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Escalation of Weighted Fuzzy Entropic Models and Their Applications for the Study of Maximum Entropy Principle

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ABSTRACT

The continued existence of mutual categories of uncertainties, viz. probabilistic and fuzzy nevertheless both are away from each other, even though contribute with a fundamental accountability in declining uncertainties and accordingly constructing the structure under learning supplementary skilled. Additionally, it is apprehended that the *maximum entropy principle* cooperate with a crucial accountability for the learning of optimization problems connected with the hypothetical information models. The current communication has been produced from this position of observation and formulates transaction with two new entropic models for discrete fuzzy distributions and additionally makes them functional for the acquaintance of *maximum entropy principle* under the situation of fuzzy constraints.

Keywords: Entropy, Fuzzy entropy, Increasing function, Decreasing function, Maximum entropy principle, Concavity.

INTRODUCTION

The entropic model well established by Shannon [22] has extraordinarily agreeable properties and make available the magnificent relevance in a sequence of disciplines. This original burst through discovery of entropic model guided the researchers to scrutinize new-fangled and pioneering entropic models. Shannon [22] structured the hypothetical environment upon introducing the crucial conception of entropy $H(P)$ attached with the discrete probability spaces**.** The fundamentally well-acknowledged perception of probabilistic entropy premeditated by Shannon [22] enriched the text of coding theory with the facilitation of numerous entropic models. This entrenched advancement arranged the stone of discrete information entropic model with astonishingly agreeable properties and was well acknowledged by subsequent appearance:

$$H(P) = -\sum_{i=1}^{n} p_i \log p_i \tag{1.1}$$

Monitoring the magnificent properties of entropic model (1.1), numerous pioneers made investigations and consequently introduced an assortment of models from application point of

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observation towards multiplicity of disciplines. These pioneers consist of Huang and Zhang [4], Parkash and Kakkar [14], Kapur [8], Markechová et al. [11] etc.

The theoretical awareness about the *maximum entropy principle* signifies that it contributes with a historic dependability for the learning of plentiful optimization problems interconnected with entropic models. On the other hand, the observation of weighted models cannot be overlooked because of their astonishingly dynamic nature. For this rationale, Parkash, Kumar, Mukesh and Kakkar [18] developed new entropic models and broadened the application area of coding theory. These discrete parametric weighted entropic models have been established by the subsequent quantitative outward show:

$$H_\alpha^1(P;W)=\frac{1}{2^{1-\alpha}-1}\left[\sum_{i=1}^n w_i p_i^\alpha-\left\{\sum_{i=1}^n w_i p_i^{\frac{1}{\alpha}}\right\}^\alpha\right],\ \alpha>1 \qquad (1.2)$$

and

$$H_{\alpha,\beta}(P;W)=\frac{1}{\beta-\alpha}\log\left[\frac{\sum_{i=1}^n w_i p_i^\alpha}{\sum_{i=1}^n w_i p_i^\beta}\right],\ \alpha<1,\beta>1 \text{ or } \alpha>1,\beta<1 \qquad (1.3)$$

Subsequent to the detection of entropic model, Zadeh [23] established the theory of fuzzy sets which took delivery of affirmation from assorted quarters and happened to be most well-liked. Observing the initiative of this hypothesis, De Luca and Termini [3] recommended fuzzy entropic model analogous to Shannon's [22] entropic model.

Recently Qin et al. [21] delivered consequential clarification about the disagreement flanked by the source and target data distributions. The authors supplemented that this learning process recommends cross-domain fault diagnosis modus operandi dependent upon enhanced multi-scale fuzzy entropic model and enhanced joint distribution alteration with the endeavor to address incompatible data distribution sandwiched between the source and target domain. Experimental consequences display that enhanced multi-scale discrete fuzzy entropic models have superior distinctive capability and transferable aptitude than numerous accessible entropy methods, and the enhanced joint distribution adaptation is additional generalization to relocate circumstances with complex data distributions.

Additionally, İnce [5] made amplification regarding the fuzzy entropy that it is second-handed to articulate the precise values of fuzziness and is described it by means of the perception of membership function. The *maximum entropy principle* endeavors to choose such function with restricted number of fuzzy values subject to restrictions engendered. Many other pioneer who made their contributions towards the development of fuzzy entropic models for discrete fuzzy distributions are Bassanezi and Roman-Flores [1], Kapur [9], Parkash [12], Parkash, Sharma and Mahajan [19, 20] etc. whereas some additional pioneer who made contributions towards the creation of discrete fuzzy divergence models in fuzzy spaces include Bhandari et al. [2], Parkash [13], Parkash and Kumar [15, 16, 17], Joshi and Kumar [7] etc.

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Jaynes [6] projected a magnificent guideline to dispense numerical probabilities through the accessibility of assured partial information. The additional pioneers who broaden the learning of this projection include Kapur and Kesavan [12], Parkash, Sharma and Mahajan [19, 20] etc. In this paper, we have prepared amplification in Jayne's [9] *maximum entropy principle* for discrete fuzzy spaces intended for studying primary fuzziness under fuzzy restrictions. To accomplish our intention, we have firstly constructed some inventive weighted trigonometric fuzzy entropic models as demonstrated in the next section.

2. GROWTH OF INNOVATIVE WEIGHTED TRIGONOMETRIC FUZZY ENTROPIC MODELS

I. We, first put forward a new-fangled weighted trigonometric fuzzy entropic model, given by the subsequent appearance:

$$H_1(A;W)=\sum_{i=1}^{n} w_i\left[Sin\frac{\beta\mu_A(x_i)}{n}+Sin\frac{\beta(1-\mu_A(x_i))}{n}-Sin\frac{\beta}{n}\right];0<\beta\le\pi,n>1 \tag{2.1}$$

Differentiating equation (2.1) w.r.t. $\mu_A(x_i)$, we get

$$\frac{\partial H_1(A;W)}{\partial\mu_A(x_i)}=\frac{\beta w_i}{n}\left[Cos\frac{\beta\mu_A(x_i)}{n}-Cos\frac{\beta\{1-\mu_A(x_i)\}}{n}\right]$$

Also

$$\frac{\partial^2 H_1(A;W)}{\partial\mu_A^2(x_i)}=-\frac{\beta^2 w_i}{n^2}\left[Sin\frac{\beta\mu_A(x_i)}{n}+Sin\frac{\beta\{1-\mu_A(x_i)\}}{n}\right]<0\ \forall i$$

This demonstrates the concavity of $H_1(A;W)$ and its highest value happens at $\mu_A(x_i)=\frac{1}{2}\forall i$.

Thus, we observe that $H_1(A;W)$ persuade the subsequent desirable properties:

(i) $H_1(A;W)$ is concave.

(ii) $H_1(A;W)$ is an increasing function of $\mu_A(x_i)$ when $0\le\mu_A(x_i)\le\frac{1}{2}$

because $\{H_1(A;W)=0\,when\,\mu_A(x_i)=0\}$ and $[H_1(A;W)]_{\max}\ge 0$ when $\mu_A(x_i)=\frac{1}{2}\forall i$

(iii) $H_1(A;W)$ is a decreasing function of $\mu_A(x_i)$ when $\frac{1}{2}\le\mu_A(x_i)\le 1$

because $[H_1(A;W)]_{\max}\ge 0$ when $\mu_A(x_i)=\frac{1}{2}\forall i$ and $\{H_1(A;W)=0\,when\,\mu_A(x_i)=0\}$

(iv) $H_1(A;W)$ does not change when $\mu_A(x_i)$ is changed to $1-\mu_A(x_i)$.

(v) $H_1(A;W)=0$ when $\mu_A(x_i)$ = 0 or 1.

(vi) $H_1(A;W)\ge 0$

The knowledge of these six properties demonstrates the soundness of the entropic model shaped in (2.1).

II. We, next propose another pioneering weighted trigonometric fuzzy entropic model, specified by the subsequent manifestation:

$$H_2(A;W)=\sum_{i=1}^{n} w_i\left[Cos\beta\mu_A(x_i)+Cos\beta(1-\mu_A(x_i))-2Cos^2\frac{\beta}{2}\right];0<\beta<\pi \tag{2.2}$$

Proceeding on comparable deliberations, we can demonstrate the legitimacy of the fuzzy entropic model wrought in (2.2).

In continuation, we have convoluted the *maximum entropy principles* through the support of entropic models (2.1) and (2.2).

3. APPLICATIONS OF DISCRETE WEIGHTED FUZZY ENTROPIC MODELS FOR THE STUDY OF MAXIMUM FUZZY PRINCIPLE

To make accessible applications, we have well deliberated the subsequent optimizational problems interconnected with our entropic models and exposed that the maximization of fuzzy entropy in every case is achieved through concavity. For the solution of these problems we employ the subsequent set of fuzzy constraints:

$$\sum_{i=1}^{n}\mu_A(x_i)=\alpha_0 \tag{3.1}$$

and

$$\sum_{i=1}^{n}\mu_A(x_i)g_r(x_i)=K;\ r=1,2,...,m\ \ and\ m+1<n \tag{3.2}$$

where $K>0$ is constant, $g_r(x_i)$ are understood to be recognized values and $\mu_A(x_i)$ are fuzzy values. Since $g_r(x_i)$ are implicitly recognized, the expected fuzzy values specified in (3.1) are presupposed to be recognized accurately.

Problem-I: Here, we reflect on optimization of weighted trigonometric fuzzy entropic models produced in equation (2.1) of the above section under the situation of fuzzy constraints (3.1) and (3.2).

To provide the solution, we reveal the subsequent Lagrangian appearance:

$$\begin{aligned}L=\sum_{i=1}^{n} w_i\left[Sin\frac{\beta\mu_A(x_i)}{n}+Sin\frac{\beta(1-\mu_A(x_i))}{n}-Sin\frac{\beta}{n}\right]+\lambda_1\left\{\sum_{i=1}^{n}\mu_A(x_i)-\alpha_0\right\}\\ +\lambda_2\left\{\sum_{i=1}^{n}\mu_A(x_i)g_r(x_i)-K\right\}\end{aligned} \tag{3.4}$$

Differentiating equation (3.4) w.r.t. $\mu_A(x_i)$, we acquire the subsequent manifestation:

$$\frac{\partial L}{\partial\mu_A(x_i)}==-\frac{2\beta w_i}{n}Sin\frac{\beta}{2n}Sin\frac{\beta}{2n}\left\{2\mu_A(x_i)-1\right\}+\left\{\lambda_1+\lambda_2 g_r(x_i)\right\} \tag{3.5}$$

Thus $\frac{\partial L}{\partial\mu_A(x_i)}=0$ gives

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$$\left\{\frac{\beta}{2n}\left\{2\mu_A(x_i)-1\right\}\right\}=Sin^{-1}\left\{\frac{n}{2\beta w_i}\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{Sin\frac{\beta}{2n}}\right\}$$

or $\mu_A(x_i)=\frac{1}{2}\left[\frac{2n}{\beta}\left\{Sin^{-1}\left\{\frac{n}{2\beta w_i}\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{Sin\frac{\beta}{2n}}\right\}\right\}+1\right]$

Applying the constraints (3.1) and (3.2), we get hold of the subsequent demonstration:

$$\frac{1}{2}\sum_{i=1}^{n}\left[\frac{2n}{\beta}\left\{Sin^{-1}\left\{\frac{n}{2\beta w_i}\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{Sin\frac{\beta}{2n}}\right\}\right\}+1\right]=\alpha_0$$

and

$$\frac{1}{2}\sum_{i=1}^{n}\left[\frac{2n}{\beta}\left\{Sin^{-1}\left\{\frac{n}{2\beta w_i}\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{Sin\frac{\beta}{2n}}\right\}\right\}+1\right]g_r(x_i)=K$$

When $\lambda_2\to 0$, we achieve the subsequent manifestation:

$$\alpha_0=\frac{1}{2}\sum_{i=1}^{n}\left[\frac{2n}{\beta}\left\{Sin^{-1}\left\{\frac{n}{2\beta w_i}\frac{\{\lambda_1\}}{Sin\frac{\beta}{2n}}\right\}\right\}+1\right]$$

and $K=\frac{1}{2}\sum_{i=1}^{n}\left[\frac{2n}{\beta}\left\{Sin^{-1}\left\{\frac{n}{2\beta w_i}\frac{\{\lambda_1\}}{Sin\frac{\beta}{2n}}\right\}\right\}+1\right]g_r(x_i)$

Thus when $\lambda_2>0$, we accomplish the subsequent materialization:

$$\left[H_1(A;W)\right]_{\max}=\sum_{i=1}^{n}\left[\begin{array}{l}\frac{1}{2}Sin\frac{\beta}{n}\left[\frac{2n}{\beta}\left\{Sin^{-1}\left\{\frac{n}{2\beta w_i}\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{Sin\frac{\beta}{2n}}\right\}\right\}+1\right]\\+Sin\frac{\beta}{n}\left[1-\frac{1}{2}\left[\frac{2n}{\beta}\left\{Sin^{-1}\left\{\frac{n}{2\beta w_i}\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{Sin\frac{\beta}{2n}}\right\}\right\}+1\right]\right]-Sin\frac{\beta}{n}\end{array}\right]$$

Let $\frac{\beta}{n}\left[\frac{2n}{\beta}\left\{Sin^{-1}\left\{\frac{n}{2\beta w_i}\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{Sin\frac{\beta}{2n}}\right\}\right\}+1\right]=\theta$

As a consequence, we acquire

$$\left[H_1(A;W)\right]_{\max}=\sum_{i=1}^{n} w_i f(\theta)$$

where $f(\theta)=\frac{1}{2}Sin\theta+Sin\frac{\beta}{n}\left\{1-\frac{n\theta}{2\beta}\right\}-Sin\frac{\beta}{n}$ and $f''(\theta)=-\left\{\frac{1}{2}Sin\theta+Sin\frac{\beta}{n}\left\{1-\frac{n\theta}{2\beta}\right\}\right\}<0$

which confirm the concavity of $f(\theta)$ and in view of the fact that sum of concave functions is also concave, we perceive that $\left[H_1(A;W)\right]_{\max}$ is concave.

Problem-II: Here, we think about optimizing the fuzzy entropic model created in equation (2.2) of the above section under the circumstances of fuzzy constraints (3.1) and (3.2).

To make available the solution, we expose the subsequent Lagrangian outward show:

$$L=\sum_{i=1}^{n} w_i\left[Cos\beta\mu_A(x_i)+Cos\beta(1-\mu_A(x_i))-2Cos^2\frac{\beta}{2}\right]+\lambda_1\left\{\sum_{i=1}^{n}\mu_A(x_i)-\alpha_0\right\}+\lambda_2\left\{\sum_{i=1}^{n}\mu_A(x_i)g_r(x_i)-K\right\} \quad (3.6)$$

Differentiating equation (3.6) w.r.t. $\mu_A(x_i)$, we get hold of the subsequent materialization:

$$\frac{\partial L}{\partial\mu_A(x_i)}=\beta w_i\left[-Sin\beta\mu_A(x_i)+Sin\beta\left(1-\mu_A(x_i)\right)\right]+\{\lambda_1+\lambda_2 g_r(x_i)\}$$

$$=2\beta w_i Cos\frac{\beta}{2}Sin\beta\frac{\{1-2\mu_A(x_i)\}}{2}+\{\lambda_1+\lambda_2 g_r(x_i)\}$$

Thus $\frac{\partial L}{\partial\mu_A(x_i)}=0$ gives $2\beta w_i Cos\frac{\beta}{2}Sin\beta\frac{\{1-\mu_A(x_i)\}}{2}=-\{\lambda_1+\lambda_2 g_r(x_i)\}$

or $\mu_A(x_i)=\frac{1}{2}\left[\frac{2}{\beta}\left\{Sin^{-1}\left\{\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{2\beta w_i Cos\frac{\beta}{2}}\right\}\right\}+1\right]$

Employing the constraints (3.1) and (3.2), we get hold of the subsequent revelation:

$$\frac{1}{2}\sum_{i=1}^{n}\left[\frac{2}{\beta}\left\{Sin^{-1}\left\{\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{2\beta w_i Cos\frac{\beta}{2}}\right\}\right\}+1\right]=\alpha_0$$

and $\frac{1}{2}\sum_{i=1}^{n}\left[\frac{2}{\beta}\left\{Sin^{-1}\left\{\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{2\beta w_i Cos\frac{\beta}{2}}\right\}\right\}+1\right]g_r\left(x_i\right)=K$

When $\lambda_2 \to 0$, we achieve the subsequent manifestation:

$$\alpha_0=\frac{1}{2}\sum_{i=1}^{n}\left[\frac{2}{\beta}\left\{Sin^{-1}\left\{\frac{\lambda_1}{2\beta w_i Cos\frac{\beta}{2}}\right\}\right\}+1\right]$$

and $K=\frac{1}{2}\sum_{i=1}^{n}\left[\frac{2}{\beta}\left\{Sin^{-1}\left\{\frac{\lambda_1}{2\beta w_i Cos\frac{\beta}{2}}\right\}\right\}+1\right]g_r\left(x_i\right)$

Thus when $\lambda_2 > 0$, we have

$$\left[H_2\left(A;W\right)\right]_{\max}=\sum_{i=1}^{n}\left[\begin{array}{l} Cos\beta\left[\frac{1}{2}\left[\frac{2}{\beta}\left\{Sin^{-1}\left\{\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{2\beta w_i Cos\frac{\beta}{2}}\right\}\right\}\right]+1\right] \\ +Cos\beta\left[\left[1-\frac{1}{2}\left[\frac{2}{\beta}\left\{Sin^{-1}\left\{\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{2\beta w_i Cos\frac{\beta}{2}}\right\}\right\}\right]\right]+1\right]-2Cos^2\frac{\beta}{2} \end{array}\right] \quad (3.7)$$

Let $\beta\left[\frac{1}{2}\left[\frac{2}{\beta}\left\{Sin^{-1}\left\{\frac{\{\lambda_1+\lambda_2 g_r(x_i)\}}{2\beta w_i Cos\frac{\beta}{2}}\right\}\right\}\right]+1\right]=\theta = \theta$

Thus $\left[H_2\left(A;W\right)\right]_{\max}=\sum_{i=1}^{n}w_i f(\theta)$

where $f(\theta)=Cos\theta+Cos\{\beta-\theta\}-2Cos^2\frac{\beta}{2}$ and $f''(\theta)=-\left\{Cos\theta+Cos\{\beta-\theta\}\right\}<0<0$

Consequently (3.7) confirms the concavity of $\left[H_2\left(A\right)\right]_{\max}$.

Concluding Remarks: The *maximum entropy principle* has established remarkable applications coupled with probabilistic spaces but under inescapable circumstances where such models cannot fit into place, we make certain the prospect of fuzzy models. Moreover, in view of the fact that the magnitude of weighted models cannot be overlooked from application point of observation, we have engendered weighted fuzzy models for discrete fuzzy distributions. Our conclusions make accessible the learning of *maximum fuzziness* under a situation of fuzzy

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restrictions. It is complementary that such learning be capable of the extension to auxiliary well acknowledged models in the fuzzy spaces.

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