JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3571-3589
https://publishoa.com
ISSN: 1309-3452

Sustainable Economic Production Quantity model for Product Return policy with Shortage

I. Antonitte Vinoline, P. Janavarthini

Department of Mathematics, Holy Cross College (Autonomous),

Tiruchirappalli - 620002, Tamilnadu, India.

Email: arulavanto@gmail.com, janavarthini.p@gmail.com

Received 2022 March 15; Revised 2022 April 20; Accepted 2022 May 10.

ABSTRACT

In this research, a new inventory model is developed to show the best Sustainable Economic Production Quantity value by considering product return policy with shortage. Sustainability is a multi-dimensional policy focusing on the environment, economic, and social impact. We propose four different sustainable economic production quantity models in this work, each of which takes into account distinct shortage scenarios. The product return are studied by considering the amount of the return as a variable. We estimated each member's profit function using an extensive method of direct accounting, determining the product's sustainability costs and return policy. Finally, the proposed models are described through a number of examples, and the acquired findings are analyzed and discussed. These findings suggest that, when compared to the three previous proposed models, the sustainable economic production quantity with partial backordering model is a more generic and realistic model that can be applied in many real-world scenarios while yielding a respectable profit.

KEYWORDS: Sustainable Economic Production Quantity Models, Shortage, Backordering, Lost sale, Return policy, Inventory management.

1. INTRODUCTION

In the modern era, Sustainable development is the new global approach for industry and industrial growth. The responsible production and consumption of commodities, particularly energy-intensive ones, may help to limit or address global warming, climate change, air pollution, water scarcity, and other related issues. Because of the importance of environmental concerns and the interconnectedness of industrial growth and environmental management, sustainable development and the principle of sustainability are receiving a lot of attention these days (Kannan[6], 2017). The present growth in global warming has prompted both consumers and manufacturers to be more concerned about emissions management social and environmental protection. In addition to cost and service, supply chains are focused on their environmental performance (Khan et al.[15], 2012).

Harris established an Economic Order Quantity (EOQ) model for estimating order quantity based on fundamental economic principles (including holding and ordering expenses) as early as 1913. Harris [9] in 1915 provided a comparable approach for determining Economic Production Quantity (EPQ) two years later, and Taft [8] published a similar formula for EPQ in 1918. Many models have been built over the years based on Harris' masterworks, but the majority of them simply modified the original EOQ model by adding other economic aspects and did not address non-economic concerns. Now, researcher are concerning on the sustainability issues in EOQ models. Bonney and Jaber [3], (2011) provide an overview of some of the environmental costs and propose an EOQ model that is both responsible and cost-effective. Wahab et al.[11], (2011) concentrated on transportation emission costs, incorporating environmental concerns in order to calculate fixed and variable carbon emission costs in order to determine the best strategy. Bouchery et al.[22], (2010) developed a basic model for sustainable lot sizing. To establish a sustainable supply chain, supply chain stakeholders must use sustainable EOQ/EPQ models to make inventory choices that are more in line with environmental concerns.

There are three basic components to the idea of sustainability: economic, environmental, and social (Lukman et al.[13], 2016; Scheel 2016; Spaiser et al. 2016). Only a few earlier publications (e.g., Bouchery et al.[2], (2012), Battini

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3571-3589
https://publishoa.com
ISSN: 1309-3452
et al.[1], (2017), and Jaber et al.[12], (2017)) address all three dimensions of sustainability. But in this research, we assume a variety of different sustainability cost functions to build the problem's total profit function. This research include the basic model, in which shortages are not permitted, when shortage are allowed the lost sale, full backordering, and partial backordering models, which can be chosen by operations managers based on the manufacturer's goal to enhance service levels.

We can also include the unpredictable situation in the concept of "Deterioration of Stored grains". Because of the insufficient holding places, shortages, returning of goods and weathering condition the quality of grains will lost in a short period. So, that in this case we can apply our proposed method of Sustainable Economic Production Quantity for product return with shortage. Sustainability is a critical concern that has been ignored in earlier research on sustainable inventory models. It may be decided which sustainable inventory model to use based on a company's shortage choices — whether they are allowed or not. The issue of inventory shortages has never been explored in earlier studies on sustainable inventory models. In order to fill a significant research need in this area, the current work focuses on constructing sustainable EPQ models with a variety of shortage options.

The model becomes more realistic and relevant to real-world settings when shortage difficulties are included in a sustainability EPQ problem. Any business that is challenged with a variety of shortages must have a strategy in place. For businesses with both sustainable and non-sustainable inventory systems, delayed sales, complete backordering, or partial backordering have effects. Turkay [21] in 2008, Bouchery et al.[22] in 2010, Bonney and Jaber[3] in 2011, Csutora et al.[4] in 2012, Glock et al. [10] in 2012, Ozlu [16] in 2013, Digiesi et al.[7] in 2013, Digiesi et al researcher formulated different EOQ/EPQ model in the recent studies. The direct accounting methodology has the higher accuracy, which is a simple and exact way for converting all sorts of sustainability factors into cost functions (Bouchery et al. [2] in 2012), may be used in additional studies. Only a few earlier efforts have been done in the context of a supply chain to answer Sustainable EOQ/EPQ problems that take into account the chain members' interactions.

Other portions of this article are as follows: In Section 2 presents the notations used throughout the work, summarizes the inventory models that have been built, and provides the best solution for each model. We formulated the mathematical model for four different type of shortages in Section 3. To illustrate the theoretical results, numerical examples are offered in Section 4. In the findings and discussion sections, the results of these cases are studied and discussed (Section 5 and 6). In Section 7 Sensitive analysis sensitivity analysis was used to explore the impact of modifications in the parameters then in Section 8 concludes with our findings and recommendations for additional investigation.

2. NOTATIONS

D_a − Rate of annual demand (units per year)

P_a − Maximum average production rate (units/year)

c − Unit price of the item ($/unit)

c' − Price per unit of waste ($/unit)

P_c − Cost of production per unit ($/unit)

S_c − Actual cost of setup ($/setup)

C_h − In a time unit, the cost of holding a unit of inventory ($/unit)

B_c − A cost of backordering an item unit in a time unit ($/unit)

G_c − Unfulfilled demand goodwill deficit ($/unit)

d_s − Dropped sale cost per unit ($d_s = (c - P_c) + G_c$) ($/unit)

γ − Market index with backordered item (percent)

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3571-3589
https://publishoa.com
ISSN: 1309-3452

$\tau -$ Inventory deterioration rate (percent)

$\rho -$ Each item per unit requires a certain amount of space (cubic meters per unit)

$\sigma -$ The weight of a supply that is no longer in use (ton per unit)

$E_{ci} -$ The average emission of carbon cost for holding inventory ($\$/m^3$)

$O_{ci} -$ For inventory obsolescence the waste collection, carbon emission cost and average disposal ($/ton)

$E_{pc} -$ The cost of carbon emissions for producing each unit ($/unit)

$SC_r -$ Return Social cost of an item ($/return)

$t -$ Average duration of transshipment item ($/hour)

$z -$ The coefficient of work stress of social cost (percent)

$N -$ Number of due times in a year for returning returned quantities of an item (number of return time schedule/year)

$M_r -$ Return rate independent of an item (percent)

$m_r -$ Return rate is determined by the refund amount (percent)

$f_a -$ Average capacity of transshipment for an item (tons/vehicle)

$SS_p -$ Setup social cost of production ($/setup)

$SS_t -$ Transshipment social cost of an item ($/hour)

$SS_h -$ Social cost of inventory holding ($/hour)

<u>Decision making Variables</u>

$IT -$ The period between two subsequent orders or the inventory cycle (time)

$F -$ The rate of time that a period has a positive inventory level (percent)

<u>Interdependent Variables</u>

$Q_p -$ Quantity of production (units per year)

$I_{max} -$ The maximum amount of inventory (units per year)

$I_{avg} -$ Inventory levels on a yearly basis (units per year)

$S_{max} -$ The maximum amount of Shortage (units per year)

$B_{max} -$ The maximum amount of Backorder (units per year)

$B_{avg} -$ The estimated annual amount of backordered items (units per year).

$Pf(IT)$: Total profit function (denoted by $Pf_{SEPQ\ Basic}(IT)$ for the basic Sustainable EPQ model, $Pf_{PBO}(IT, F)$ for the Sustainable EPQ – PBO model and the $Pf_{LS}(IT, F)$ for the lost sale Sustainable EPQ model) ($/year)

$FTP -$ Function of Total Profit ($/year)

$FTC -$ Function of Total Cost ($/year)

$PF_c -$ Cost function of production ($/year)

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3571-3589
https://publishoa.com
ISSN: 1309-3452

EF_{pc} − Cost function of "carbon emission of production" (\$/year)

SSF_h − Cost function of inventory holding for an item (\$/year)

SF_c − Function of Setup cost (\$/year)

CF_h − Function of holding cost (\$/year)

EF_{ci} − Cost function of "carbon emission of holding inventory" (\$/year)

OBF_{ci} − The function of inventory obsolescence cost (\$/year)

OF_{ci} − Cost function of "carbon emission of obsolescence inventory" (\$/year)

BF_c − Cost function of Backordering (\$/year)

GF_c − Cost function of Goodwill loss (\$/year)

SCF − Social cost function (\$/year)

SCF_h − Social cost function of holding inventory (\$/year)

SCF_w − Work stress of social cost function (\$/period)

SCF_r − Social cost function of return item (\$/period)

SCF_t − Social cost function of transshipment (\$/period)

SCF_p − Social cost function of production setup of an item (\$/period)

In the model development process, the basic principles are:

1. This model is single product, single period and single transshipment mode.
2. Yearly demand of a product is deterministic.
3. Production capacity is confined and the product maximum average production rate is P_a
4. Cost of transshipment is included in the unit price of the item c
5. The supplier returns "returned products" to the retailer according to predetermined due dates and times throughout the year, and all connected expenses are charged to his/her account.
6. For each returned product, the supplier must pay the unit sale price (c) to the retailer.
7. The scrap value c' of all returned items will be sold by the supplier immediately.
8. The supplier is expected to incur work stress as a societal cost

3. Sustainable EPQ model

Modelling the basic Sustainable EPQ model without shortage we define the $Pf_{SEPQ\ Basic}(IT)$ are as follows

$$Pf_{SEPQ}\,(IT) = \text{FTP} - PF_c - EF_{pc} - SF_c - CF_h - EF_{ci} - OBF_{ci} - SSF_h - OF_{ci} - SCF_r - SCF_p - SCF_h - SCF_w - SCF_r - SCF_t \quad \ldots(1)$$

$$= \mathrm{c}D_a - D_aP_c - E_{pc}D_a - \frac{S_c}{IT} - C_hI_{avg} - E_{ci}\rho I_{avg} - \tau(c-c')I_{avg} - \tau\sigma I_{avg}O_{ci} - (c-c')R_rD_a - \frac{SS_p}{IT} - SS_hI_{avg} - \frac{wP_c}{ITD_a} - NSC_r - 2SS_tt\left(\left[\frac{D_a}{f_a}\right]+1\right) - 2SS_tt\left(\left[\frac{R_rD_a}{f_a}\right]+1\right)$$

from Pentico et al. (2009),

$$I_{avg} = \frac{D_aIT}{2}\left(1-\frac{D_a}{P_a}\right) \quad \ldots(2)$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3571-3589
https://publishoa.com
ISSN: 1309-3452

We utilize three environmental characteristics to calculate the cost function of holding inventory(EF_{ci}), inventory obsolescence cost (OBF_{ci}), and emission of production (EF_{pc}). The average carbon emission cost for these three characteristics is carbon emission cost of carbon, average disposal, trash collection, and holding a unit of inventory (C_h), inventory obsolescence (O_{ci}), as well as the carbon emission cost for producing each unit (E_{pc}). While the basic SEPQ model's optimal inventory cycle may be calculated by optimizing the following yearly profit function:

$$Pf_{SEPQ}(IT) = \mathrm{c}D_a - D_aP_c - E_{pc}D_a - \frac{S_c}{IT} - C_h\frac{D_aIT}{2}\left(1-\frac{D_a}{P_a}\right) - E_{ci}\rho\frac{D_aIT}{2}\left(1-\frac{D_a}{P_a}\right) - \tau(c-c')\frac{D_aIT}{2}\left(1-\frac{D_a}{P_a}\right) - \tau\sigma\frac{D_aIT}{2}\left(1-\frac{D_a}{P_a}\right)O_{ci} - (c-c')R_rD_a - \frac{SS_p}{IT} - SS_h\frac{D_aIT}{2}\left(1-\frac{D_a}{P_a}\right) - \frac{wP_c}{ITD_a} - NSC_r - 2SS_tt\left(\left[\frac{D_a}{f_a}\right]+1\right) - 2SS_tt\left(\left[\frac{R_rD_a}{f_a}\right]+1\right) \quad \ldots(3)$$

To make the notation easier to understand, we consider,

$$C'_h = C_h\left(1-\frac{D_a}{P_a}\right) \quad \ldots(4)$$

$$E'_{ci} = E_{ci}\left(1-\frac{D_a}{P_a}\right) \quad \ldots(5)$$

$$c'' = (c-c')\left(1-\frac{D_a}{P_a}\right) \quad \ldots(6)$$

$$O'_{ci} = \left(1-\frac{D_a}{P_a}\right)O_{ci} \quad \ldots(7)$$

$$SS'_h = SS_h\left(1-\frac{D_a}{P_a}\right) \quad \ldots(8)$$

The return function, which indicates the percentage of items returned to the distribution center by users, may be defined as follows:

$$R_r = M_r + m_r * \frac{r}{v} \quad \ldots(9)$$

R_rD_a is the total return amount of a product in a year, and is the total return quantity of a product in each of N equal "return periods" is $\frac{R_rD_a}{N}$ As a result, the average quantity of product I returned in each period is $\frac{R_rD_a}{2N}$.

So the profit function changes to,

$$Pf_{SEPQ}(IT) = \mathrm{c}D_a - D_aP_c - E_{pc}D_a - \frac{S_c}{IT} - C'_h\frac{D_aIT}{2} - E'_{ci}\rho\frac{D_aIT}{2} - \tau c''\frac{D_aIT}{2} - \tau\sigma\frac{D_aIT}{2}O'_{ci} - (c-c')R_rD_a - \frac{SS_p}{IT} - SS'_h\frac{D_aIT}{2} - \frac{zP_c}{ITD_a} - NSC_r - 2SS_tt\left(\left[\frac{D_a}{f_a}\right]+1\right) - 2SS_tt\left(\left[\frac{R_rD_a}{f_a}\right]+1\right) \quad \ldots(10)$$

To find IT_{SEPQ}, we must first show that the profit function is concave.

Theorem 1

The profit function from equation (10) is concave.

Proof. We take the first partial derivative of Pf_{SEPQ} with regards to IT we have:

$$\frac{dPf}{dIT} = \frac{S_c}{IT^2} - \frac{D_a}{2}\left[C'_h + E'_{ci}\rho + \tau c'' + \tau\sigma O'_{ci} + SS'_h\right] + \frac{SS_p}{IT^2} + \frac{zP_c}{IT^2D_a} \quad \ldots(11)$$

Where, the second partial derivative of Pf_{SEPQ} with regards to IT

$$\frac{d^2Pf}{d^2IT} = \frac{-2}{IT^3}\left[S_c + SS_p + \frac{zP_c}{D_a}\right] \le 0$$

The profit function is strictly concave because it is always negative.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3571-3589
https://publishoa.com
ISSN: 1309-3452
Taking the first derivative to zero produces the best period length as shown below, since the profit function is concave.

$$IT_{SEPQ} = \sqrt{\frac{2(D_aS_c+D_aSS_p+zP_c)}{{D_a}^2z}} \qquad \ldots(12)$$

where, $w = C'_h + E'_{ci}\rho + \tau c'' + \tau\sigma O'_{ci} + SS'_h$ …(13)

Maximizing the profit function, as shown in Equation (10), is the same as reducing the cost function. Production and emission costs are not included in the following calculation since they are unrelated to the length of the period.

$$\text{FTP}_{SEPQ} = \frac{1}{IT}\left[S_c + SS_p + \frac{zP_c}{D_a}\right] + C'_h\frac{D_aIT}{2} + E'_{ci}\rho\frac{D_aIT}{2} + \tau c''\frac{D_aIT}{2} + \tau\sigma\frac{D_aIT}{2}O'_{ci} + SS'_h\frac{D_aIT}{2}$$
…(14)

Substituting equation (12) into equation (14) after a little more calculation, we've obtained to

$$\text{FTP}_{SEPQ} = \sqrt{2w(D_aS_c + D_aSS_p + zP_c)} \qquad \ldots(15)$$

As a result, the maximum profit is

$$Pf_{SEPQ}(IT) = (\text{c} - P_c - E_{pc})D_a - \sqrt{2w(D_aS_c + D_aSS_p + zP_c)} - (c-c')R_rD_a - NSC_r - 2SS_tt\left(\left[\frac{D_a}{f_a}\right]+1\right) - 2SS_tt\left(\left[\frac{R_rD_a}{f_a}\right]+1\right) \qquad \ldots(16)$$

3.1. Sustainable EPQ model with lost sales

In this part, we look at the Sustainable EPQ model, which assumes that all sales are lost due to shortages. The profit function in this case is as follows:

$$Pf_{Lostsale}(IT,F) = \text{FTP} - PF_c - EF_{pc} - SF_c - CF_h - EF_{ci} - OBF_{ci} - SSF_h - OF_{ci} - SCF_r - SCF_p - SCF_h - SCF_w - SCF_r - SCF_t - GF_c \qquad \ldots(17)$$

$$Pf_{Lostsale}(IT,F) = \text{c}D_a - D_aP_c - E_{pc}D_a - \frac{S_c}{IT} - C_hI_{avg} - E_{ci}\rho I_{avg} - \tau(c-c')I_{avg} - \tau\sigma I_{avg}O_{ci} - (c-c')R_rD_a - \frac{SS_p}{IT} - SS_hI_{avg} - \frac{wP_c}{ITD_a} - NSC_r - 2SS_tt\left(\left[\frac{D_a}{f_a}\right]+1\right) - 2SS_tt\left(\left[\frac{R_rD_a}{f_a}\right]+1\right) - D_aG_c(1-F)$$
…(18)

from Pentico et al. (2009),

$$I_{avg} = \frac{D_aITF^2}{2}\left(1-\frac{D_a}{P_a}\right) \qquad \ldots(19)$$

Substituting I_{avg} into profit function equation (18) we have

$$Pf_{Lostsale}(IT,F) = \text{c}D_a - D_aP_c - E_{pc}D_a - \frac{S_c}{IT} - C_h\frac{D_aITF^2}{2}\left(1-\frac{D_a}{P_a}\right) - E_{ci}\rho\frac{D_aITF^2}{2}\left(1-\frac{D_a}{P_a}\right) - \tau(c-c')\frac{D_aITF^2}{2}\left(1-\frac{D_a}{P_a}\right) - \tau\sigma\frac{D_aITF^2}{2}\left(1-\frac{D_a}{P_a}\right)O_{ci} - (c-c')R_rD_a - \frac{SS_p}{IT} - SS_h\frac{D_aITF^2}{2}\left(1-\frac{D_a}{P_a}\right) - \frac{wP_c}{ITD_a} - NSC_r - 2SS_tt\left(\left[\frac{D_a}{f_a}\right]+1\right) - 2SS_tt\left(\left[\frac{R_rD_a}{f_a}\right]+1\right) - D_aG_c(1-F) \qquad \ldots(20)$$

Substituting equation (4) to (8) and equation (13) into equation (20) we obtain

$$Pf_{lostsale}(IT,F) = (\text{c} - P_c - E_{pc})D_aF - \frac{1}{IT}\left[S_c + SS_p + \frac{zP_c}{D_a}\right] - \frac{wD_aITF^2}{2} - (c-c')R_rD_a - NSC_r - 2SS_tt\left(\left[\frac{D_a}{f_a}\right]+1\right) - 2SS_tt\left(\left[\frac{R_rD_a}{f_a}\right]+1\right) - D_aG_c(1-F) \qquad \ldots(21)$$

To find $Pf_{lostsale}(IT,F)$,we must first show that the profit function is concave.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3571-3589
https://publishoa.com
ISSN: 1309-3452
Theorem 2

The profit function from equation (21) is concave.

Proof : From equation (21) we know

$$Pf_{Lostsale}(IT,F) = (\mathrm{c} - P_c - E_{pc})D_aF - \frac{1}{IT}\left[S_c + SS_p + \frac{zP_c}{D_a}\right] - \frac{wD_aITF^2}{2} - (c-c')R_rD_a - NSC_r - 2SS_tt\left(\left[\frac{D_a}{f_a}\right]+1\right) - 2SS_tt\left(\left[\frac{R_rD_a}{f_a}\right]+1\right) - D_aG_c(1-F)$$

Taking first partial derivative of $Pf_{lostsale}(IT,F)$ with respect to IT

$$\frac{dPf_{lostsale}}{dIT} = -\left(\frac{2}{IT^2}\left[S_c + SS_p + \frac{zP_c}{D_a}\right] + \frac{wD_aF^2}{2}\right) \qquad \ldots(22)$$

Taking first partial derivative of $Pf_{Lostsale}(IT,F)$ with respect to F

$$\frac{dPf_{lostsale}}{dF} = (\mathrm{c} - P_c - E_{pc})D_a - wD_aITF - D_aG_c \qquad \ldots(23)$$

Taking second partial derivative of $Pf_{Lostsale}(IT,F)$ with respect to IT

$$\frac{\partial^2 Pf_{Lostsale}}{\partial^2 F} = -wD_aIT \qquad \ldots(24)$$

Taking second partial derivative of $Profit\ function_{lostsale}(IT,F)$ with respect to F

$$\frac{\partial^2 Pf_{Lostsale}}{\partial F \partial IT} = \frac{\partial^2 Pf_{Lostsale}}{\partial IT \partial F} = -(wD_aITF) \qquad \ldots(25)$$

To demonstrate the mentioned profit function's concavity, we must show that

$$[IT,F]H\begin{bmatrix} IT \\ F \end{bmatrix} \leq 0$$

Where, $H = \begin{bmatrix} \frac{\partial^2 Pf_{Lostsale}}{\partial^2 IT} & \frac{\partial^2 Pf_{Lostsale}}{\partial IT \partial F} \\ \frac{\partial^2 Pf_{Lostsale}}{\partial F \partial IT} & \frac{\partial^2 Pf_{Lostsale}}{\partial^2 F} \end{bmatrix}$

Thus we have,

$$[IT,F]\begin{bmatrix} -\left(\frac{2}{IT^2}\left[S_c + SS_p + \frac{zP_c}{D_a}\right] + \frac{wD_aF^2}{2}\right) & -wD_aITF \\ -wD_aITF & -wD_aIT \end{bmatrix}\begin{bmatrix} IT \\ F \end{bmatrix}$$

$$= \left[\frac{2}{IT^2}\left[S_c + SS_p + \frac{zP_c}{D_a}\right] + \frac{wD_aF^2}{2} - wD_aITF^2 - 2wD_aIT\right]\begin{bmatrix} IT \\ F \end{bmatrix}$$

$$= -\frac{2}{IT}\left[S_c + SS_p + \frac{zP_c}{D_a}\right] - 4wD_aITF^2 \leq 0$$

Because of the concavity of the profit function stated in Equation (21), evaluating the partial derivative of the profit function with respect to period length yields the following:

$$\frac{dPf}{dIT} = \frac{1}{IT^2}\left[S_c + SS_p + \frac{zP_c}{D_a}\right] - \frac{wD_aF^2}{2} \qquad \ldots(26)$$

$$IT = \frac{1}{D_aF}\sqrt{\frac{2(D_aS_c + D_aSS_p + zP_c)}{w}} \qquad \ldots(27)$$

Substituting IT into the profit function, from (21) we have

$Pf_{Lostsale}(F) = Pf_{Lostsale}(T(F), F) = (c - P_c - E_{pc})D_a F - F\sqrt{2w(D_a S_c + D_a SS_p + zP_c)} - (c - c')R_r D_a - NSC_r - 2SS_t t\left(\left[\frac{D_a}{f_a}\right] + 1\right) - 2SS_t t\left(\left[\frac{R_r D_a}{f_a}\right] + 1\right) - D_a G_c(1 - F)$

$$Pf_{Lostsale}(F) = \left[(d_s - E_{pc})D_a - \sqrt{2w(D_a S_c + D_a SS_p + zP_c)}\right]F - (c - c')R_r D_a - NSC_r - 2SS_t t\left(\left[\frac{D_a}{f_a}\right] + 1\right) - 2SS_t t\left(\left[\frac{R_r D_a}{f_a}\right] + 1\right) - D_a G_c(1 - F) \quad \ldots(28)$$

Linear function ($Pf_{Lostsale}(F)$) with respect to the variable F. The maximum profit is calculated by considering the function's slope $Pf_{Lostsale}(F)$. we have:

Case 1:

If $(d_s - E_{pc})D_a \geq \sqrt{2w(D_a S_c + D_a SS_p + zP_c)}$ when $F = 1$, then we obtain the maximum profit. This profit is taken by

$$Pf = Pf_{Lostsale}(F) = \left[(d_s - E_{pc})D_a - \sqrt{2w(D_a S_c + D_a SS_p + zP_c)}\right]F - (c - c')R_r D_a - NSC_r - 2SS_t t\left(\left[\frac{D_a}{f_a}\right] + 1\right) - 2SS_t t\left(\left[\frac{R_r D_a}{f_a}\right] + 1\right) - D_a G_c(1 - F) \quad \ldots(29)$$

The effective inventory cycle in this situation is

$$IT = \sqrt{\frac{2(D_a S_c + D_a SS_p + zP_c)}{{D_a}^2 w}} \quad \ldots(30)$$

Case 2:

If $(d_s - E_{pc})D_a < \sqrt{2w(D_a S_c + D_a SS_p + zP_c)}$ when $F = 0$, then we obtain the maximum profit and the effective inventory cycle $IT = \infty$ it means there are no inventory on hand and sales are constantly lost.

3.2. Sustainable EPQ model with full backordering

From equation (1) the profit function changes as follows,

$$Pf_{BO}(IT, F) = \text{FTP} - PF_c - EF_{pc} - SF_c - CF_h - EF_{ci} - OBF_{ci} - SSF_h - OF_{ci} - SCF_r - SCF_p - SCF_h - SCF_w - SCF_r - SCF_t - BF_c \quad \ldots(31)$$

$$= cD_a - D_a P_c - E_{pc}D_a - \frac{S_c}{IT} - C_h I_{avg} - E_{ci}\rho I_{avg} - \tau(c - c')I_{avg} - \tau\sigma I_{avg}O_{ci} - (c - c')R_r D_a - \frac{SS_p}{IT} - SS_h I_{avg} - \frac{wP_c}{ITD_a} - NSC_r - 2SS_t t\left(\left[\frac{D_a}{f_a}\right] + 1\right) - 2SS_t t\left(\left[\frac{R_r D_a}{f_a}\right] + 1\right) - B_c B_{avg}$$

Where, from Pentico et al. (2009),

$$I_{avg} = \frac{D_a ITF^2}{2}\left(1 - \frac{D_a}{P_a}\right) \quad \ldots(32)$$

$$B_{avg} = \frac{D_a IT(1 - F^2)}{2}\left(1 - \frac{D_a}{P_a}\right) \quad \ldots(33)$$

Substituting equation (32) and (33) in equation (31). We have

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3571-3589
https://publishoa.com
ISSN: 1309-3452

$$Pf_{BO}(IT,F) = cD_a - D_aP_c - E_{pc}D_a - \frac{S_c}{IT} - C_h\frac{D_aITF^2}{2}\left(1-\frac{D_a}{P_a}\right) - E_{ci}\rho\frac{D_aITF^2}{2}\left(1-\frac{D_a}{P_a}\right) - \tau(c-c')\frac{D_aITF^2}{2}\left(1-\frac{D_a}{P_a}\right) - \tau\sigma O_{ci}\frac{D_aITF^2}{2}\left(1-\frac{D_a}{P_a}\right) - (c-c')R_rD_a - \frac{SS_p}{IT} - SS_h\frac{D_aITF^2}{2}\left(1-\frac{D_a}{P_a}\right) - \frac{wP_c}{ITD_a} - NSC_r - 2SS_tt\left(\left[\frac{D_a}{f_a}\right]+1\right) - 2SS_tt\left(\left[\frac{R_rD_a}{f_a}\right]+1\right) - B_c\frac{D_aIT(1-F^2)}{2}\left(1-\frac{D_a}{P_a}\right) \quad ...(34)$$

From equation (13),

$$Pf_{BO}(IT,F) = \left(c - P_c - E_{pc}\right)D_a - \frac{1}{IT}\left(S_c + SS_p + \frac{zP_c}{D_a}\right) - \frac{wD_aITF^2}{2} - (c-c')R_rD_a - NSC_r - 2SS_tt\left(\left[\frac{D_a}{f_a}\right]+1\right) - 2SS_tt\left(\left[\frac{R_rD_a}{f_a}\right]+1\right) - B_c\frac{D_aIT(1-F^2)}{2}\left(1-\frac{D_a}{P_a}\right) \quad \ldots(35)$$

Minimizing the following function is equivalent to maximizing the objective function shown in Equation (35).

$$\pi(IT,F) = \frac{\beta_1}{IT} + (\beta_2F^2 - 2\beta_3F + \beta_3)\,IT \quad \ldots(36)$$

The new variables are,

$$\beta_1 = S_c + SS_p + \frac{zP_c}{D_a} > 0 \quad \ldots(37)$$

$$\beta_2 = w + \left(1-\frac{D_a}{P_a}\right)B_c > 0 \quad \ldots(38)$$

$$\beta_3 = \left(1-\frac{D_a}{P_a}\right)B_c > 0 \quad \ldots(39)$$

Theorem 3

The cost function Equation (36) in convex.

Proof:

The cost function indicated in Equation (36) is similar to the functions proposed by Taleizadeh (2014a, 2014b), and the convexity is unaffected by the notations modifications. As a result of these research, it is simple to demonstrate that the objective function represented in Equation (35) is convex.

We acquire the optimal values of choosing variables after proving the cost function's convexity by setting the initial partial derivatives of equation (36) with respect to F and IT equal to zero. As a result, we have:

$$\frac{\partial\pi(IT,F)}{F} = (2F\beta_2 - 2\beta_3)\,IT = 0 \quad \ldots(40)$$

Thus,

$$F = \frac{\beta_3}{\beta_2} = \frac{\left(1-\frac{D_a}{P_a}\right)B_c}{w+\left(1-\frac{D_a}{P_a}\right)B_c} \quad \ldots(41)$$

In addition, we have

$$\frac{\partial\pi(IT,F)}{IT} = \frac{\beta_1}{IT^2} + (\beta_2F^2 - 2\beta_3F + \beta_3) \quad \ldots(42)$$

Hence the maximum length of period is,

$$IT = \sqrt{\frac{\beta_1}{\beta_2F^2 - 2\beta_3F + \beta_3}} \quad \ldots(43)$$

Substituting the values for $\beta_1, \beta_2\ , \beta_3$ after some algebra. We get

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3571-3589
https://publishoa.com
ISSN: 1309-3452

$$IT = \sqrt{\frac{2\left(w+\left(1-\frac{D_a}{P_a}\right)B_c\right)\left(S_c+SS_p+\frac{zP_c}{D_a}\right)}{w\left(1-\frac{D_a}{P_a}\right)B_cD_a}} \quad \ldots(44)$$

3.3. Sustainable EPQ model with partial backordering

$$Pf_{PBO}(IT,F) = \text{FTP} - PF_c - EF_{pc} - SF_c - CF_h - EF_{ci} - OBF_{ci} - SSF_h - OF_{ci} - SCF_r - SCF_p - SCF_h - SCF_w - SCF_r - SCF_t - GF_c - BF_c \quad \ldots(45)$$

$$= \text{c}D_a - D_aP_c - E_{pc}D_a - \frac{S_c}{IT} - C_hI_{avg} - E_{ci}\rho I_{avg} - \tau(c-c')I_{avg} - \tau\sigma I_{avg}O_{ci} - (c-c')R_rD_a - \frac{SS_p}{IT} - SS_hI_{avg} - \frac{wP_c}{ITD_a} - NSC_r - 2SS_tt\left(\left[\frac{D_a}{f_a}\right]+1\right) - 2SS_tt\left(\left[\frac{R_rD_a}{f_a}\right]+1\right) - D_aG_c(1-F) - B_cB_{avg}$$
$\ldots(46)$

Where, from Pentico et al. (2009),

$$I_{avg} = \frac{D_aITF^2}{2}\left(1-\frac{D_a}{P_a}\right) \quad \ldots(47)$$

$$B_{avg} = \frac{\gamma D_aIT(1-F)^2}{2}\left(1-\frac{\gamma D_a}{P_a}\right) \quad \ldots(48)$$

Substituting equation (47) and (48) in equation (46) we get

$$Pf_{PBO}(IT,F) = \text{c}D_a - D_aP_c - E_{pc}D_a - \frac{S_c}{IT} - C_h\frac{D_aITF^2}{2}\left(1-\frac{D_a}{P_a}\right) - E_{ci}\rho\frac{D_aITF^2}{2}\left(1-\frac{D_a}{P_a}\right) - \tau(c-c')\frac{D_aITF^2}{2}\left(1-\frac{D_a}{P_a}\right) - \tau\sigma\frac{D_aITF^2}{2}\left(1-\frac{D_a}{P_a}\right)O_{ci} - (c-c')R_rD_a - \frac{SS_p}{IT} - SS_h\frac{D_aITF^2}{2}\left(1-\frac{D_a}{P_a}\right) - \frac{wP_c}{ITD_a} - NSC_r - 2SS_tt\left(\left[\frac{D_a}{f_a}\right]+1\right) - 2SS_tt\left(\left[\frac{R_rD_a}{f_a}\right]+1\right) - D_aG_c(1-F) - B_c\frac{\gamma D_aIT(1-F)^2}{2}\left(1-\frac{\gamma D_a}{P_a}\right) \quad \ldots(49)$$

For more simplification we have,

$$B'_c = B_c\left(1-\frac{\gamma D_a}{P_a}\right) \quad \ldots(50)$$

Substituting equation (50) and (4) to (8) into equation (49) we obtain

$$Pf_{PBO}(IT,F) = (\text{c} - P_c - E_{pc})D_a[(1-F)\gamma + F] - \frac{1}{IT}\left(S_c + SS_p + \frac{zP_c}{D_a}\right) - (c-c')R_rD_a - \frac{wD_aITF^2}{2} - NSC_r - 2SS_tt\left(\left[\frac{D_a}{f_a}\right]+1\right) - 2SS_tt\left(\left[\frac{R_rD_a}{f_a}\right]+1\right) - D_aG_c(1-F)(1-\gamma) - \frac{B'_c\gamma D_aIT(1-F)^2}{2}$$
$\ldots(51)$

The following step is to maximize function, to identify the optimal values for IT and F. We must first verify the concavity of the profit function before obtaining the optimum values of decision variables.

Theorem 4

The function of profit shown in equation (51) is concave.

Proof:

The profit function illustrated in Equation (51) is identical to the function provided by Pentico et al. (2009), with minor notational differences that have no effect on the convexity. As a result of this study, it is simple to demonstrate that the objective function represented in Equation (51) is concave.

Equation (51)'s partial derivatives with respect to the decision variables may be obtained and utilized to identify the optimal values due to its concavity. So far, we've got:

$$\frac{\partial Pf}{\partial F} = \left(c - P_c - E_{pc} + G_c\right)D_a(1-\gamma) - wD_aITF^2 + \gamma D_aITB'_c(1-F)^2 \quad \ldots(52)$$

We have,

$$d_s = (c - P_c) + G_c \quad \ldots(53)$$

Taking equation (51) equal to zero we get,

$$F = \frac{(1-\gamma)(d_s - E_{pc}) + \gamma ITB'_c}{IT(w + \gamma B'_c)} \quad \ldots(54)$$

Taking partial derivative with respect to IT

$$\frac{\partial Pf}{\partial IT} = \frac{D_aS_c + D_aSS_p + zP_c}{IT^2 D_a} + \frac{wD_aF^2}{2} - \frac{\gamma D_aB'_c(1-F)^2}{2} \quad \ldots(55)$$

Equation (54) is reduced to

$$\frac{2(D_aS_c + D_aSS_p + zP_c)}{IT^2 D_a} = wD_aF^2 + \gamma D_aB'_c(1-F)^2 \quad \ldots(56)$$

Ultimately we get

$$IT = \sqrt{\frac{2(D_aS_c + D_aSS_p + zP_c)}{wD_a{}^2F^2 + D_a{}^2\gamma B'_c(1-F)^2}} \quad \ldots(57)$$

Substituting equation (54) into equation (57)

$$IT = \sqrt{\frac{2\left(S_c + SS_p + \left[\frac{zP_c}{D_a}\right]\right)(w + \gamma B'_c)}{wD_a{}^2\gamma B'_c} - \frac{(1-\gamma)^2\left(d_s - E_{pc}\right)^2}{w\gamma B'_c}} \quad \ldots(58)$$

Equation (58) might be used to calculate F value from equation (54). By putting the findings of Equations (58) and (54) into Equation, the greatest profit $Pf_{PBO}(IT, F)$ may be found (51).

3.4. A partial backordering case solution technique for optimality

With slight adjustments in decision variable coefficients, the profit function illustrated in Equation (36) is equivalent to the profit function of San José et al. (2009). They devised a method for determining the best inventory policy, which we modified for our partial backordering model (proposed in section 3.4). The following solution algorithm, based on San José et al. (2009), can be used to identify the independent and dependent decision variables:

Step 1

Calculate the values $\Delta = (1-\gamma)^2\left(d_s - E_{pc}\right)^2 D_a{}^2 - 2w(D_aS_c + D_aSS_p + zP_c)$ and $\varepsilon = \gamma B'_c$

If $\Delta > 0$ the optimal strategy is $F = 1$ and $IT = \sqrt{\frac{2\left(D_aS_c + D_aSS_p + zP_c\right)}{D_a{}^2 w}}$ the maximum profit is

$Pf_{PBO}(IT, F) = \left(c - P_c - E_{pc}\right)D_a - \sqrt{2w\left(D_aS_c + D_aSS_p + zP_c\right)} - (c - c')R_rD_a - NSC_r - 2SS_tt\left(\left[\frac{D_a}{f_a}\right] + 1\right) - 2SS_tt\left(\left[\frac{R_rD_a}{f_a}\right] + 1\right)$. Then move to step 4

If $\Delta = 0$ go to step 2

If $\Delta < 0$ go to step 3

Step 2

If $\varepsilon > 0$ the optimal strategy $F = 1$ and $IT = \sqrt{\frac{2(D_a S_c + D_a SS_p + zP_c)}{{D_a}^2 w}}$ the maximum profit is

$Pf_{PBO}(IT, F) = (\mathrm{c} - P_c - E_{pc})D_a - \sqrt{2w(D_a S_c + D_a SS_p + zP_c)} - (c - c')R_r D_a - NSC_r - 2SS_t t\left(\left[\frac{D_a}{f_a}\right] + 1\right) - 2SS_t t\left(\left[\frac{R_r D_a}{f_a}\right] + 1\right)$. Then move to step 4

If $\varepsilon = 0$ and $\gamma = 0$ and the maximum profit may be made at any stage in the inventory cycle by adding value $Pf_{PBO}(IT, F) = -D_a G_c(1 - F) - NSC_r - 2SS_t t\left(\left[\frac{D_a}{f_a}\right] + 1\right) - 2SS_t t\left(\left[\frac{R_r D_a}{f_a}\right] + 1\right)$ then move to step 4

Step 3

If $\varepsilon > 0$ the optimal strategy (IT, F) is obtained from equation (54) and (58) and the maximum profit $Pf_{PBO}(IT, F)$ is calculated from equation (51) then move to step 4

If $\varepsilon = 0$ and the optimal strategy $F = 0$, $IT = \infty$ and $Pf_{PBO}(IT, F) = -D_a G_c(1 - F) - NSC_r - 2SS_t t\left(\left[\frac{D_a}{f_a}\right] + 1\right) - 2SS_t t\left(\left[\frac{R_r D_a}{f_a}\right] + 1\right)$ no inventory is kept in this situation, and sales are always lost. Then move to step 4

Step 4

Finally, using D_aIT and I_{max} = FD_aIT$\left(1 - \left[\frac{D_a}{P_a}\right]\right)$ calculate total demand per cycle and maximum inventory level, respectively. Furthermore, C = (1 − F)D_aIT$\left(1 - \left[\frac{D_a}{P_a}\right]\right)$ and B_{max} = γC may be used to calculate the maximum values of stock-out and backordered, respectively.

Finally, use Q_p = D_aIT [(1−F)+ F] to calculate the production quantity

4. NUMERICAL EXAMPLES

We will use numerical examples to demonstrate how to implement the optimal policies for the SEPQ inventory models (with shortage and return policy) and how the suggested solution technique works. This section's purpose is to prove how the models described in this article may be used to solve various SEPQ situations (with shortages and return policy). Each of these examples can assist readers understand how to choose and utilize any of the models we've provided.

Ex 1

This is a manufacturing system with these factors:

$C_h = 2.5, O_{ci} = 13, c' = 5, c = 10, \tau = 0.1, \rho = 1.7, \gamma = 0.45, \sigma = 2, B_c = 3, P_a = 100, d_s = 4, E_{pc} = 0.0168, S_c = 20, G_c = 1, P_c = 7, SS_h = 0.003, v = 0.86, SC_r = 0.14, SS_p = 0.17, M_r = 1\%, m_r = 10\%, N = 12, f = 4, E_{ci} = 0.55, B_c = 3, D_a = 40, z = 10\%, SS_t = 0.001, t = 1.25$ respectively.

So, $d_s = (c - P_c) + G_c = 4$

From equation (4) to (8) as follow

$C'_h = C_h\left(1 - \frac{D_a}{P_a}\right) = 2.5\left(1 - \frac{40}{100}\right) = 1.5$

$$E'_{ci} = E_{ci}\left(1 - \frac{D_a}{P_a}\right) = 0.55\left(1 - \frac{40}{100}\right) = 0.33$$

$$c'' = (c - c')\left(1 - \frac{D_a}{P_a}\right) = (10 - 5)\left(1 - \frac{40}{100}\right) = 3$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3571-3589
https://publishoa.com
ISSN: 1309-3452

$O'_{ci} = \left(1 - \frac{D_a}{P_a}\right) O_{ci} = 13\left(1 - \frac{40}{100}\right) = 7.8$

$SS'_h = SS_h \left(1 - \frac{D_a}{P_a}\right) = 0.003\left(1 - \frac{40}{100}\right) = 0.00018$

Where, from Soleymanfar, Vahid Reza, et al. (2021),

$\lambda = C_h + E_{ci} * \rho + SS_h = 2.5 + 0.55 * 1.7 + 0.0003 = 3.4353$

$r = \frac{s}{2} - \frac{vM_r}{2m_r} - \frac{\lambda}{4N} = \frac{10}{2} - \frac{0.86*1\%}{2*10\%} - \frac{3.4353}{4*12} = 4.885$

From equation (9)

$$R_r = M_r + m_r * \frac{r}{v} = 1\% + 10\% * \frac{4.885}{0.86} = 5.68 * 10^{-3}$$

From equation (13) we have

$w = C'_h + E'_{ci}\rho + \tau c'' + \tau\sigma O'_{ci} + SS'_h$

$w = 1.5 + 0.33 * 1.7 + 0.1 * 3 + 0.1 * 2 * 7.8 + 0.00018$

$w = 3.92118$ \$/unit

From equation (50) we get

$B'_c = B_c\left(1 - \frac{\gamma D_a}{P_a}\right) = 3\left(1 - \frac{0.45*40}{100}\right) = 2.46$ \$/unit. After that, we use our suggested solution algorithm to get the optimum solution. We begin by calculating the system parameters Δ and ϵ, which are as follows:

$\Delta = (1-\gamma)^2\left(d_s - E_{pc}\right)^2 {D_a}^2 - 2w(D_a S_c + D_a SS_p + zP_c) = 1316.4$ and $\varepsilon = \gamma B'_c = 1.107$. as $\Delta > 0$ then the optimum solution is $F = 1$ and $IT = \sqrt{\frac{2(D_a S_c + D_a SS_p + zP_c)}{{D_a}^2 w}} = 0.50$. The profit function $Pf_{PBO}(IT, F) = \left(c - P_c - E_{pc}\right)D_a - \sqrt{2w\left(D_a S_c + D_a SS_p + zP_c\right)} - (c - c')R_r D_a - NSC_r - 2SS_t t\left(\left[\frac{D_a}{f_a}\right] + 1\right) - 2SS_t t\left(\left[\frac{R_r D_a}{f_a}\right] + 1\right) = 36.91$ \$/year.

Finding the values of interdependent variables are

$D_a IT = 20$ units

$I_{max} = F D_a IT\left(1 - \left[\frac{D_a}{P_a}\right]\right) = 12$

$(1-F)\, D_a IT\left(1 - \left[\frac{D_a}{P_a}\right]\right) = 0$

$B_{max} = \gamma C = 0$

$Q_p = D_a IT\ [(1-F) + F] = 20$ units

Ex 2

We now assume that all parameter settings are comparable to those in Example 1, but we change the γ and choose a new $\gamma = 0.5$. Repeating the procedure again we get $B'_c = B_c\left(1 - \frac{\gamma D_a}{P_a}\right) = 2.4$ \$/unit

$\Delta = (1-\gamma)^2\left(d_s - E_{pc}\right)^2 {D_a}^2 - 2w(D_a S_c + D_a SS_p + zP_c) = -10.8 < 0$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3571-3589
https://publishoa.com
ISSN: 1309-3452

$$IT = \sqrt{\frac{2(S_c + SS_p + \left[\frac{zP_c}{D_a}\right])(w + \gamma B'_c)}{w{D_a}^2\gamma B'_c} - \frac{(1-\gamma)^2(d_s - E_{pc})^2}{w\gamma B'_c}} = 0.510$$

$$F = \frac{(1-\gamma)(d_s - E_{pc}) + \gamma IT B'_c}{IT(w + \gamma B'_c)} = 0.996$$

From equation (51) we get the total profit $Pf_{PBO}(IT, F) = 37.39$ \$/year. Thus we have

$D_a IT = 20.4\ units$

$I_{max} = \mathrm{F}D_a\mathrm{IT}\left(1 - \left[\frac{D_a}{P_a}\right]\right) = 12.19$

$C = (1-\mathrm{F})\ D_a\mathrm{IT}\left(1 - \left[\frac{D_a}{P_a}\right]\right) = 0.05$

$B_{max} = \gamma\mathrm{C} = 0.025$

Q_p = D_aIT [(1−F) + F]= 20.4 units

Ex 3

Finally, we assume that all of the parameters have values comparable to those in Example 1, but that $\gamma = 0$. We're using the Sustainable EPQ model of lost sales in this example. The optimal strategy depends on the values $(d_s - E_{pc})D_a = 159.33$ and

$$\sqrt{2w(D_aS_c + D_aSS_p + zP_c)} = 79.57$$

As $(d_s - E_{pc})D_a > \sqrt{2w(D_aS_c + D_aSS_p + zP_c)}$ then the optimal strategy is given by $F = 1$

and $IT_{SEPQ} = \sqrt{\frac{2(D_aS_c + D_aSS_p + zP_c)}{{D_a}^2 z}} = 0.506$. Also from equation (28) the maximum profit is $Pf_{Lostsale}(F) = \left[(d_s - E_{pc})D_a - \sqrt{2w(D_aS_c + D_aSS_p + zP_c)}\right]F - (c - c')R_rD_a - NSC_r - 2SS_t t\left(\left[\frac{D_a}{fa}\right] + 1\right) - 2SS_t t\left(\left[\frac{R_rD_a}{fa}\right] + 1\right) - D_aG_c(1 - F) = 36.58$ \$/year

5. RESULTS

Table 1 summaries the outcomes of various situations. The total profit function is the target function for optimizing the proposed models in this study. As a result, we focus on examining the overall profit amount of each model in the multiple situations described in our examples.

Table 1 A summary of the findings

Numerical Example	Related Model	γ	IT	F	Total Profit (\$/year)
1	$SEPQ$	0.45	0.51	1	36.91
2	$SEPQ_{PBO}$	0.5	0.510	0.996	37.39
3	$SEPQ_{LS}$	0	0.506	1	36.58

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3571-3589
https://publishoa.com
ISSN: 1309-3452
The cost effective values of γ, IT, and F are represented in Figures 2 and 3 compare the overall profit of various models. As previously stated, the SEPQ-Basic model's maximum profit is

$$Pf_{SEPQ}(IT) = (c - P_c - E_{pc})D_a - \sqrt{2w(D_aS_c + D_aSS_p + zP_c)} - (c - c')R_rD_a - NSC_r - 2SS_tt\left(\left[\frac{D_a}{f_a}\right] + 1\right) - 2SS_tt\left(\left[\frac{R_rD_a}{fa}\right] + 1\right) = 36.91\$/\text{year}$$

By using the Matlab 2D plotting we obtain the results in figure1and figure 2

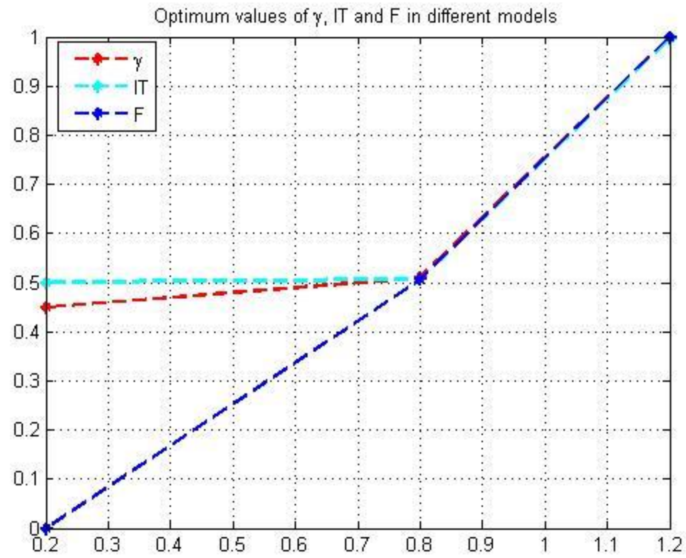

Figure 1

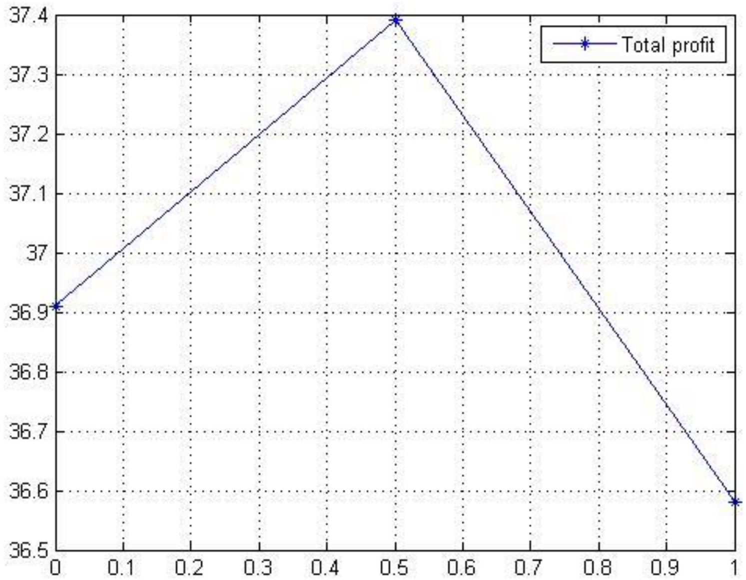

Figure 2

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3571-3589
https://publishoa.com
ISSN: 1309-3452

6. DISCUSSION

The findings of numerical examples are discussed in this section. As shown in Example 1, the best policy is the same for both the SEPQ and SEPQPBO models if = 0.45, and the total profit is the same (36.91 $/year). However, if = 0.50, the SEPQPBO model has a superior outcome since its total profit PBO (IT, F) = 37.39 $/year is larger than one of the SEPQ models, as demonstrated in Example 2.

In addition, as shown in Example 4, the total profit of the SEPQ Lost sales model and the SEPQ model is nearly same. However, the overall profit of the SEPQ Lost sales model is smaller than the SEPQ PBO model, which has a profit of 37.39 $ per year. Since $\gamma = 0$ there will be no backorders and all orders will be denied.

It is self-evident that if we can save all orders in a shortage full backordering model, we will earn more sales than in a partial backordering one. However, in the real world, where all firms compete (many consumers aren't loyal enough to limit their business to just one), the SEPQ-partial backordering model (SEPQ PBO) is a realistic model that takes both environmental and socio - economic factors into account. The overall profit of the SEPQ PBO model is greater than the SEPQ and SEPQ lost sale models, as shown in Figure 2.

When backordering is feasible to make greater profit, the SEPQ PBO model, which is based on the assumption that only a fraction of orders will be backlogged can be employed. The SEPQ PBO model is a sustainable EPQ model that takes into account shortages and return policies. It achieves a fair total profit amount using the IT and F values depending on the provided parameter values.

7. SENSITIVE ANALYSIS

We discussed sensitivity analysis in this part. In the previously mentioned numerical example, sensitivity analysis was used to explore the impact of modifications (under or over estimate) particularly for Sustainable EPQ Partial Backordering model in various inventory parameters, as well as the impact of optimal solutions for various variables and overall cost. This analysis was carried out by modifying (raising and lowering) the parameters from – 20% to + 20%, one parameter at a time, while maintaining the original values of the other parameters. Table 2 shows the numerical outcomes of this data analysis.

Table 2 Sensitive analysis of Sustainable EPQ Partial Backordering model for various parameters.

Parameters	% Changes	% Changes in		
		IT	F	Total Profit ($/year)
D_a	-20%	0.277	1.64	54.85
	-10%	0.4	1.206	45.10
	$+10\%$	1.1	0.59	25.81
	$+20\%$	1.17	0.567	19.61
S_c	-20%	0.691	0.797	25.73
	-10%	0.607	0.875	15.546
	$+10\%$	0.389	1.233	-37.37
	$+20\%$	0.205	2.130	-183.32
SS_p	-20%	0.51	0.996	37.39
	-10%	0.51	0.996	37.39
	$+10\%$	0.51	0.996	37.39
	$+20\%$	0.51	0.996	37.39
P_c	-20%	0.51	0.996	37.39
	-10%	0.51	0.996	37.39

	$+10\%$	0.51	0.996	37.39
	$+20\%$	0.51	0.996	37.39
d_s	-20%	$-$	$-$	$-$
	-10%	0.285	1.73	33.269
	$+10\%$	0.647	0.775	33.38
	$+20\%$	0.750	0.648	30.49
E_{pc}	-20%	0.51	0.996	37.39
	-10%	0.51	0.996	37.39
	$+10\%$	0.51	0.996	37.39
	$+20\%$	0.51	0.996	37.39
w	-20%	0.596	0.769	30.6
	-10%	0.557	0.868	34.72
	$+10\%$	0.350	1.465	41.45
	$+20\%$	0.324	1.707	48.07
B'_c	-20%	0.51	0.996	37.39
	-10%	0.51	0.996	37.39
	$+10\%$	0.51	0.996	37.39
	$+20\%$	0.51	0.996	37.39
γ	-20%	0.71	0.687	39.83
	-10%	0.63	0.794	37.98
	$+10\%$	0.24	2.04	40
	$+20\%$	$-$	$-$	$-$

Table 2 allows us to make the following observations:

The cycle length of the system (IT) is less sensitive with respect to parameters (SS_p), (P_c) (B'_c) and (E_{pc}). These mentioned parameters hardly have any effect in the optimal cycle length (IT). Highly sensitive with respect to Annual demand (D_a), Market index (γ) and Dropped sale (d_s). As a result, both of these variables have a significant influence on the inventory system's cycle length. In comparison to the other characteristics, however, it's quite sensitive.

The Rate of time of the system (F) is less sensitive with respect to parameters (d_s) and (D_a). These mentioned parameters hardly have any effect in the optimal cycle length(IT). Highly sensitive with respect to Setup cost (S_c) and Market index (γ). As a result, both of these variables have a significant influence on the inventory system's cycle length. In comparison to the other characteristics, however, it's quite sensitive.

The total profit (Pf) is highly sensitive with respect to the annual demand (D_a). Less sensitive with respect to Setup cost (S_c) i.e., it has less effect in the system whereas moderately sensitive the rest of the parameters. This shows that the supplier should give much concentration on the annual demand (D_a) instead of all other cost.

8. CONCLUSION

In this research, we discussed about four different types of Sustainable EPQ model with shortage and return policy in production system. Sustainability issues are included by environmental, social and economic cost such as Cost function of carbon emission of production, Cost function of inventory holding for an item, Cost function of carbon emission of obsolescence inventory, Work stress of social cost function and so on. The inventory models are Sustainable EPQ model with Fundamental Sustainable EPQ, Lost sale, Partial Backordering and Full Backordering models. Among these models Sustainable EPQ Partial Backordering model gained reasonable profit satisfying all sustainability conditions when compared to other models. Because of their adaptable and simple computing techniques, these new models may be valuable for organizations seeking environmentally conscious manufacturing systems. In terms of economic, social and environmental factors, our suggested sustainable EPQ models cover all of the major shortage scenarios for product return policy. The future scope of this research is to extend the single product Sustainable EPQ model to multiproduct

Sustainable EPQ inventory model. It can also be extend using software and considering other inventory conditions such as quantity discount and uncertainty model.

9. REFERENCE

1. Battini, D., Glock, C. H., Grosse, E. H., Persona, A., & Sgarbossa, F. (2017). Ergo-lot-sizing: An approach to integrate ergonomic and economic objectives in manual materials handling. International Journal of Production Economics, 185, 230–239.
2. Bouchery, Y., Ghafari, A., Jemai, Z., & Dallery, Y. (2012). Including sustainability criteria into inventory models. European Journal of Operational Research, 222, 229–240
3. Bonney, M., & Jaber, M. (2011). Environmentally responsible inventory models: Non-classical. International Journal of Production Economics, 133(1), 43–53.
4. Csutora, M., Dobos, I., Vörösmarty, G. (2012). A sustainable Economic Order Quantity Model With Carbon Footprint. 17th International Symposium on Inventories. Budapest: International Society for Inventory Research (ISIR)
5. D W Pentico, M J Drake, C Toews. The deterministic EPQ with partial backordering: A new approach, Omega, 2009, 37: 624-636.
6. D. Kannan (2017). Role of multiple stakeholders and the critical success factor theory for the sustainable supplier selection process. International Journal of Production Economics (in press)
7. Digiesi, S., Mossa, G., Mummolo, G. (2013). A Sustainable Order Quantity Model under Uncertain Product Demand, 7th IFAC Conference on Manufacturing Modelling, Management and Control. St. Petersburg: International Federation of Automatic Control, 664–669.
8. E W Taft. The Most Economical Production Lot, Iron Age, 1918, 101: 1410–1412.
9. F W Harris. What quantity to make at once? Operation and costs, in the library of factory management, Chicago: A. W. Shaw Company, 1915, Vol. V: 47-52.
10. Glock, C. H., Jaber, M. Y., & Searcy, C. (2012). Sustainability strategies in an EPQ model with price and quality sensitive demand. The International Journal of Logistics Management, 23(3), 340–359.
11. H Jawad, M Y Jaber, M Bonney. The Economic Order Quantity model revisited: an Extended Exergy Accounting approach, Journal of Cleaner Production, 2014, http://dx.doi.org/10.1016/j.jclepro.2014.06.079
12. Jaber, M. Y., Bonney, M., & Jawad, H. (2017). Comparison between economic order/manufacture quantity and just-in-time models from a thermodynamics point of view. Computers & Industrial Engineering, 112, 503–510.
13. Lukman, R. K., Glavic, P., Carpenter, A., Virtic, P. (2016). Sustainable consumption and productionResearch, experience and development- The Europe we want, Journal of Cleaner Production, 138 (Part 2), 139–147.
14. M I M Wahab, S M H Mamun, P Ongkunaruk. EOQ models for a coordinated two-level international supply chain considering imperfect items and environmental impact, International Journal of Production Economics, 2011, 134(1): 151-158.
15. M Khan, M Y Jaber, C H Glock. Impact of Learning on the Enviromental Performance of a Two Level Supply Chain, 17th International Symposium on Inventories. Budapest: International Society for Inventory Research (ISIR), 2012.
16. Ozlu, H. (2013). Joint Decisions on Inventory Replenishment and Emission Reduction Investment Under Diferent Emission Regulations, MSc. Thesis. Graduate School of Engineering and Science, Bilkent University
17. Scheel, C. (2016). Beyond Sustainability. Transforming industrial zero-valued residues into increasing economic returns. Journal of Cleaner Production, 131, 376–386.
18. Soleymanfar, Vahid Reza, et al. "Sustainable EOQ and EPQ models for a two-echelon multi-product supply chain with return policy. " Environment, Development and Sustainability (2021): 1-27
19. Spaiser, V., Ranganathan, S., Swain, R. B., & Sumpter, D. J. T. (2016). The sustainable development oxymoron: Quantifying and modelling the incompatibility of sustainable development goals. International Journal of Sustainable Development & World Ecology, 24(6), 457–470
20. Taleizadeh, Ata Allah, Vahid Reza Soleymanfar, and Kannan Govindan. "Sustainable economic production quantity models for inventory systems with shortage." *Journal of cleaner production* 174 (2018): 1011-1020.
21. Turkay, M. (2008). Environmentally conscious supply chain management, Process Systems Engineering (pp. 45–86). WILEY-VCH.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 3571-3589
https://publishoa.com
ISSN: 1309-3452

22. Y Bouchery, A Ghaffari, Z Jemai, Y Dallery. How to Incorporate Sustainability Issues into Traditional Operations, Research Report, Laboratoire Génie Industriel, Paris: Ecole Centrale Paris, 2010.