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Multiplicative Triple Fibonacci Sequence of Fourth Order Under Nine Specific Schemes

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Abstract
K.T. Atanassov are Firstly established the Coupled Fibonacci Sequence in 1985. In 1987, The essence of Fibonacci Triple Sequences are examined. Fibonacci Sequence stand out as a kind of super sequence with amazing properties. This is the meteoric expansion in the province of Fibonacci Sequence. Leonardo de Pisa foremost Fibonacci's observation on the growth of the rabbit population as a result in 1202.
Triple Fibonacci Sequence are hype in the last years, but Multiplicative Triple Sequence of Recurrence Relations are less known. Extravagant work has been done to course on Fibonacci Triple Sequence in Additive form. In 1995, Multiplicative Coupled Fibonacci Sequence are treated. Our wish of this paper to offer some results of Multiplicative Triple Fibonacci Sequence of fourth order under nine specific schemes.
Keywords- Fibonacci Sequence, Multiplicative Triple Fibonacci sequence

1. **Introduction**
The Fibonacci Triple Sequence is a current guidance in universality of Coupled Fibonacci sequence. Fibonacci sequence and their abstract principle have umpteen tempting utilization and properties to every field of science. The best motive for this relevance is Koshy's book [9]. The Coupled Fibonacci Sequence was first installed by K.T. Atanassov [4] and also investigated many inquisitive properties and a modern protocol of generalization of Fibonacci Sequence [2,5,6].
J.Z. Lee and J.S. Lee ratified Firstly Additive Triple Sequence [3]. K.T. Atanassov lay out new notion for Additive Triple Fibonacci Sequence [7,8] and called 3-Fibonacci Sequence or 3-F Sequence.

Let $\{\mathfrak{P}_i\}_{i=0}^{\infty}$ $\{\mathfrak{Q}_i\}_{i=0}^{\infty}$ and $\{\mathfrak{R}_i\}_{i=0}^{\infty}$ be three infinite sequences and called 3-F Sequence or Triple Fibonacci Sequence with initial value a, b, c, d, e and f.
If $\mathfrak{P}_0 = a, \mathfrak{Q}_0 = b, \mathfrak{R}_0 = c, \mathfrak{P}_1 = d, \mathfrak{Q}_1 = e, \mathfrak{R}_1 = f$, then nine different schemes of Multiplicative Triple Fibonacci Sequence are as follows:

First Scheme:

$$\mathfrak{P}_{n+2} = \mathfrak{Q}_{n+1}.\mathfrak{R}_n$$
$$\mathfrak{Q}_{n+2} = \mathfrak{R}_{n+1}.\mathfrak{P}_n$$
$$\mathfrak{R}_{n+2} = \mathfrak{P}_{n+1}.\mathfrak{Q}_n$$

Second Scheme:

$$\mathfrak{P}_{n+2} = \mathfrak{R}_{n+1}.\mathfrak{Q}_n$$
$$\mathfrak{Q}_{n+2} = \mathfrak{P}_{n+1}.\mathfrak{R}_n$$
$$\mathfrak{R}_{n+2} = \mathfrak{Q}_{n+1}.\mathfrak{P}_n$$

Third Scheme:

$$\mathfrak{P}_{n+2} = \mathfrak{P}_{n+1}.\mathfrak{Q}_n$$
$$\mathfrak{Q}_{n+2} = \mathfrak{Q}_{n+1}.\mathfrak{R}_n$$
$$\mathfrak{R}_{n+2} = \mathfrak{R}_{n+1}.\mathfrak{P}_n$$

Fourth Scheme:

$$\mathfrak{P}_{n+2} = \mathfrak{Q}_{n+1}.\mathfrak{P}_n$$
$$\mathfrak{Q}_{n+2} = \mathfrak{R}_{n+1}.\mathfrak{Q}_n$$
$$\mathfrak{R}_{n+2} = \mathfrak{P}_{n+1}.\mathfrak{R}_n$$

Fifth Scheme:

$$\mathfrak{P}_{n+2} = \mathfrak{P}_{n+1}.\mathfrak{R}_n$$
$$\mathfrak{Q}_{n+2} = \mathfrak{Q}_{n+1}.\mathfrak{P}_n$$
$$\mathfrak{R}_{n+2} = \mathfrak{R}_{n+1}.\mathfrak{Q}_n$$

Sixth Scheme:

$$\mathfrak{P}_{n+2} = \mathfrak{R}_{n+1}.\mathfrak{P}_n$$
$$\mathfrak{Q}_{n+2} = \mathfrak{P}_{n+1}.\mathfrak{Q}_n$$
$$\mathfrak{R}_{n+2} = \mathfrak{Q}_{n+1}.\mathfrak{R}_n$$

Seventh Scheme:

$$\mathfrak{P}_{n+2} = \mathfrak{P}_{n+1}.\mathfrak{P}_n$$
$$\mathfrak{Q}_{n+2} = \mathfrak{Q}_{n+1}.\mathfrak{Q}_n$$
$$\mathfrak{R}_{n+2} = \mathfrak{R}_{n+1}.\mathfrak{R}_n$$

Eighth Scheme:

$$\mathfrak{P}_{n+2} = \mathfrak{Q}_{n+1}.\mathfrak{Q}_n$$
$$\mathfrak{Q}_{n+2} = \mathfrak{R}_{n+1}.\mathfrak{R}_n$$
$$\mathfrak{R}_{n+2} = \mathfrak{P}_{n+1}.\mathfrak{P}_n$$

Ninth Scheme:

$$\mathfrak{P}_{n+2} = \mathfrak{R}_{n+1}.\mathfrak{R}_n$$
$$\mathfrak{Q}_{n+2} = \mathfrak{P}_{n+1}.\mathfrak{P}_n$$
$$\mathfrak{R}_{n+2} = \mathfrak{Q}_{n+1}.\mathfrak{Q}_n$$

O.P. Sikhwal, M. Singh, S. Bhatnagar [1] studied numerous results of second order.
In this paper, we encourage some results on Multiplicative Triple Fibonacci Sequence of fourth order under nine specific schemes.

2. Multiplicative Triple Fibonacci Sequence of third order:
Let $\{\mathfrak{P}_i\}_{i=0}^{\infty}$ $\{\mathfrak{Q}_i\}_{i=0}^{\infty}$ and $\{\mathfrak{R}_i\}_{i=0}^{\infty}$ be three infinite sequences and called 3-F Sequence or Triple Fibonacci Sequence with initial value a, b, c, d, e, f, g, h and i be given.
If $\mathfrak{P}_0 = a, \mathfrak{Q}_0 = b, \mathfrak{R}_0 = c, \mathfrak{P}_1 = d, \mathfrak{Q}_1 = e, \mathfrak{R}_1 = f, \mathfrak{P}_2 = g, \mathfrak{Q}_2 = h, \mathfrak{R}_2 = i$ then there are twenty-seven different schemes of Multiplicative Triple Fibonacci Sequence.

3. Multiplicative Triple Fibonacci Sequence of fourth order:
Let $\{\mathfrak{P}_i\}_{i=0}^{\infty}$ $\{\mathfrak{Q}_i\}_{i=0}^{\infty}$ and $\{\mathfrak{R}_i\}_{i=0}^{\infty}$ be three infinite sequences and called 3-F Sequence or Triple Fibonacci Sequence with initial value a, b, c, d, e, f, g, h, i, j, k and l be given.
If $\mathfrak{P}_0 = a, \mathfrak{Q}_0 = b, \mathfrak{R}_0 = c, \mathfrak{P}_1 = d, \mathfrak{Q}_1 = e, \mathfrak{R}_1 = f, \mathfrak{P}_2 = g, \mathfrak{Q}_2 = h, \mathfrak{R}_2 = i, \mathfrak{P}_3 = j, \mathfrak{Q}_3 = k, \mathfrak{R}_3 = l$ Then there are 81 schemes of Multiplicative Triple Fibonacci Sequence of fourth order. In this paper, we are presenting some identities of fourth order under nine specific schemes and these nine schemes are as follows:

First Scheme:

$$\mathfrak{P}_{n+3} = \mathfrak{P}_{n+2}.\mathfrak{P}_{n+1}.\mathfrak{P}_n$$
$$\mathfrak{Q}_{n+3} = \mathfrak{Q}_{n+2}.\mathfrak{Q}_{n+1}.\mathfrak{Q}_n$$
$$\mathfrak{R}_{n+3} = \mathfrak{R}_{n+2}.\mathfrak{R}_{n+1}.\mathfrak{R}_n$$

Second Scheme:

$$\mathfrak{P}_{n+3} = \mathfrak{Q}_{n+2}.\mathfrak{Q}_{n+1}.\mathfrak{Q}_n$$
$$\mathfrak{Q}_{n+3} = \mathfrak{R}_{n+2}.\mathfrak{R}_{n+1}.\mathfrak{R}_n$$
$$\mathfrak{R}_{n+3} = \mathfrak{P}_{n+2}.\mathfrak{P}_{n+1}.\mathfrak{P}_n$$

Third Scheme:

$$\mathfrak{P}_{n+3} = \mathfrak{R}_{n+2}.\mathfrak{R}_{n+1}.\mathfrak{R}_n$$
$$\mathfrak{Q}_{n+3} = \mathfrak{P}_{n+2}.\mathfrak{P}_{n+1}.\mathfrak{P}_n$$
$$\mathfrak{R}_{n+3} = \mathfrak{Q}_{n+2}.\mathfrak{Q}_{n+1}.\mathfrak{Q}_n$$

Fourth Scheme:

$$\mathfrak{P}_{n+3} = \mathfrak{P}_{n+2}.\mathfrak{Q}_{n+1}.\mathfrak{R}_n$$
$$\mathfrak{Q}_{n+3} = \mathfrak{Q}_{n+2}.\mathfrak{R}_{n+1}.\mathfrak{P}_n$$
$$\mathfrak{R}_{n+3} = \mathfrak{R}_{n+2}.\mathfrak{P}_{n+1}.\mathfrak{Q}_n$$

Fifth Scheme:

$$\mathfrak{P}_{n+3} = \mathfrak{R}_{n+2}.\mathfrak{P}_{n+1}.\mathfrak{Q}_n$$
$$\mathfrak{Q}_{n+3} = \mathfrak{P}_{n+2}.\mathfrak{Q}_{n+1}.\mathfrak{R}_n$$

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$$\mathfrak{R}_{n+3} = \mathfrak{Q}_{n+2}.\mathfrak{R}_{n+1}.\mathfrak{P}_n$$

Sixth Scheme:

$$\mathfrak{P}_{n+3} = \mathfrak{Q}_{n+2}.\mathfrak{R}_{n+1}.\mathfrak{P}_n$$
$$\mathfrak{Q}_{n+3} = \mathfrak{R}_{n+2}.\mathfrak{P}_{n+1}.\mathfrak{Q}_n$$
$$\mathfrak{R}_{n+3} = \mathfrak{P}_{n+2}.\mathfrak{Q}_{n+1}.\mathfrak{R}_n$$

Seventh Scheme:

$$\mathfrak{P}_{n+3} = \mathfrak{P}_{n+2}.\mathfrak{R}_{n+1}.\mathfrak{Q}_n$$
$$\mathfrak{Q}_{n+3} = \mathfrak{Q}_{n+2}.\mathfrak{P}_{n+1}.\mathfrak{R}_n$$
$$\mathfrak{R}_{n+3} = \mathfrak{R}_{n+2}.\mathfrak{Q}_{n+1}.\mathfrak{P}_n$$

Eighth Scheme:

$$\mathfrak{P}_{n+3} = \mathfrak{Q}_{n+2}.\mathfrak{P}_{n+1}.\mathfrak{R}_n$$
$$\mathfrak{Q}_{n+3} = \mathfrak{R}_{n+2}.\mathfrak{Q}_{n+1}.\mathfrak{P}_n$$
$$\mathfrak{R}_{n+3} = \mathfrak{P}_{n+2}.\mathfrak{R}_{n+1}.\mathfrak{Q}_n$$

Nineth Scheme:

$$\mathfrak{P}_{n+3} = \mathfrak{R}_{n+2}.\mathfrak{Q}_{n+1}.\mathfrak{P}_n$$
$$\mathfrak{Q}_{n+3} = \mathfrak{P}_{n+2}.\mathfrak{R}_{n+1}.\mathfrak{Q}_n$$
$$\mathfrak{R}_{n+3} = \mathfrak{Q}_{n+2}.\mathfrak{P}_{n+1}.\mathfrak{R}_n$$

Table for first scheme with few terms as below:

n	$\mathfrak{P}_n$	$\mathfrak{Q}_n$	$\mathfrak{R}_n$
0	a	b	c
1	d	e	f
2	g	h	i
3	j	k	l
4	$adgj$	$behk$	$cfil$
5	$ad^2g^2j^2$	$be^2h^2k^2$	$cf^2i^2l^2$

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Table for second scheme with few terms as below:

n	$\mathfrak{P}_n$	$\mathfrak{Q}_n$	$\mathfrak{R}_n$
0	a	b	c
1	d	e	f
2	g	h	i
3	j	k	l
4	$behk$	$cfil$	$adgj$
5	$be^2h^2k^2$	$cf^2i^2l^2$	$ad^2g^2j^2$

Table for third scheme with few terms as below:

n	$\mathfrak{P}_n$	$\mathfrak{Q}_n$	$\mathfrak{R}_n$
0	a	b	c
1	d	e	f
2	g	h	i
3	j	k	l
4	$cfil$	$adgj$	$behk$
5	$cf^2i^2l^2$	$ad^2g^2j^2$	$be^2h^2k^2$

4. Main Result:

We can use any of above mentioned nine schemes to prove theorem 1,2 and 3.

Theorem 1: For every natural number $n \geq 2$,

$$(\mathfrak{P}_0\mathfrak{Q}_0\mathfrak{R}_0)^n(\mathfrak{P}_1\mathfrak{Q}_1\mathfrak{R}_1)^{n+1}(\mathfrak{P}_2\mathfrak{Q}_2\mathfrak{R}_2)^{n+2}(\mathfrak{P}_3\mathfrak{Q}_3\mathfrak{R}_3)^{n+3}$$
$$= (\mathfrak{P}_3\mathfrak{Q}_3\mathfrak{R}_3)(\mathfrak{P}_4\mathfrak{Q}_4\mathfrak{R}_4)^{n-2}(\mathfrak{P}_6\mathfrak{Q}_6\mathfrak{R}_6)$$

Proof: We prove this above result by induction method:

For $n = 2$ then $(\mathfrak{P}_0\mathfrak{Q}_0\mathfrak{R}_0)^2(\mathfrak{P}_1\mathfrak{Q}_1\mathfrak{R}_1)^3(\mathfrak{P}_2\mathfrak{Q}_2\mathfrak{R}_2)^4(\mathfrak{P}_3\mathfrak{Q}_3\mathfrak{R}_3)^5$

$$= (\mathfrak{P}_1\mathfrak{Q}_1\mathfrak{R}_1)(\mathfrak{P}_2\mathfrak{Q}_2\mathfrak{R}_2)^2(\mathfrak{P}_3\mathfrak{Q}_3\mathfrak{R}_3)^3(\mathfrak{P}_4\mathfrak{Q}_4\mathfrak{R}_4)^2$$

(By First scheme)

$$= (\mathfrak{P}_2\mathfrak{Q}_2\mathfrak{R}_2)(\mathfrak{P}_3\mathfrak{Q}_3\mathfrak{R}_3)^2(\mathfrak{P}_4\mathfrak{Q}_4\mathfrak{R}_4)(\mathfrak{P}_5\mathfrak{Q}_5\mathfrak{R}_5)$$

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(By First scheme)

$$= (\mathfrak{P}_3\mathfrak{Q}_3\mathfrak{R}_3)(\mathfrak{P}_6\mathfrak{Q}_6\mathfrak{R}_6)$$

The result is true for $n = 2$

Let us assume the result is true for $n \geq 2$

Then

$$(\mathfrak{P}_0\mathfrak{Q}_0\mathfrak{R}_0)^{n+1}(\mathfrak{P}_1\mathfrak{Q}_1\mathfrak{R}_1)^{n+2}(\mathfrak{P}_2\mathfrak{Q}_2\mathfrak{R}_2)^{n+3}(\mathfrak{P}_3\mathfrak{Q}_3\mathfrak{R}_3)^{n+4}$$
$$= (\mathfrak{P}_0\mathfrak{Q}_0\mathfrak{R}_0)(\mathfrak{P}_1\mathfrak{Q}_1\mathfrak{R}_1)(\mathfrak{P}_2\mathfrak{Q}_2\mathfrak{R}_2)(\mathfrak{P}_3\mathfrak{Q}_3\mathfrak{R}_3)(\mathfrak{P}_0\mathfrak{Q}_0\mathfrak{R}_0)^{n}(\mathfrak{P}_1\mathfrak{Q}_1\mathfrak{R}_1)^{n+1}$$
$$(\mathfrak{P}_2\mathfrak{Q}_2\mathfrak{R}_2)^{n+2}(\mathfrak{P}_3\mathfrak{Q}_3\mathfrak{R}_3)^{n+3}$$
$$= (\mathfrak{P}_4\mathfrak{Q}_4\mathfrak{R}_4)(\mathfrak{P}_3\mathfrak{Q}_3\mathfrak{R}_3)(\mathfrak{P}_4\mathfrak{Q}_4\mathfrak{R}_4)^{n-2}(\mathfrak{P}_6\mathfrak{Q}_6\mathfrak{R}_6)$$

(By hypothesis)

$$= (\mathfrak{P}_3\mathfrak{Q}_3\mathfrak{R}_3)(\mathfrak{P}_4\mathfrak{Q}_4\mathfrak{R}_4)^{n-1}(\mathfrak{P}_6\mathfrak{Q}_6\mathfrak{R}_6) \quad \text{(By first scheme)}$$

Thus, the result is true for $n + 1$. Hence by induction method the result is true for any positive integer $n \geq 2$.

Theorem 2: For every even integer $n \geq 0$,

$$(\mathfrak{P}_n\mathfrak{Q}_n\mathfrak{R}_n)^{[\frac{n}{2}]}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+3}$$
$$= (\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^2(\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})$$

Proof: We prove the above result by induction method:

For $n = 0$ then

$$(\mathfrak{P}_0\mathfrak{Q}_0\mathfrak{R}_0)(\mathfrak{P}_1\mathfrak{Q}_1\mathfrak{R}_1)(\mathfrak{P}_2\mathfrak{Q}_2\mathfrak{R}_2)^2(\mathfrak{P}_3\mathfrak{Q}_3\mathfrak{R}_3)^3 = (\mathfrak{P}_2\mathfrak{Q}_2\mathfrak{R}_2)(\mathfrak{P}_3\mathfrak{Q}_3\mathfrak{R}_3)^2(\mathfrak{P}_4\mathfrak{Q}_4\mathfrak{R}_4)$$

The result is true for $n = 0$

Let us assume the result is true for $n \geq 0$

Then

$$(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n+2}{2}]}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n+2}{2}]+1}(\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})^{[\frac{n+2}{2}]+2}(\mathfrak{P}_{n+5}\mathfrak{Q}_{n+5}\mathfrak{R}_{n+5})^{[\frac{n+2}{2}]+3}$$
$$=$$
$$(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+5}\mathfrak{Q}_{n+5}\mathfrak{R}_{n+5})^{[\frac{n}{2}]+4}$$

………..(*)

Now we will bring each part of this equation by solving and put its value here.

$$(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+1}$$
$$= (\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n-1}\mathfrak{Q}_{n-1}\mathfrak{R}_{n-1})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n-2}\mathfrak{Q}_{n-2}\mathfrak{R}_{n-2})^{[\frac{n}{2}]+1}$$

………(a)

$$(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+2}$$
$$= (\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n-1}\mathfrak{Q}_{n-1}\mathfrak{R}_{n-1})^{[\frac{n}{2}]+2}$$

………(b)

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$$(\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})^{[\frac{n}{2}]+3}$$
$$= (\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]+3}$$

………(c)

$$(\mathfrak{P}_{n+5}\mathfrak{Q}_{n+5}\mathfrak{R}_{n+5})^{[\frac{n}{2}]+4}$$
$$= (\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})^{[\frac{n}{2}]+4}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+4}(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+4}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+4}$$

………(d)

now putting the value of equation (a), (b), (c) and (d) in equation (*), we get

$$(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+5}\mathfrak{Q}_{n+5}\mathfrak{R}_{n+5})^{[\frac{n}{2}]+4}$$
$$= (\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n-1}\mathfrak{Q}_{n-1}\mathfrak{R}_{n-1})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n-2}\mathfrak{Q}_{n-2}\mathfrak{R}_{n-2})^{[\frac{n}{2}]+1}$$
$$(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n-1}\mathfrak{Q}_{n-1}\mathfrak{R}_{n-1})^{[\frac{n}{2}]+2}$$
$$(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]+3}$$
$$(\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})^{[\frac{n}{2}]+4}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+4}(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+4}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+4}$$
$$= (\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})^{[\frac{n}{2}]+4}$$
$$(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+4}$$
$$(\mathfrak{P}_{n-1}\mathfrak{Q}_{n-1}\mathfrak{R}_{n-1})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+4}$$
$$(\mathfrak{P}_{n-2}\mathfrak{Q}_{n-2}\mathfrak{R}_{n-2})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n-1}\mathfrak{Q}_{n-1}\mathfrak{R}_{n-1})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+4}$$

Now we use given hypothesis for every line,

$$= (\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})(\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})^{n-2}(\mathfrak{P}_{n+5}\mathfrak{Q}_{n+5}\mathfrak{R}_{n+5})$$
$$(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{n-2}(\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})$$
$$(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{n-2}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})$$
$$(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{n-2}(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})$$
$$= (\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})(\mathfrak{P}_{n+5}\mathfrak{Q}_{n+5}\mathfrak{R}_{n+5})^{n-2}(\mathfrak{P}_{n+6}\mathfrak{Q}_{n+6}\mathfrak{R}_{n+6})$$

Thus, the result is true for $n+2$. Hence by induction method the result is true for any positive even integer $n \geq 0$.

Theorem 3: For every odd integer $n \geq 1$,

$$(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+3}$$
$$= (\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{2}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})$$

Proof: We prove the above result by induction method:

For $n = 1$ then $(\mathfrak{P}_1\mathfrak{Q}_1\mathfrak{R}_1)^0(\mathfrak{P}_2\mathfrak{Q}_2\mathfrak{R}_2)(\mathfrak{P}_3\mathfrak{Q}_3\mathfrak{R}_3)^2(\mathfrak{P}_4\mathfrak{Q}_4\mathfrak{R}_4)^3$

$$= (\mathfrak{P}_2\mathfrak{Q}_2\mathfrak{R}_2)(\mathfrak{P}_3\mathfrak{Q}_3\mathfrak{R}_3)^2(\mathfrak{P}_4\mathfrak{Q}_4\mathfrak{R}_4)$$

The result is true for every odd no. $n = 1$

Let us assume the result is true for every odd no. $n \geq 0$

Then

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$$(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n+2}{2}]}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n+2}{2}]+1}(\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})^{[\frac{n+2}{2}]+2}(\mathfrak{P}_{n+5}\mathfrak{Q}_{n+5}\mathfrak{R}_{n+5})^{[\frac{n+2}{2}]+3}$$
$$=$$
$$(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+5}\mathfrak{Q}_{n+5}\mathfrak{R}_{n+5})^{[\frac{n}{2}]+4}$$

………..(*)

Now we will bring each part of this equation by solving and put its value here.

$$(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+1} = (\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n-1}\mathfrak{Q}_{n-1}\mathfrak{R}_{n-1})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n-2}\mathfrak{Q}_{n-2}\mathfrak{R}_{n-2})^{[\frac{n}{2}]+1}$$

………(a)

$$(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+2} = (\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n-1}\mathfrak{Q}_{n-1}\mathfrak{R}_{n-1})^{[\frac{n}{2}]+2}$$

………(b)

$$(\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})^{[\frac{n}{2}]+3} = (\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]+3}$$

………(c)

$$(\mathfrak{P}_{n+5}\mathfrak{Q}_{n+5}\mathfrak{R}_{n+5})^{[\frac{n}{2}]+4} = (\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})^{[\frac{n}{2}]+4}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+4}(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+4}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+4}$$

………(d)

now putting the value of equation (a), (b), (c) and (d) in equation (*), we get

$$(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+5}\mathfrak{Q}_{n+5}\mathfrak{R}_{n+5})^{[\frac{n}{2}]+4}$$
$$= (\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n-1}\mathfrak{Q}_{n-1}\mathfrak{R}_{n-1})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n-2}\mathfrak{Q}_{n-2}\mathfrak{R}_{n-2})^{[\frac{n}{2}]+1}$$
$$(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n-1}\mathfrak{Q}_{n-1}\mathfrak{R}_{n-1})^{[\frac{n}{2}]+2}$$
$$(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]+3}$$
$$(\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})^{[\frac{n}{2}]+4}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+4}(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+4}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+4}$$
$$= (\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})^{[\frac{n}{2}]+4}$$
$$(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{[\frac{n}{2}]+4}$$
$$(\mathfrak{P}_{n-1}\mathfrak{Q}_{n-1}\mathfrak{R}_{n-1})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{[\frac{n}{2}]+4}$$
$$(\mathfrak{P}_{n-2}\mathfrak{Q}_{n-2}\mathfrak{R}_{n-2})^{[\frac{n}{2}]+1}(\mathfrak{P}_{n-1}\mathfrak{Q}_{n-1}\mathfrak{R}_{n-1})^{[\frac{n}{2}]+2}(\mathfrak{P}_{n}\mathfrak{Q}_{n}\mathfrak{R}_{n})^{[\frac{n}{2}]+3}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{[\frac{n}{2}]+4}$$

Now we use given hypothesis for every line,

$$= (\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})^{n-2}(\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})$$
$$(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})^{n-2}(\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})$$

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$$(\mathfrak{P}_n\mathfrak{Q}_n\mathfrak{R}_n)(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})^{n-2}(\mathfrak{P}_{n+2}\mathfrak{Q}_{n+2}\mathfrak{R}_{n+2})$$
$$(\mathfrak{P}_{n-1}\mathfrak{Q}_{n-1}\mathfrak{R}_{n-1})(\mathfrak{P}_n\mathfrak{Q}_n\mathfrak{R}_n)^{n-2}(\mathfrak{P}_{n+1}\mathfrak{Q}_{n+1}\mathfrak{R}_{n+1})$$
$$= (\mathfrak{P}_{n+3}\mathfrak{Q}_{n+3}\mathfrak{R}_{n+3})(\mathfrak{P}_{n+4}\mathfrak{Q}_{n+4}\mathfrak{R}_{n+4})^{n-2}(\mathfrak{P}_{n+5}\mathfrak{Q}_{n+5}\mathfrak{R}_{n+5})$$

Thus, the result is true for $n+2$. Hence by induction method the result is true for any positive odd integer $n \geq 1$.

We can use first scheme from above mentioned nine schemes to prove theorem 4 and theorem 5.

Theorem 4: For every integer $n \geq 0$,

(a) $\sqrt{\prod_{k=0}^{10n+4} \mathfrak{P}_k} = \mathfrak{P}_4\mathfrak{P}_{14} \dots \dots \mathfrak{P}_{10n+4}$

(b) $\sqrt{\prod_{k=0}^{10n+4} \mathfrak{Q}_k} = \mathfrak{Q}_4\mathfrak{Q}_{14} \dots \dots \mathfrak{Q}_{10n+4}$

(c) $\sqrt{\prod_{k=0}^{10n+4} \mathfrak{R}_k} = \mathfrak{R}_4\mathfrak{R}_{14} \dots \dots \mathfrak{R}_{10n+4}$

Proof: We prove the above result by induction method:

For $n = 1$ then

$$\sqrt{\prod_{k=0}^{14} \mathfrak{P}_k} = \sqrt{\mathfrak{P}_0\mathfrak{P}_1\mathfrak{P}_2\mathfrak{P}_3\mathfrak{P}_4\mathfrak{P}_5\mathfrak{P}_6\mathfrak{P}_7\mathfrak{P}_8\mathfrak{P}_9\mathfrak{P}_{10}\mathfrak{P}_{11}\mathfrak{P}_{12}\mathfrak{P}_{13}\mathfrak{P}_{14}}$$
$$= \sqrt{\mathfrak{P}_4^2\mathfrak{P}_9^2\mathfrak{P}_{14}^2}$$
$$= \mathfrak{P}_4\mathfrak{P}_9\mathfrak{P}_{14}$$

The result is true for every odd no. $n = 1$

Let us assume the result is true for every odd no. $n \geq 0$

Then

$$\sqrt{\prod_{k=0}^{10n+14} \mathfrak{P}_k}$$
$$= \sqrt{10n + 5.10n + 6.10n + 7.10n + 8.10n + 9.10n + 10.10n + 11.10n + 12.10n + 13.10n + 14 \prod_{k=0}^{10n+4} \mathfrak{P}_k}$$
$$= \sqrt{\mathfrak{P}_{10n+9}^2\mathfrak{P}_{10n+14}^2}\mathfrak{P}_4\mathfrak{P}_{14} \dots \dots \mathfrak{P}_{10n+4}$$
$$= \mathfrak{P}_4\mathfrak{P}_{14} \dots \dots \mathfrak{P}_{10n+14}$$

Thus, the result is true for $n+1$. Hence by induction method the result is true for any positive odd integer $n \geq 0$.

Theorem 5: For every integer $n \geq 1$,

(a) $\sqrt{\prod_{k=0}^{10n-1} \mathfrak{P}_k} = \mathfrak{P}_9\mathfrak{P}_{19} \dots \dots \mathfrak{P}_{10n-1}$

(b) $\sqrt{\prod_{k=0}^{10n-1} \mathfrak{Q}_k} = \mathfrak{Q}_9 \mathfrak{Q}_{19} \dots \dots \mathfrak{Q}_{10n-1}$

(c) $\sqrt{\prod_{k=0}^{10n-1} \mathfrak{R}_k} = \mathfrak{R}_9 \mathfrak{R}_{19} \dots \dots \mathfrak{R}_{10n-1}$

Proof: The proof of this theorem can be done by mathematical induction same as above theorem.

5. Conclusion:

Extremely work has been performed on Multiplicative Triple Fibonacci Sequence. In this paper, we have to picture some outcome of Multiplicative Triple Fibonacci Sequence of fourth order under nine specific schemes.

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