-\beta\eta)^2}{2}\log(1-\beta\eta) - \frac{(1-\beta\eta)^2}{4} - (1-\beta\eta)\log(1-\beta\eta) + (1-\beta\eta)\right\}\right] \tag{9}$$

$$k = \frac{1}{\varphi(z)}\left(\frac{q}{2\pi} + \frac{c\eta^2}{2}\right)$$

Where

$$\varphi(z)=\left[-\frac{\eta^3}{12\beta}-\frac{\eta^2}{4\beta}\log(1-\beta\eta)+\frac{1}{2\beta^3}\left\{\frac{(1-\beta\eta)^2}{2}\log(1-\beta\eta)-\frac{(1-\beta\eta)^2}{4}-(1-\beta\eta)\log(1-\beta\eta)+(1-\beta\eta)\right\}\right]$$

$$\left(\frac{\partial p}{\partial z}-\frac{\sin\theta}{F}\right)(1+\lambda_1)=\frac{1}{\varphi(z)}\left(\frac{q}{2\pi}+\frac{c\eta^2}{2}\right)$$

$$\left(\frac{\partial p}{\partial z}-\frac{\sin\theta}{F}\right)=\frac{1}{\varphi(z)(1+\lambda_1)}\left(\frac{q}{2\pi}+\frac{c\eta^2}{2}\right)$$

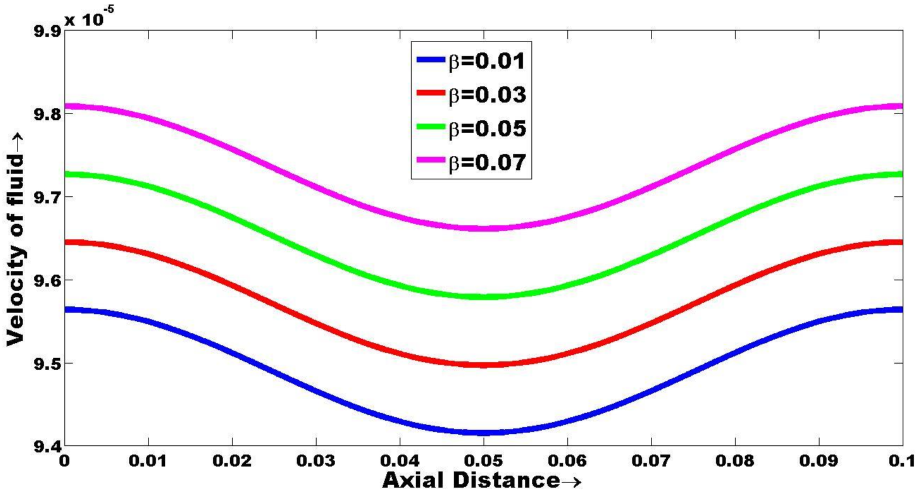
$$\frac{\partial p}{\partial z}=\frac{1}{\varphi(z)(1+\lambda_1)}\left(\frac{q}{2\pi}+\frac{c\eta^2}{2}\right)+\frac{\sin\theta}{F}$$

$$u_2=-c-\frac{1}{2\beta}\left(\frac{\partial p}{\partial z}-\frac{\sin\theta}{F}\right)(1+\lambda_1)\left[\eta-r+\log\frac{(1-\beta\eta)}{(1-\beta r)}\right]$$

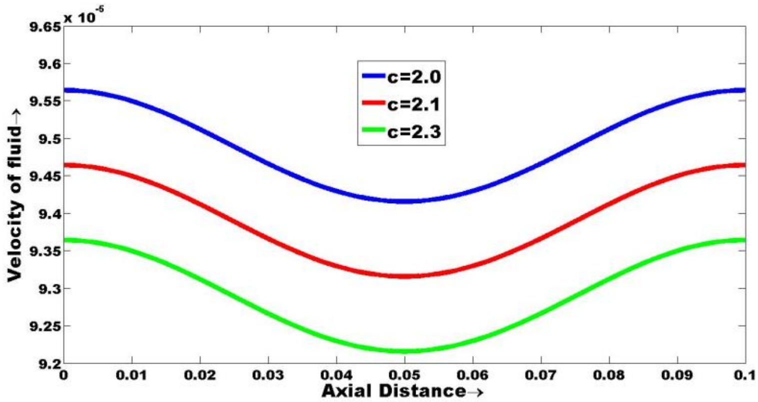
$$u_2=-c-\frac{(1+\lambda_1)}{2\beta}\left[\frac{1}{\varphi(z)(1+\lambda_1)}\left(\frac{q}{2\pi}+\frac{c\eta^2}{2}\right)\right]\left[\eta-r+\log\frac{(1-\beta\eta)}{(1-\beta r)}\right]$$

$$u_2=-c-\frac{1}{2\beta}\left[\frac{1}{\varphi(z)}\left(\frac{q}{2\pi}+\frac{c\eta^2}{2}\right)\right]\left[\eta-r+\log\frac{(1-\beta\eta)}{(1-\beta r)}\right] \quad (10)$$

3. Results and Discussion:



Graph 1: The relationship between the fluid's velocity and the axial distance for a range of different viscosity coefficient values



Graph 2: Changes in fluid speed with axial distance for varied values of wave speed

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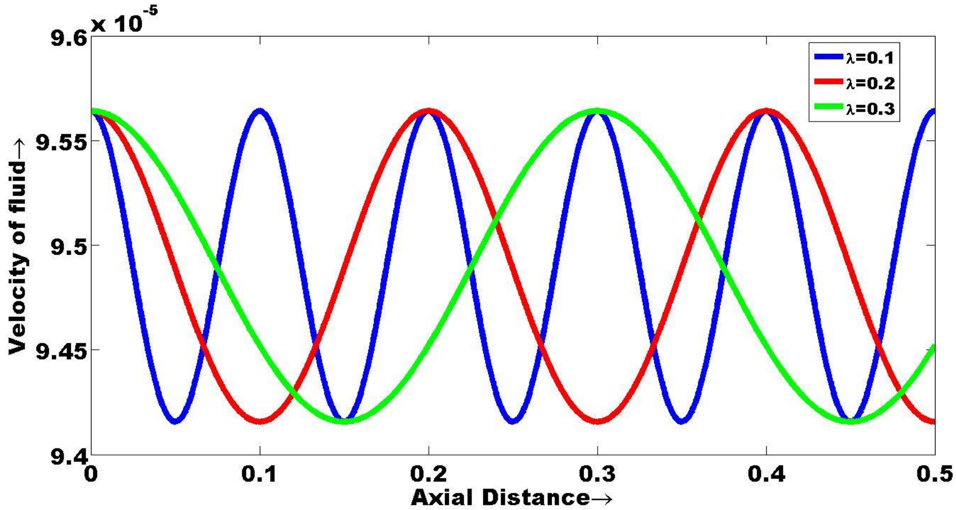
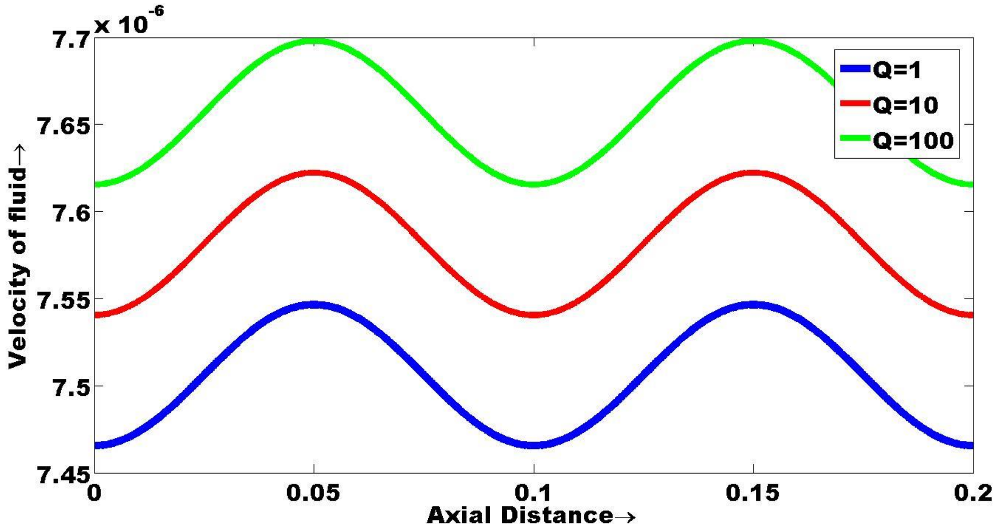
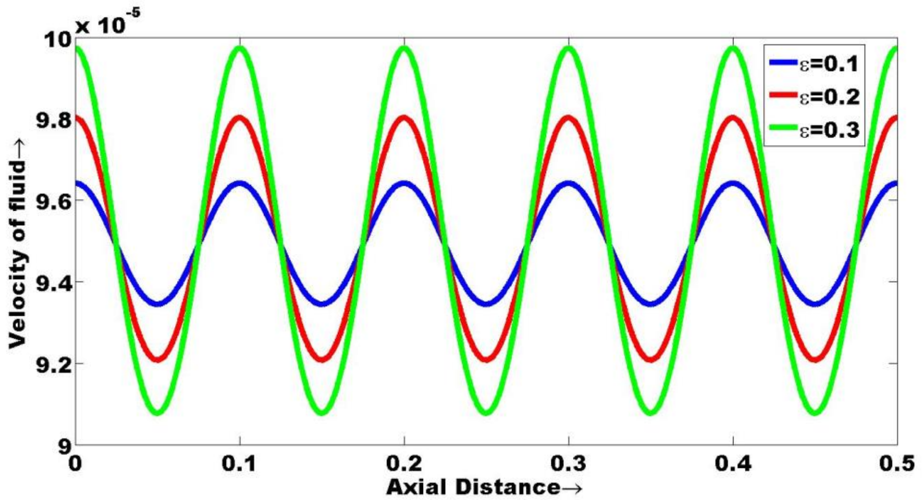


Graph-3: The relationship between the axial distance and the fluid's velocity at different values of the wave length



Graph-4: The fluid's velocity varies with axial distance for a variety of flow rates



Graph-5: The relationship between the fluid's velocity and the axial distance, given waves with varying amplitudes and varied values

Following the successful verification of the code through the use of non-Newtonian results in an inclined tube, the code was put to use to explore the effect of various parameters on flow characteristics. In this section, the results of numerical computations carried out with the MATLAB software are displayed in the form of graphs, namely Graphs (1) through (5). We use a methodical approach to investigating the impacts of the viscosity coefficient (β) , wave velocity (c), wave length (λ) and amplitude of the wave (ε). The velocity of the fluid is plotted against the axial distance in graphs (1) to (5). The velocity of the fluid is depicted in graph (1) for a range of different viscosity coefficient values, including 0.01, 0.03, 0.05, and 0.7. According to what can be gleaned from looking at this graph, the speed of the fluid goes up as the viscosity coefficients go up. The velocity of the fluid is depicted in graph (2) for a number of different wave velocity values, including $c = 2.0, 2.1, 2.3$. According to what can be seen in this graph, the speed of the fluid goes up whenever the wave velocity goes down. The velocity of the fluid is depicted in graph (3) for a number of different wave length values $\lambda = 0.1, 0.2, 0.3$. This graph demonstrates that when the wave length is amplified, there is a corresponding rise in the span of the velocity of the fluid. The graph (4) provides an explanation of how the velocity of the fluid changes depending on the flow rate. It can be seen from looking at this graph that the speed of the fluid gets faster as the flow rate gets higher. The velocity of the fluid is depicted in graph (5) for three different values of the amplitude of the wave: 0.1, 0.2, and 0.3. It can be seen from this graph that the amplitude of the velocity increases as the amplitude of the wave increases.

4. Concluding Remarks-

In order to investigate the flow that is caused by sinusoidal peristaltic waves utilizing the long wavelength approximation, a non-Newtonian fluid model that obeys Jeffrey's through-inclined-tube liquid flow has been implemented. It is easy to see that the magnitude of the velocity is at its lowest point for a non-Newtonian fluid in the centre of the axial distance. Furthermore, the magnitude of the velocity drops as the wave velocity increases, but it grows as the viscosity coefficient rises. When dealing with a non-Newtonian fluid, the amplitude of the velocity of the fluid will grow in proportion to the length of the wave. The velocity of the fluid exhibits a sinusoidal effect, increasing with the axial velocity until the midpoint of the axial distance, at which point it begins to decrease again before increasing again. It is highly believed that the findings of the analysis can be used to discuss the peristaltic flow of blood and other physiological fluids. This belief is based on the fact that it is possible to apply the findings.

REFERENCES-

1. Akbar N.S., Butt A. W. (2015): “Heat transfer analysis for the peristaltic flow of Herschel–Bulkley fluid in a nonuniform inclined channel”, Journal of Physical Sciences, 70(1):23-32.
2. Bhatt S. S., Medhavi A., Gupta R. S., Singh, U. P. (2017): “Effects of heat transfer during peristaltic transport in nonuniform channel with permeable walls” Journal of Heat Transfer, 139,1-6.
3. Divya B.B., Manjunatha G., Rajashekhar C.,Vaidya H.,Prasad K.V. (2021): Analysis of temperature dependent properties of a peristaltic MHD flow in a non-uniform channel: A Casson fluid model, Ain Shams Engineering Journal, 12(2):2181-2191
4. Hayat T., Ambreen A., Ali N. (2008): “Peristaltic transport of a Johnson- Segalman fluid in an asymmetric channel, Mathematical and Computer Modelling”, 47:380-400
5. Latha R., Kumar B.R., Makinde O.D. (2018): “Effects of heat dissipation on the peristaltic flow of Jeffery and Newtonian fluid through an asymmetric channel with porous medium” Defect and Diffusion Forum, 387:218–243
6. Mahmood W., Sajid M., Ali N., Sadiq M.N. (2022): “A new interfacial condition for the peristaltic flow of a micropolar fluid”, Ain Shams Engineering Journal, 13(5):101744
7. Manjunatha G., Rajashekhar C., Vaidya H., Prasad K.V., Vajravelu K. (2020): “Impact of heat and mass transfer on the peristaltic mechanism of Jeffrey fluid in a non-uniform porous channel with variable viscosity and thermal conductivity”, Journal of Thermal Analysis and Calorimetry, 139:1213–1228.
8. Pandey S.K., Singh A. (2019): “Peristaltic transport of Herschel–Bulkley fluids in tubes of variable cross section induced by dilating peristaltic waves: application to sliding hiatus hernia”, International Journal of Dynamics and Control, 7:407-418

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 3689-3695
https://publishoa.com
ISSN: 1309-3452
9. Ranjit N.K., Shit G.C., Sinha A. (2017): "Transportation of ionic liquids in a porous micro-channel induced by peristaltic wave with joule heating and wall-slip conditions", Chemical Engineering Science, 171:545 – 557.
10. Reddy R. H., Kavitha A., Sreenadh S., Saravana, R. (2011): "Effect of induced magnetic field on peristaltic transport of a carreau fluid in an inclined channel filled with porous material", International Journal of Mechanical and Materials Engineering, 6(2):240-249.
11. Saravana R., Reddy R. H., Goud J. S., Sreenadh S. (2016): "MHD peristaltic flow of a hyperbolic tangent fluid in a non-uniform channel with heat and mass transfer", IOP Conference Series: Materials Science and Engineering, 263:1-15.
12. Shit G.C., Ranjit N.K. (2016): "Role of slip velocity on peristaltic transport of couple stress fluid through an asymmetric non-uniform channel: Application to digestive system", Journal of Molecular Liquids, 221: 305-315.