JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 4333-4337
https://publishoa.com
ISSN: 1309-3452

Effects of Exponential Demand and Time Proportional Deterioration on Inventory System

Mohan. A[1], T. Iswarya[2]
[1&2] Department of Mathematics
Vels Institute of Science, Technology & Advanced Studies,
Chennai, Tamilnadu, India
Mail ID: iswarya.sbs@velsuniv.ac.in
Received: 2022 March 15; **Revised:** 2022 April 20; **Accepted:** 2022 May 10

Abstract:

This paper deal with deterministic inventory model for deteriorating item with shortages that are partially lost and partially backloggingand time proportional deteriorating items. This model discusses about the demand rate which is exponential decay rate. The holding cost is linear and the deterioration cost is both time dependent. To represent the solution and application of the model, numerical analysis has been provided. By finding the minimum value for the total cost, the model is solved.

1. Introduction

Inventory management primarily deals with stocking and storing of produced goods. Inventory management is required at many locations like facility or supply network so that plans can be made for producing, stocking and storing of materials.

The scope of inventory management deals with lead time, holding cost, production cost, cost of carrying the stocks, asset management, predicting, replenishment, deteriorating and defective goods, returns, quality and quantity management, demand and shortage forecasting, maintaining the physical inventory and physical space for the inventory that is kept in the available space.

The papers of Sandeep Kumar [14], Mishra & Singh [12] are taken and made more realistic by making the demand exponential and the deterioration rate time proportional. Both the deterioration rate and the holding cost that is used in this model is time dependent. Shortages are allowed. Shortages are partially lost and partially backlogging. All the cost used in the model are optimized which gives the minimal value for the Total Cost

2. Assumptions

The following assumptions are made to establish the model.

1. Demand is exponential decay rate, $D = Me^{-\lambda t}$ where M stands for the demand rate and λ stands for rate of change of demand.
2. Shortages are partially lost and partially backlogged and is given by $\pi(x) = e^{-\lambda x}$ and $\pi(T) \geq 0$ where λ is a parameter

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 4333-4337
https://publishoa.com
ISSN: 1309-3452

3. Deterioration rate is time proportional $\theta(t) = \theta t$ where $0 < \theta < 1$
4. The holding cost is a time dependent and it is given by a linear function. i.e., H(t) = a + bt (a>0, b>0).

3. MODEL FORMULATION:

The changes of the inventory level $[0, t_1]$ and shortage period $[t_1, T]$ is given by the following differential equations.

$$\frac{dI_1(t)}{dt} + \theta(t)I_1(t) = -E(t)$$

$$\frac{dI_2(t)}{dt} = -E(t)$$

Where $\theta(t) = \theta t$ and $E(t) = Me^{-\lambda t}$

The boundary conditions are

$$I_1(t) = I_2(t) = 0 \text{ at } t = t_1$$

3.1.PROBLEM DEFINITION

The Inventory diminishes due to the deterioration and demand during $[0, t_1]$. Therefore, the inventory level during the time period $[0, t_1]$, is given by the differential equation as follows

$$\frac{dI_1(t)}{dt} + \theta(t)I_1(t) = -E(t) \; 0 \le t \le T$$

Solving (1) with the boundary conditions, we have

$$I_1(t) = \frac{M}{(\theta - \lambda)}\left[e^{(\theta-\lambda)T - \theta t} - e^{-\lambda t}\right], 0 \le t \le T$$

Inventory level during shortage period $[t_1, T]$ which is partially backlogged depends on the demand. The inventory during $[t_1, T]$ at this stage can be represented by a differential equation as follows.

$$\frac{dI_2(t)}{dt} = -Me^{-\lambda t}$$

The boundary conditions are given as $I_2(t_1)$ =0 and $I_2(t)$ = IB

$$\int dI_2(t) = \int_{t_1}^{t} -Me^{-\lambda t}\, dt$$

$$I_2(t) = -M \int_{t_1}^{t} e^{-\lambda t} dt$$

We get the solution as

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 4333-4337
https://publishoa.com
ISSN: 1309-3452

$$I_2(t) = \frac{M}{(\theta-\lambda)}\left[e^{(\theta-\lambda)T} - 1\right]$$

Therefore, the total cost function has the following elements:

3.3 Holding cost:

$$HC = \int_0^{t_1} H(t) I_1(\mathrm{t})\mathrm{dt}$$

$$= \int_0^{t_1} -\frac{M}{(\theta-\lambda)}\left[e^{(\theta-\lambda)T-\theta t} - e^{-\lambda t}\right](\mathrm{a}+\mathrm{bt})\,\mathrm{dt}$$

$$HC = -\frac{M}{(\theta-\lambda)}\left\{-\frac{ae^{((\theta-\lambda)t_1}}{\lambda}\left[e^{\lambda t} + e^{-\lambda t}\right] + \frac{be^{(\theta-\lambda)t}}{\lambda}\left[2te^{\lambda t} + e^{\lambda t} - e^{-\lambda t}\right]\right\}$$

3.4 ShortageCost during $[t_1, T]$:

$$SC = \int_{t_1}^{T} -\frac{M}{(\theta-\lambda)}\left[e^{(\theta-\lambda)T-\theta t} - e^{-\lambda t}\right] e^{-\lambda x}(t_1 - t)dt$$

$$SC = t_1 e^{\lambda x} e^{(\theta-\lambda)t_1}\left\{e^{\lambda t} - e^{-\lambda t} + \frac{t^2}{2}\left[e^{\lambda t} - e^{-\lambda t}\right]\right\}$$

3.5 Deterioration Cost:

$CD = C[I_1(0) - I_2(0)]$

$CD = CMe^{-\lambda t}\left\{\frac{1}{(\theta-\lambda)}\left[e^{\theta t} - e^{-\lambda t}\right] + \frac{1}{\lambda}\left[1 - e^{-\lambda T}\right]\right\}$

3.6 Ordering Cost per order:

CO = A

3.7Total cost per time unit:

$$TC(t_1, T) = \frac{1}{T}[HC + SC + CO + CD]$$

$TC(t_1, T) =$
$-\frac{M}{(\theta-\lambda)}\left\{-\frac{ae^{((\theta-\lambda)t_1}}{\lambda}\left[e^{\lambda t} + e^{-\lambda t}\right] + \frac{be^{(\theta-\lambda)t}}{\lambda}\left[2te^{\lambda t} + e^{\lambda t} - e^{-\lambda t}\right]\right\} + t_1 e^{\lambda x} e^{(\theta-\lambda)t_1}\left\{e^{\lambda t} - e^{-\lambda t} +\right.$
*t22eλt−e−λt+CMe−λt1θ−λeθt−e−λt+1λ1−e−λT+*A

The Necessary condition for the total cost per time unit to be minimized is

$$\frac{\partial TC}{\partial t_1} = 0 \; and \; \frac{\partial TC}{\partial T} = 0$$

Provided

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 4333-4337
https://publishoa.com
ISSN: 1309-3452

$$\left(\frac{\partial^2 TC}{\partial t_1^2}\right)\left(\frac{\partial^2 TC}{\partial T^2}\right)-\left(\frac{\partial^2 TC}{\partial t_1 \partial T}\right)^2 > 0 \; and \; \left(\frac{\partial^2 TC}{\partial t_1^2}\right) > 0$$

4. Numerical Example and Sensitivity Analysis

We assume the following values with proper units for the above model:

R=500, M=400,$\lambda = 0.01$,a=0.25, C=1,$\theta = 0.7$. Using these values in the above given we obtain the minimum total cost per unit time TC=721.46

5. Conclusion

In this paper, the demand considered is exponential decay rate and linear time holding cost. We can use this model where the inventories depend upon not only the time but also the season of sale. For example, clothes of certain fashion, food and some fashion items used by women.From the above result we can conclude that the above model gives the analytical solution for minimal total cost. This model can be reconstructed using the fuzzy numbers which will give more realistic solution.

6. References

1. Abad, P.L., Optimal price and order-size for a reseller under partial backlogging. Computers and Operation Research, 28, 53- 65.
2. Abad, P.L., Optimal pricing and lot-sizing under conditions of perishability and partial backordering. Management Science, 42, 1093-1104.
3. Abu Hashan Md Mashud. An EOQ deteriorating inventory model with different types of demand and fully backlogged shortages. International Journal of Logistics Systems and Management, Vol. 36, No. 1, 2020
4. Alfares, H. K., 2007. Inventory model with stock-level dependent demand rate and variable holding cost.Int. J. Production Economics 108 (2007) 259-265
5. Dye CY, Ouyang LY, and Hsieh TP (2007) Deterministic inventory model for deteriorating items with capacity constraint and time proportional Backlogging rate. European Journal of Operational Research178 (3):789-807
6. Ghare, P. M. and Schrader, G. F., A model for an exponentially decaying inventory. Journal of Industrial Engineering, 14, 238-243.
7. Goyal, S. K. and B. C. Giri, Recent trends in modeling of deteriorating inventory. European Journal of Operational Research, 134, 1-16.
8. Huang and Hsu. A note on a buyer-vendor EOQ model with changeable lead-time in supply chain, Journal of Information & Optimization Sciences, 29, 305–310.
9. Kuo-Chen Hung, An inventory model with generalized type demand, deterioration and backorder rates, European Journal of Operational Research, 208(3), 239-242.
10. Lee, W-C. and Wu, J-W. (2004) A Note on EOQ model for items with mixtures of exponential distribution deterioration, Shortages and time-varying demand, Quality and Quantity, 38, 457-473.

11. Mishra & L. Singh. Deteriorating inventory model for time dependent demand and holding cost with partial backlogging. International Journal of Management Science and Engineering Management, X(X): 1-5, 2011
12. Mishra, V.K. and Singh, L.S., Deteriorating inventory model with time dependent demand and partial backlogging, Applied Mathematical Sciences, 4(72), 3611-3619.
13. R. Li, H. Lan and J. R. Mawhinney, "A Review on Deteriorating Inventory Study," Journal of Service Science and Manage- ment, Vol. 3, No. 1, 2010, pp.117-129.
14. Sandeep Kumar, Study of an Inventory Model with Time Varying Holding cost, Exponential Decaying Demand and Constant Deterioration, Motherhood International Journal of Multidisciplinary Research & Development, Volume I, Issue IV, May 2017, pp. 01-09
15. W. Ouyang and X. Cheng, "An Inventory Model for Deteriorating Items with Exponential Declining Demand and Partial Backlogging," Yugoslav Journal of Operation Research, Vol. 19, No. 2, 2005, pp. 277-288.