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Reliability Estimation of the Topp-Leone G- Pareto Distribution: Properties and Application

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Abstract:

The singular probability distributions that exist are insufficient to model naturally occurring events. Including the exponential distribution and the one-parameter Pareto distribution as a model for modelling real data. Therefore, there was a need to generalize and expand the probability distributions to develop the effectiveness of the distributions by adding one or more new parameters. Therefore, in this paper, we will introduce (Topp Leone G-Pareto distribution (TLG-P)) as a new model with two parameters (β,θ) based on the proposed family (Topp Leone G family).Therefore, the sum of the mathematical properties such as the central and eccentric moments and the moment-generating function. We will used maximum likelihood method. Moreover, the practical application of real data based on the proposed method. We can be employed for showing this distribution better compared to other distributions.

1. INTRODUCTION

In the past decade, statisticians have developed many families of univariate distributions through the method of adding parameters to distributions. The likes byTopp & Leone. [1]Who pioneered the method of adding parameter to distributions and proposed the exponentiated -G classMarshall and Olkin [2] developed marshall-olkin-G family. In the recent years, introduced other methods of developing families of distributions. Statisticians have developed new families of univariate distributions through the methods of Alzaatreh et al. [3]. Thus, a new distribution obtained by extending any of these new families of distributions with some common distributions to get more flexibility in modelling real life data in some areas such as finance, economics, engineering, agriculture, medicine and biological sciences. Although, statisticians aim at developing a new family of univariate distributions in order to building distributions with a skewed function to the right or left, the function is symmetric, or the function is inverse in the form of -J shape. To make the kurtosis more flexible when compared to that of any baseline distribution. To construct heavy-tailed distributions for modelling various real data sets. To provide persistently better fits than other generated distributions with the same underlying model. Therefore,statisticianshave developed many families of univariate distributions through the methodsby adding parameters to distributions.Such as, the exponentiated generalized-G by Yousef et al. [4]. New Weibull-G by Tahir et al. [5].Transmuted andexponentiated generalized-G by Pourdarvishet al. [6]. Kumaraswamy marshall-olkin by Handique et al. [7]. Zografos-Balakrishnan odd log-logistic family by Gauss et al. [8]. Kumaraswamy transmuted-G by Afify et al. [9]. Burr X-G by Al-Babtain et al. [10]. Generalized odd generalized exponential family by Alizadeh et al. [11]. Beta transmuted-H by Afify et al. [12]. Topp leone odd log-logistic family by Barito et al. [13]. Beta Weibull-G by Yousef et al.[14]. Type I general exponential class of distributions by Hamedani et al. [15]. The transmuted Weibull-G family of distributions by Alizadeh et al. [16]. The gamma-Weibull-G family of distributions by Oluyede[17]. Exponentiated kumaraswamy – G by Cordeiro et al. [18]. Burr-X exponential G-Family of distributions by Sanusi[19]. In this paper we willsuggesta new distribution named is(Top Leone G- Pareto distribution (TLG-P)) with two parameters depending by proposed family (Topp Leone Generator family (TLG-F)**)** mathematical characteristics,To be more flexible in representing real data.We will also discuss some of the main characteristics Such as, quintile function, moment, use(maximum likelihood method (MLE)) implemented. Theapplication for factual data canworkingdirectly to show the better fitting as Compared to other distributions mentioned in the statistical literature.

2. Methodology: Pareto distribution, Topp-Leone G- Pareto Distribution,Topp-Leone distribution, likelihood method,Topp Leone Generator family (TLG-F).

3. Topp Leone – G Family

If the Topp Leon distribution has a cumulative distribution function and a probability density function, Buckley [1]which aregive respectively:

$$\mathrm{F}_{TL}(y,\theta) = x^{\theta}(2-x)^{\theta} \quad ; 0 < x < 1 \,, \theta > 0 \qquad (1)$$

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$$f_{TL}(y,\theta) = 2\theta x^{\theta-1}(1-x)(2-x)^{\theta-1} \quad ;0 < x < 1\,,\theta > 0 \tag{2}$$

It should be The Topp Leone - G Family (TL-GF) is being present Buckley [6], by employing the CDF and PDF respectively:

$$\mathrm{F}(\mathrm{x},\theta,\alpha) \;= \left(\mathrm{G}(\mathrm{x},\alpha)\right)^{\vartheta}(2-\mathrm{G}(\mathrm{x},\alpha))^{\vartheta}(\mathbf{3})$$

$$\mathrm{f}(\mathrm{x},\theta) \;= 2\theta \mathrm{g}(\mathrm{x},\alpha)[1-\;\mathrm{G}(\mathrm{x},\alpha)][\,\mathrm{G}(\mathrm{x},\alpha)]^{\vartheta-1}[\,2-\mathrm{G}(\mathrm{x},\alpha)]^{\vartheta-1}(\mathbf{4})$$

Respectively for $\mathrm{x} > 0, \boldsymbol{\vartheta} > 0$and α is the shape parameter.

4. The Pareto Distribution

The cumulative distribution function and probability density function can be write for the Pareto distribution Buckley [20] re given by the following:

$$\mathbf{G}(\mathrm{x};\beta)_p = 1-\mathrm{x}^{-\beta} \quad ; \; \mathrm{x} \geq 1\,,\beta > 0 \tag{5}$$

And

$$\mathrm{f}(\mathrm{x};\beta)_p = \frac{\beta}{\mathrm{x}^{\beta+1}} \quad ; \; \mathrm{x} \geq 1\,,\beta > 0 \tag{6}$$

5. Topp Leone – G ParetoDistributions (TL-GP)

In thissection, we will suggest a new distribution using the Pareto distribution and the Topp Leone G-family (TLG-f), thisdistribution called(Topp Leone – G Pareto (TL-GP)).

The CDF and the PDF of this distribution(TLG-P)get it by the following:

$$\mathrm{F}(\mathrm{x},\theta) \;= (G(x))^{\theta}(2-G(x))^{\theta} \tag{7}$$

Now we substitute an equation (5) in equation (7)

$$\mathrm{F}_{\mathrm{TLGP}}\,(\mathrm{x},\theta,\beta) = \left(1-\mathrm{x}^{-\beta}\right)^{\theta} \;\; [2-(1-\mathrm{x}^{-\beta})]^{\theta}$$

$$\mathrm{F}_{\mathrm{TLGP}}\,(\mathrm{x},\theta,\beta) = \left(1-\mathrm{x}^{-\beta}\right)^{\theta}(1+\mathrm{x}^{-\beta})^{\theta} \quad ; \; \mathrm{x} > 0 \;\;,\theta,\beta > 0 \tag{8}$$

The pdf of the (TLG-P) is:

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$$\frac{dF_{TLG-P}(x,\theta,\beta)}{dx} = \frac{d}{dx}(1-x^{-\beta})^{\theta}(1+x^{-\beta})^{\theta}$$

$$= \frac{d}{dx}[(1-x^{-\beta})(1+x^{-\beta})]^{\theta}$$

$$= \frac{d}{dx}[(1-x^{-2\beta})]^{\theta}$$

$$=\theta[(1-x^{-2\beta})]^{\theta-1}\left(2\beta\, x^{-2\beta-1}\right)$$

Then the PDF of the TLG-P distribution given by:

$$f_{TLG-P}(x,\theta,\beta) = \begin{cases} 2\theta\beta x^{-2\beta-1}(1-x^{-2\beta})^{\theta-1} & ;\ x \geq 1\ ,\theta,\beta > 0 \\ 0 & ;\quad O.w \end{cases} \tag{9}$$

Where β and θ are the shape parameters.

The pdf plots of the TLG-P distribution displayed in Fig. 1.

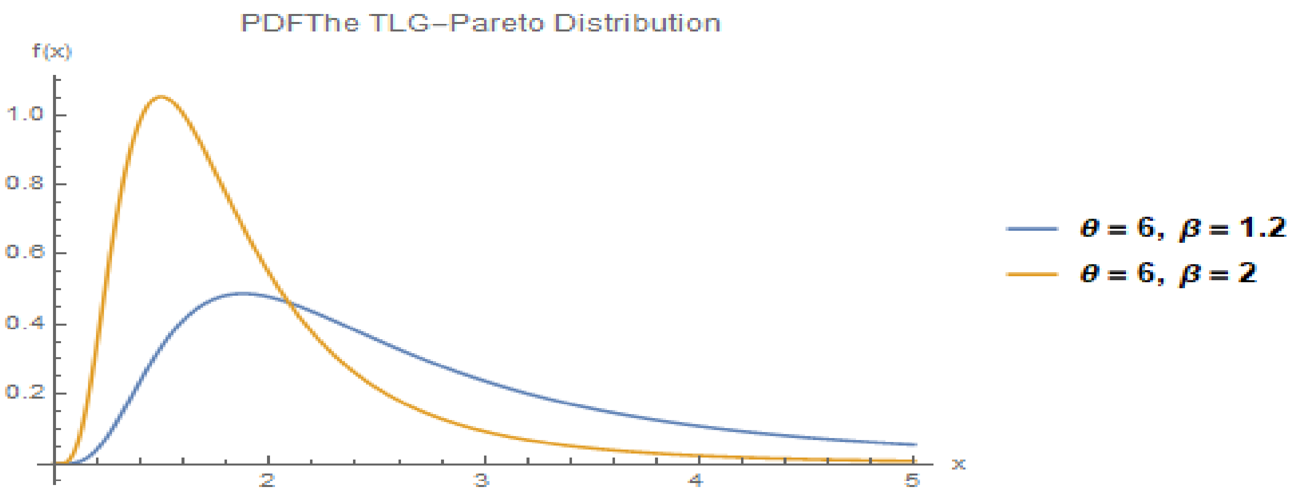

FIGURE 1.PDFTLG-Pareto Distribution

The cumulative distribution function (CDF)of the TLG − P distribution:

$$F_{TLPG}\,(x;\beta;\theta) \quad = P_r(X \leq x)$$

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$$F_{TLPG}(x;\beta;\theta) = \int_{1}^{x} f_{TLPG}(x;\beta;\theta)\,.\,dx$$

$$F_{TLPG}(x;\beta;\theta) = \begin{cases} \left(\mathbf{1}-\mathbf{x}^{-\boldsymbol{\beta}}\right)^{\boldsymbol{\theta}}(\mathbf{1}+\mathbf{x}^{-\boldsymbol{\beta}})^{\boldsymbol{\theta}} & ;\ x \geq 1\ , \theta, \beta > 0 \\ 0 & ;\quad O.w \end{cases} > 0 \qquad (10)$$

The CDF plots of the TLG − P distribution are displayin Fig. 2.

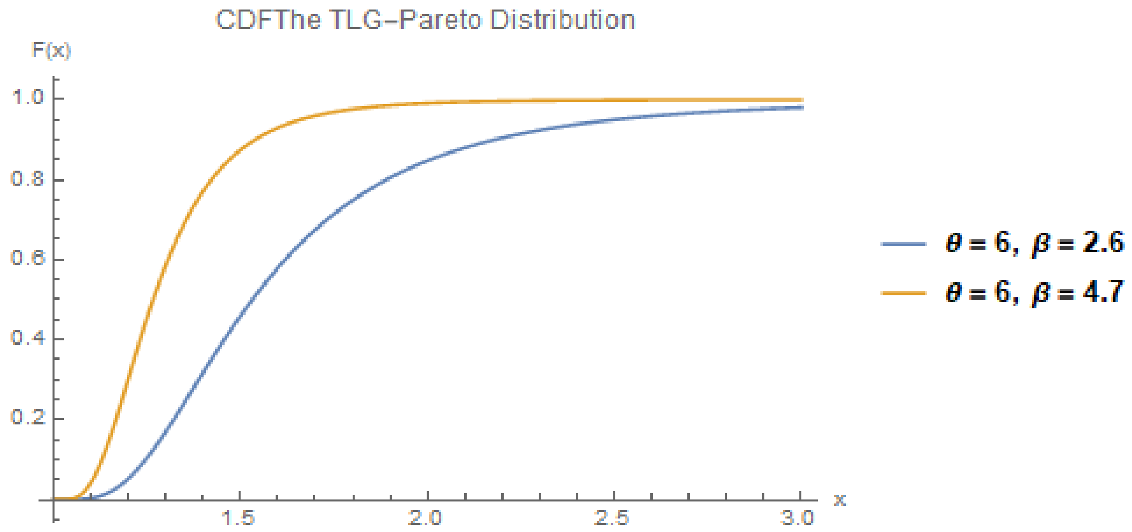

FIGURE 2.CDFTLG-Pareto Distribution

The reliability or Survival function given as:

$$R(x;\beta;\theta)_{TLPG} = 1 - F_{TLG-P}(x;\beta;\theta)$$

$$R(x;\beta;\theta)_{TLPG} = 1 - (1 - x^{-2\beta})^{\theta} (11)$$

The reliability or Survival plots of the TLG − P distribution are display in Fig. 2.

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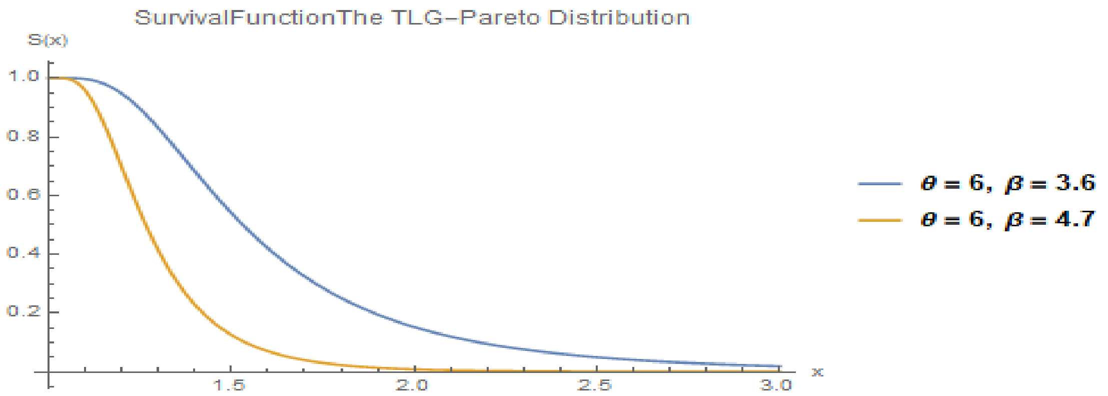

The hazard function of TLG − P distribution (Fig. 4) can obtained as:
FIGUARE 3. Survival function of the TLG-Pareto distribution.

$$H(x;\beta;\theta) = \frac{f(x;\beta;\theta,P)}{S(x;\beta;\theta)}$$

$$H(x;\beta;\theta) = \frac{2\theta\beta x^{-2\beta-1}(1-x^{-2\beta})^{\theta-1}}{1-(1-x^{-2\beta})^{\theta}} \qquad (12)$$

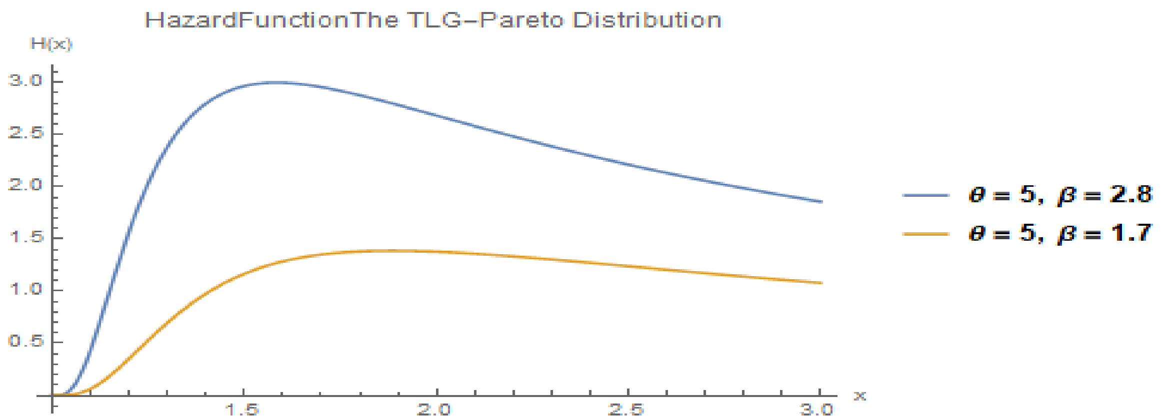

FIGURE 4.Hazard Function The TLG-Pareto Distribution

6. Statistical Properties of TLG-P

Decentralized Moment about origin

Let x be a random variable follows TLG-P distribution with parameterβ, θThe r^{th} raw moment of a continuous random variable x denoted byμ_r' is defined as:

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$$\mu_r = E(x^r) = \int_1^{\infty} x^r f(x) dx \tag{13}$$

$$E(x^r) = \int_1^{\infty} x^r 2\theta\beta x^{-2\beta-1}(1-x^{-2\beta})^{\theta-1} dx$$

$$E(x^r) = 2\theta\beta \int_1^{\infty} x^{r-2\beta-1}(1-x^{-2\beta})^{\theta-1} dx$$

Using the binomial expansion, we get the following:

$$(1-x^{-2\beta})^{\theta-1} = \sum_{k=0}^{\theta-1} \binom{\theta-1}{k} (-1)^k (x^{-2\beta})^k$$

$$(1-x^{-2\beta})^{\theta-1} = \sum_{k=0}^{\theta-1} \binom{\theta-1}{k} (-1)^k x^{-2\beta k}$$

Therefore$E(x^r) = 2\theta\beta \sum_{k=0}^{\theta-1} \binom{\theta-1}{k} (-1)^k x^{-2\beta k} \int_1^{\infty} x^{r-2\beta-1} dx$

$$E(x^r) = 2\theta\beta \sum_{k=0}^{\theta-1} \binom{\theta-1}{k} (-1)^k \int_1^{\infty} x^{r-2\beta k-2\beta-1} dx$$

$$\int_1^{\infty} x^{r-2\beta k-2\beta-1} dx = \frac{x^{-2\beta k-2\beta+r}}{-2\beta k-2\beta+r}$$

$$\int_1^{\infty} x^{r-2\beta k-2\beta-1} dx = \frac{1}{(-2\beta k-2\beta+r)} \left[\frac{1}{x^{(2\beta k+2\beta-r)}}\right]_1^{\infty}$$

$$= -\frac{1}{(2\beta k+2\beta-r)} \left[\frac{1}{\infty} - 1\right]$$

$$= \frac{1}{(2\beta k+2\beta-r)}$$

Therefore, the decentralized torque of the new distribution (TLG-Pareto) given as follows:

$$\mu_r' = E(x^r) = 2\theta\beta \sum_{k=0}^{\theta-1} \binom{\theta-1}{k} (-1)^k \frac{1}{(2\beta k+2\beta-r)} \tag{14}$$

Now we get the first, second, third and fourth moments, respectively of TLG-Pareto Distribution distribution by putting r = 1,2,3,4... In Equation (14) as:

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When r=one, the first eccentric moment (arithmetic mean) of the TLG-Pareto distribution is as follows:

$$\mu_1' = \mathrm{E}(\mathrm{x}^1) = 2\theta\beta \sum_{\mathrm{k}=0}^{\theta-1} \binom{\theta-1}{\mathrm{k}} (-1)^{\mathrm{k}} \frac{1}{(2\beta\mathrm{k}+2\beta-1)} \qquad (15)$$

When r=two, then the second eccentric moment of the (TLG-Pareto) distribution is as follows:

$$\mu_2' = \mathrm{E}(\mathrm{x}^2) = 2\theta\beta \sum_{\mathrm{k}=0}^{\theta-1} \binom{\theta-1}{\mathrm{k}} (-1)^{\mathrm{k}} \frac{1}{(2\beta\mathrm{k}+2\beta-2)} (16)$$

The variance of the new distribution (TLG-Pareto) given as follows:-

$$\mathrm{V}(\mathrm{x}) = \mathrm{E}(\mathrm{x}^2) - \left(\mathrm{E}(\mathrm{x})\right)^2 (17)$$

$$\mathrm{V}(\mathrm{x}) = 2\theta\beta \sum_{\mathrm{k}=0}^{\theta-1} \binom{\theta-1}{\mathrm{k}} (-1)^{\mathrm{k}} \frac{1}{(2\beta\mathrm{k}+2\beta-1)} - \left(2\theta\beta \sum_{\mathrm{k}=0}^{\theta-1} \binom{\theta-1}{\mathrm{k}} (-1)^{\mathrm{k}} \frac{1}{(2\beta\mathrm{k}+2\beta-2)}\right)^2 \qquad (18)$$

Where r=3

$$\mu_3' = \mathrm{E}(\mathrm{x}^3) = 2\theta\beta \sum_{\mathrm{k}=0}^{\theta-1} \binom{\theta-1}{\mathrm{k}} (-1)^{\mathrm{k}} \frac{1}{(2\beta\mathrm{k}+2\beta-3)} (19)$$

Where r=4

$$\mu_4' = \mathrm{E}(\mathrm{x}^4) = 2\theta\beta \sum_{\mathrm{k}=0}^{\theta-1} \binom{\theta-1}{\mathrm{k}} (-1)^{\mathrm{k}} \frac{1}{(2\beta\mathrm{k}+2\beta-4)} (20)$$

Central Moment about mean

Let x denote the random variable follows TLG-P Distribution then moments about the mean of μ_r is:

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$$\mu_r = E(x-\mu)^r = \int_1^{\infty} (x-\mu)^r f_{\mathrm{TLG-P}}(x;\beta;\theta)dx \tag{21}$$

$$\mathrm{E}(\mathrm{x}-\mu_1)^{\mathrm{r}} = 2\theta\beta \sum_{\mathrm{k}=0}^{\mathrm{r}} \binom{\mathrm{r}}{\mathrm{k}} -{\mu_1}^{\mathrm{r}}(\mu_{\mathrm{r-k}}) \int_1^{\infty} \mathrm{x}^{\mathrm{r}-2\beta\mathrm{k}-2\beta-1}\mathrm{dx}$$

$$\mathrm{E}(\mathrm{x}-\mu_1)^{\mathrm{r}} = 2\theta\beta \sum_{\mathrm{k}=0}^{\mathrm{r}} \binom{\mathrm{r}}{\mathrm{k}} -{\mu_1}^{\mathrm{r}}(\mu_{\mathrm{r-k}}) \frac{1}{(2\beta\mathrm{k}+2\beta-\mathrm{r})} \tag{22}$$

Now we get the first, second, third and fourth moments, respectively of TLG-Pareto Distribution distribution by putting r = 1,2,3,4... in Equation (22) as:

Where r=1

$$\mathrm{E}(\mathrm{x}-\mu_1)^1 = 2\theta\beta \sum_{\mathrm{k}=0}^{1} \binom{1}{\mathrm{k}} -{\mu_1}^1(\mu_{1-\mathrm{k}}) \frac{1}{(2\beta\mathrm{k}+2\beta-1)} \tag{23}$$

Where r=2

$$\mathrm{E}(\mathrm{x}-\mu_2)^2 = 2\theta\beta \sum_{\mathrm{k}=0}^{2} \binom{2}{\mathrm{k}} -{\mu_2}^2(\mu_{2-\mathrm{k}}) \frac{1}{(2\beta\mathrm{k}+2\beta-2)} \tag{24}$$

Where r=3

$$\mathrm{E}(\mathrm{x}-\mu_3)^3 = 2\theta\beta \sum_{\mathrm{k}=0}^{3} \binom{3}{\mathrm{k}} -{\mu_3}^3(\mu_{3-\mathrm{k}}) \frac{1}{(2\beta\mathrm{k}+2\beta-3)} \tag{25}$$

COEFFICIENTS OF VARIATION (CV)

$$\mathrm{C.V} = \frac{\sigma}{\mu} \times 100$$

$$\mathrm{C.V} = \frac{\sqrt{2\theta\beta \sum_{\mathrm{k}=0}^{2}\binom{2}{\mathrm{k}} -{\mu_2}^2(\mu_{2-\mathrm{k}}) \frac{1}{(2\beta\mathrm{k}+2\beta-2)}}}{2\theta\beta \sum_{\mathrm{k}=0}^{\theta-1}\binom{\theta-1}{\mathrm{k}}(-1)^{\mathrm{k}} \frac{1}{(2\beta\mathrm{k}+2\beta-1)}} \times 100 \tag{26}$$

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COEFFICIENT OF SKEWEDNESS (C.S)

$$C.S = \frac{E(x-\mu)^3}{\sigma^3}$$

$$C.S = \frac{2\theta\beta\sum_{k=0}^{3}\binom{3}{k} - \mu_3{}^3(\mu_{3-k})\frac{1}{(2\beta k+2\beta-3)}}{\left[2\theta\beta\sum_{k=0}^{2}\binom{2}{k} - \mu_2{}^2(\mu_{2-k})\frac{1}{(2\beta k+2\beta-2)}\right]^{3/2}} \qquad (27)$$

COEFFICIENT OF KURTOSIS (KS)

$$K.S = \frac{E(x-\mu)^4}{\sigma^4}$$

$$K.S = \frac{2\theta\beta\sum_{k=0}^{4}\binom{4}{k} - \mu_4{}^4(\mu_{4-k})\frac{1}{(2\beta k+2\beta-4)}}{\left(2\theta\beta\sum_{k=0}^{2}\binom{2}{k} - \mu_2{}^2(\mu_{2-k})\frac{1}{(2\beta k+2\beta-2)}\right)^{2}} \qquad (28)$$

7. PARAMETER ESTIMATION

Maximum Likelihood Estimators

Let x1, x2, x3, x4,........,x_n be a random sample of size n from TLG-P distribution.

The likelihood function, of TLG-P distribution given by:

$$L = \prod_{i=1}^{n} f(x_i, \emptyset)$$

$$L(x_1, x_2, \ldots\ldots.x_n, \emptyset) = f\left(x_{1,}, \emptyset\right).f(x_2, \emptyset), \ldots\ldots, f(x_n, \emptyset) \qquad (29)$$

$$L = \prod_{i=1}^{n} 2\theta\beta x^{-2\beta-1}(1-x^{-2\beta})^{\theta-1} \qquad (30)$$

$$\ln Lf(X,\theta,\beta) = n\ln(\theta) + n\ln(2) + n\ln\beta - (2\beta+1)\sum_{i=1}^{n}\ln(x) + (\theta-1)\sum_{i=1}^{n}\ln(1-x^{-2\beta}) \qquad (31)$$

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The maximum likelihood estimates of parameters of (TLG-P), canbe get by maximizing the log-likelihood function given in (21), so, taking the derivative for unknown parameters and further proceed as follow:

$$\frac{\partial \ln Lf(X,\theta)}{\partial\theta} = \frac{n}{\theta} + \sum_{i=1}^{n} \ln(1 - xi^{-2\beta}) = 0$$

$$\hat{\theta} = \frac{-n}{\sum_{i=1}^{n} \ln(1 - xi^{-2\beta})} \quad (32)$$

$$\frac{\partial \ln Lf(X,\theta)}{\partial\beta} = \frac{n}{\beta} + \frac{2\ln(xi)}{x^{2\beta} - 1} = 0$$

$$\hat{\beta} = -\frac{\left(nx^{2\beta} - 1\right)}{2\ln(xi)} \quad (33)$$

The MLE of distribution parameters θ and β can be calculated by finding the maximum value for equation (32),(33) finding.

8. APPLICATION OF theTopp-Leone G- Pareto (TLG-P) DISTRIBUTION

The flexibility and performance of (**TLG-P**) Distribution are evaluated on competing models vizone parameter exponential distribution (ED).) one parameter Lindely distribution(LD), Akash distribution, Shankar distribution ardhanadistribution, sujatha distribution and Pareto distribution Here, the distribution is fitted to data set for the number of hours the patients people with virus (covid-19), were in hospital before death for AL Hussein Educational Hospital in Karbala, for sample size (n=97) (see table 1.). The performance of the distribution compared with (TLG-P) Distribution. Exponential distribution (ED), one parameter Lindely distribution(LD), Akash distribution, Shankar distribution ardhana distribution, sujatha distribution and Pareto distribution for the data set using Akaike Information Criterion (AIC), Akaike Information Criterion (AICC) Distribution with the lowest AIC, AICC considered the most flexible and superior distribution for a given data set, The results are present in the tables (2).

TABLE1: Data set for the number of hours patients were in hospital before death

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1.1,1.1,1.3,1.4,1.5,1.5,1.4,1.6,1.8,1.6,1.7,1.7,1.8,1.8,1.9,2,2,2.2,2.2,2.3,2.5,2.3,2.4,2.5,2.5,
2.3,2.3,2.3,2.4,2.4,2.5,2.6,2.6,2.6,2.6,2.6,2.7,2.8,2.8,2.8,2.9,2.9,3,3,3,3,3.1,3.1,3.1,3.2,3.3,
3.3,3.3,3.4,3.4,3.4,3.4,3.4,3.5,3.5,3.5,3.6,3.6,3.6,3.6,3.6,3.6,3.8,3.8,3.9,4,4,4,4,4,4,4.2,4.5,
4.2,4.3,4.4,4.5,4.6,4.7,4.8,4.8,4.9,5,5.2,5.3,5.5,6.3,6.2,6,7,7.2,8

To choose the best model within the set of models that compared with the new distribution, the best is the model corresponding to the lowest value for Akaike Information Criterion (AIC) and Akaike Information Correct (AIC_c) (see tabul 2.), the general formula for (AIC) and (AIC_c)are:

$$\mathrm{AIC} = -2\log\left(\frac{\hat{\theta}_{\mathrm{MLE}}}{x}\right) + 2K$$

Where:

$\log\left(\frac{\hat{\theta}_{MLE}}{x}\right)$: Value of the logarithm maximum likelihood function.

K: Estimated number of parameters.

And

$$\mathrm{AIC_c} = \mathrm{AIC} + \frac{2K(K+1)}{N-K-1}$$

Where, AIC: Akaike Information Criterion.

K: Estimated number of parameters.

N: sample size

TABLE 2. Parameters estimates and goodness – of – fits by Akaike Information Criterion (AIC), Akaike Information Criterion Corrected (AICC) .

Distributions	MLE	X^2Anderson-D		Cramer- V		AIC	AIC_C
		statistic	P-Value	Statistic	P-Value		

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TLG-P	$\hat{\beta} = 1.0217$ $\hat{\theta} = 5.2647$	3.2944	0.0695	0.29585	0.1388	357.07	357.20
Exp	$\hat{\lambda} = 0.521$	16.45	0.2102	0.8758	0.1488	457.27	457.31
Akash	$\hat{\alpha} = 0.76908$	7.244	0.8106	0.3285	0.91254	374.90	374.94
Pareto	$\hat{\beta} = 4.1401$	6.3422	0.0687	3.9764	0.16921	401.33	401.37

CONCLUSION

In this paper,the new Topp-Leone G- Pareto (TLG-P) distribution,some of the properties are derive and discussed like moments, reliability analysis, and hazard rate. The method of maximum likelihood estimation is use for determining the parameters. The performance of the new model is determined by fitting to real-life data using the goodness of fit criteria such as AIC, AIC_C.The value of P-Value tests (Cramer -von mises, Anderson-Darling) greater than the moral level (0.05), and this leads not to reflect the hypothesis of the notice (the appropriateness of the real data for probability distributions under study. It is found that Topp-Leone G- Pareto (TLG-P) distribution gives a better fit to the data set as compared (TLG-P) one parameter exponential distribution (ED), Akash distribution, and Pareto distribution Further) depending on the values of AIC, AICC in the table (2).

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