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# Seismic Wave Propagation at Plane Interface of Two Micropolar Mediums

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**Abstract**

The objective of this paper is to study the refraction and reflection of longitudinal (LD) waves at plane interface between micropolar elastic solid (MES) and electro-microelastic solid (EMS) half-spaces. A LD wave is taken to be impinge obliquely at the plane interface between MES and EMS half spaces. The ratios of amplitude of different type of refracted and reflected waves have been obtained numerically and results have been depicted graphically with the help of MATLAB Graphical routines. This has been observed that the mentioned amplitude ratios depend on the material properties and angle of incidence of incident waves. Also few particular cases have been discussed and then obtained result is compared with the exist ones. This study is very useful for the researchers pursing the research in the field of wave propagation and solid mechanics.

*Keywords*: Micropolar elastic solid, electro-microelastic solid, longitudinal waves, reflection, refraction, amplitude ratios, angle of incidence.

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## 1.Introduction

The molecular and atomic structure of the materials is ignored in classical theory of elasticity. When experiments were performed on the construction materials like steel, aluminium, concrete etc., results obtained using classical theory of elasticity were matched with experimental results. But various discrepancies were observed near holes, and cracks, where stress gradients were considerable. Thus, it was observed that microstructure plays a significant role in refraction and reflection of waves. When elastic waves propagate from one medium to another they exhibit different behaviour. In this research article, behaviour of longitudinal wave is observed when it propagates from micropolar elastic solid to electro-microelastic solid.

Voigt[1] proposed the description of the discrepancies of classical theory of elasticity by introducing moment vector along with force vector in translation of motion. Then Cosserat and Cosserat [2] presented a theory according to which the material particles are capable of rotation and linear displacement during the deformation of material. The micropolar elasticity theory was given by Cosserat. Eringen and his colleagues [3]–[6] developed the micropolar theory of elasticity that is being used on these type of materials, also for the problems where the classical theory of elasticity fails due to material microstructure. Micropolar elastic materials may be imagined as the materials with dumbbell type molecules or the materials whose molecules are rigid short cylinders. Micropolar theory of elasticity has its importance due to its application in many physical substances like concrete with muddy fluids and sand, chopped fibre composites, foams, the blood rigid cells of animal, porous materials etc. Tomar and Gogna[7] studied the coefficient of refraction and reflection at the interface of two micropolar solid half-spaces that at the time when coupled wave incidence on interface don't have the same elastic properties. Some relevant literature work in the same field has been done by many other researchers like Poonia et. al [8] , Kumari et. al [9] , Singh [10], and Bijarnia et. al[11] .

In the present paper the propagation of waves in MES and EMS half-spaces is discussed. Also, the ratio of amplitudes of different types of refracted and reflected waves is calculated for specific models, and graphical results are represented corresponding to the incident wave's angle of incidence.

## 2.Fundamental equations and constitutive relations

*For MES half-space (Medium $M_1$)*

Eringen's [4], in micropolar elastic medium equation of motion are as follow:

$$({c_1}^2+{c_3}^2)\nabla^2\phi=\frac{\partial^2\phi}{\partial t^2}, \tag{1}$$

$$({c_2}^2+{c_3}^2)\nabla^2 U+{c_3}^2\nabla\times\Phi=\frac{\partial^2 U}{\partial t^2}, \tag{2}$$

$$({c_4}^2\nabla^2-2{\omega_0}^2)\Phi+{\omega_0}^2\nabla\times U=\frac{\partial^2\Phi}{\partial t^2}, \tag{3}$$

and

$$c_1^2=\frac{\lambda+2\mu}{\rho},\ c_2^2=\frac{\mu}{\rho},\ c_3^2=\frac{\kappa}{\rho},\ c_4^2=\frac{\gamma}{\rho j},\ \omega_0^2=\frac{\kappa}{\rho j}, \tag{4}$$

Equation (1) corresponding to LD wave moving with velocity $V_1$ and defined as ${V_1}^2={c_1}^2+{c_3}^2$ given by Parfitt and Eringen [12] and the equations in (2) and (3) represents coupled equations in the vector potentials U & $\Phi$. The waves named as coupled transverse and micro-rotations corresponds to these equations. If $\frac{\omega^2}{{\omega_0}^2}>20$, there exists 2 set of coupled-wave that propagates with velocities $\frac{1}{\lambda_1}$ and $\frac{1}{\lambda_2}$ such that:

$${\lambda_1}^2=\frac{1}{2}\left[B-\sqrt{B^2-4C}\right],\quad {\lambda_2}^2=\frac{1}{2}\left[B+\sqrt{B^2-4C}\right], \tag{5}$$

$$B=\frac{q(p-2)}{\omega^2}+\frac{1}{({c_2}^2+{c_3}^2)}+\frac{1}{{c_4}^2},\ C=\left(\frac{1}{{c_4}^2}-\frac{2q}{\omega^2}\right)\frac{1}{({c_2}^2+{c_3}^2)},\ p=\frac{\kappa}{\mu+\kappa},\ q=\frac{\kappa}{\gamma}. \tag{6}$$

Taking the components of micro- rotation and displacement as below to consider the 2D problem

$$\Phi=(0,\Phi_2,0),\qquad U=(u,0,w), \tag{7}$$

$$u_1=\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial z},\qquad u_3=\frac{\partial\phi}{\partial z}+\frac{\partial\psi}{\partial x}, \tag{8}$$

and stresses components are represented as

$$t_{zz}=(\lambda+2\mu+\kappa)\frac{\partial^2\phi}{\partial z^2}+\lambda\frac{\partial^2\phi}{\partial x^2}+(2\mu+\kappa)\frac{\partial^2\psi}{\partial x\,\partial z}, \tag{9}$$

$$t_{zx}=(2\mu+\kappa)\frac{\partial^2\phi}{\partial x\,\partial z}-(\mu+\kappa)\frac{\partial^2\psi}{\partial z^2}+\mu\frac{\partial^2\psi}{\partial x^2}-\kappa\Phi_2, \tag{10}$$

$$m_{zy}=\gamma\frac{\partial\Phi_2}{\partial z}, \tag{11}$$

*For EMS half-space (Medium $M_2$)*

In continuous theory of microstretch elasticity the electromagnetic fields are described first by Eringen [3], and due to absence of thermal effect, microstretch continuum and magnetic flux vector will be exposed exclusively to electric field. As a result, these types of continuous materials are referred to as EMS medium given by

$$\bar{t}_{kl}=\left(\bar{\lambda}_0\bar{\psi}+\bar{\lambda}u_{r,r}\right)\bar{\delta}_{kl}+\bar{\mu}\left(\bar{u}_{k,l}+\bar{u}_{l,k}\right)+\bar{\kappa}\left(\bar{u}_{l,k}-\bar{\epsilon}_{klr}\bar{\Phi}_r\right), \tag{12}$$

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$$\overline{m}_{kl} = \alpha\overline{\Phi}_{r,r}\overline{\delta}_{kl} + \beta\overline{\Phi}_{k,l} + \gamma\overline{\Phi}_{l,k} + b_0\overline{\epsilon}_{lkm}\overline{\Phi}_{,r}\ , \tag{13}$$

$$\overline{m}_{k} = \alpha_0\overline{\psi}_{,k} + \lambda_2 E_k - b_0\overline{\epsilon}_{klm}\overline{\Phi}_{l,m}, \tag{14}$$

$$\overline{D}_k = \left(1 + \chi^{\overline{E}}\right)E_k + \lambda_3\overline{\epsilon}_{lmk}\overline{\Phi}_{l,m} + \lambda_2\overline{\psi}_{,k}, \tag{15}$$

where $\overline{t}_{kl}$ , $\overline{m}_{kl}$ , $\overline{m}_k$ , $\overline{D}_k$ ; $\bar{\lambda}, \overline{\mu}$ ; $\overline{\kappa}$ , $\alpha$ , $\beta$ , $\gamma$ ; $b_0$ , $\lambda_0$ , $\alpha_0$ ; $\chi^{\overline{E}}$ , $\lambda_2$ , $\lambda_3$ ; $\overline{u}_k$ , $\overline{\Phi}_k$ , $\overline{\psi}$ and $E_k$ are force stress tensor, couple stress, microstretch vector, dielectric displacement vector; Lame's constants; micropolar constants; microstretch constants; dielectric susceptibility, coupling constants; displacements, micropolar rotation vector, scalar microstretch and electric field vector respectively. For homogeneous electro-microelastic and isotropic solid medium, the field equations in section (7) of Eringen [3] are as follows:

$$(\overline{c}_1^2 + \overline{c}_3^2)\nabla\,\nabla.\mathbf{\overline{u}} - (\overline{c}_2^2 + \overline{c}_3^2)\nabla\times\ \nabla\times\mathbf{\overline{u}} + \overline{c}_3^2\ \nabla\times\overline{\Phi} + \bar{\lambda}_0\ \nabla\overline{\psi} = \ddot{\mathbf{\overline{u}}}, \tag{16}$$

$$(\overline{c}_4^2 + \overline{c}_5^2)\nabla\,\nabla.\overline{\Phi} - \overline{c}_4^2\nabla\times\ \nabla\times\overline{\Phi} + \overline{\omega}_0^2\ \nabla\times\mathbf{\overline{u}} - 2\overline{\omega}_0^2\overline{\Phi} = \ddot{\overline{\Phi}}, \tag{17}$$

$$\overline{c}_6^2\nabla^2\overline{\psi} - \overline{c}_7^2\overline{\psi} - \overline{c}_8^2\ \nabla.\mathbf{\overline{u}} + \overline{c}_9^2\ \nabla.\overline{E} = \ddot{\overline{\psi}}, \tag{18}$$

$$\nabla.\overline{D} = 0, \tag{19}$$

$$\nabla\times\overline{E} = 0, \tag{20}$$

where

$$\overline{c}_1^2 = \frac{\bar{\lambda} + 2\overline{\mu}}{\overline{\rho}}\ , \quad \overline{c}_2^2 = \frac{\overline{\mu}}{\overline{\rho}}\ , \quad \overline{c}_3^2 = \frac{\overline{\kappa}}{\overline{\rho}}\ , \quad \overline{c}_4^2 = \frac{\overline{\gamma}}{\overline{\rho}\overline{j}}\ , \quad \overline{c}_5^2 = \frac{\overline{\alpha} + \overline{\beta}}{\overline{\rho}\overline{j}}\ , \quad \overline{c}_6^2 = \frac{2\overline{\alpha}_0}{\overline{\rho}\overline{j}}\ ,$$

$$\overline{c}_7^2 = \frac{2\bar{\lambda}_1}{3\ \overline{\rho}\overline{j}}\ , \quad \overline{c}_8^2 = \frac{2\lambda_0}{3\ \overline{\rho}\overline{j}_0}\ , \quad \overline{c}_9^2 = \frac{2\bar{\lambda}_2}{\overline{\rho}\overline{j}}\ , \overline{\omega}_0^2 = \frac{\overline{c}_3^2}{\overline{j}} = \frac{\overline{\kappa}}{\overline{\rho}\overline{j}}\ , \quad \bar{\lambda}_0 = \frac{\lambda_0}{\overline{\rho}}. \tag{21}$$

Now, let's introduce the scalar potentials $\xi$, $\overline{q}$ and $\epsilon$; and the vector potentials $\overline{U}, \overline{\Pi}$ as:

$$\overline{U} = \nabla\overline{q} + \nabla\times\overline{U}\ ,\ \ \Phi = \nabla\xi + \nabla\times\overline{\Pi}\ ,\ \ \overline{E} = -\nabla\epsilon\ ,\ \ \nabla.\overline{U} = \nabla.\overline{\Pi} = 0, \tag{22}$$

Now, by using these into equations (16) -(20), we obtain following equations

$$(\overline{c}_1^2 + \overline{c}_3^2)\nabla^2\overline{q} + \bar{\lambda}_0\overline{\psi} = \ddot{\overline{q}} \tag{23}$$

$$(\overline{c}_6^2 - \overline{c}_{10}^2)\nabla^2\overline{\psi} - \overline{c}_7^2\overline{\psi} - \overline{c}_8^2\nabla^2\overline{q} = \ddot{\overline{\psi}} \tag{24}$$

$$(\overline{c}_2^2 + \overline{c}_3^2)\nabla^2\overline{U} + \overline{c}_3^2\ \nabla\times\overline{\Pi} = \ddot{\overline{U}} \tag{25}$$

$$\overline{c}_4^2\nabla^2\overline{\Pi} - 2\overline{\omega}_0^2\overline{\Pi} + \overline{\omega}_0^2\ \nabla\times\overline{U} = \ddot{\overline{\Pi}} \tag{26}$$

$$(\overline{c}_4^2 + \overline{c}_5^2)\nabla^2\xi - 2\overline{\omega}_0^2\xi = \ddot{\xi} \tag{27}$$

$$\nabla^2\epsilon = \frac{\bar{\lambda}_2}{1+\chi^{\overline{E}}}\nabla^2\overline{\psi} \tag{28}$$

where $\overline{c}_{10}^2 = \frac{2\bar{\lambda}_2^2}{\overline{\rho}\overline{j}_0(1+\chi^{\overline{E}})}$ .

Here, in scalar potentials $\overline{q}$ and $\overline{\psi}$ the equations (23) & (24) are coupled, and in the scalar potentials $\epsilon$ and $\overline{\psi}$ the equation (28) is also coupled. In vector potentials $\overline{U}$ & $\overline{\Pi}$ equations (25) & (26) are coupled. Further in scalar potential $\xi$ the equation (27) is uncoupled.

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### 3.Formulation of the Problem

Assume the positive direction of unit vector $\bar{\mathbf{n}}$ to be the form of plane wave propagation that is given by:

$$\{\bar{q}, \bar{\psi}, \bar{U}, \bar{\Pi}\} = \{a_1, b_1, A_0, B_0\} exp\{ik(\bar{\mathbf{n}}.\bar{\mathbf{r}} - \bar{V}t)\} \quad (29)$$

Here $a_1$ and $b_1$ are using for complex constant; $A_0$ and $B_0$ are stand for complex constant vectors; $\bar{V}$, $\bar{\mathbf{r}}$, k and ω having their usual meaning. Using the terms of $\bar{q}$ and $\bar{\psi}$ from (29) in equations (23) and (24), after that removing $a_1$ and $b_1$, consequently getting the equation

$$\bar{A}\bar{V}^4 - \bar{B}\bar{V}^2 + \bar{C} = 0 \quad (30)$$

Where $\bar{A} = 1 - \frac{\bar{\lambda}_1 \Omega}{3\bar{\kappa}}\left(\frac{j}{\bar{j}_0}\right)$, $\bar{B} = \left(\bar{c}_1^2 + \bar{c}_3^2 - \frac{\lambda_0 \bar{\lambda}_0}{\bar{\lambda}_1}\right)\bar{A} + \bar{c}_6^2 - \bar{c}_{10}^2 + \frac{\lambda_0 \bar{\lambda}_0}{\bar{\lambda}_1}$ , $\bar{C} = (\bar{c}_1^2 + \bar{c}_3^2)(\bar{c}_6^2 - \bar{c}_{10}^2)$ and $\Omega = \frac{2\bar{\omega}_0^2}{\omega}$. Equation (30) is quadratic in $\bar{V}^2$ and the roots of above equation are represented by:

$$\bar{V}_{1,2}^2 = \frac{1}{2\bar{A}}\left[\bar{B} \pm \sqrt{(\bar{B}^2 - 4\bar{A}\bar{C})}\right] \quad (31)$$

where '+'sign for the velocity $\bar{V}_1^2$ and '-' sign for the velocity $\bar{V}_2^2$.

It can be seen that from equations (23) and (29) the constants $a_1$ and $b_1$ both are related to one-another by the relation

$$b_1 = \zeta a_1 \quad (32)$$

Where $\zeta = \frac{\omega^2}{\bar{\lambda}_0}\left[\frac{\bar{c}_1^2 + \bar{c}_3^2}{\bar{V}^2} - 1\right]$ is coupling parameter between $\bar{q}$ & $\bar{\psi}$.

With the help of the expression of $\bar{q}$ & $\bar{\psi}$ form the (23) into (16), the vector of displacement $\bar{\mathbf{u}}$ is found as

$$\bar{\mathbf{u}} = ika_1 \bar{\mathbf{n}} \exp\{ik(\bar{\mathbf{n}}.\bar{\mathbf{r}} - \bar{V}t)\}. \quad (33)$$

Above result represents that both vectors $\bar{\mathbf{u}}$, and $\bar{\mathbf{n}}$ are parallel.

The equation (25) and (26) represent the two sets of coupled transverse waves that propagates and the corresponding velocities $\bar{V}_3^2$ and $\bar{V}_4^2$ produced by Parfitt and Eringen [12]

$$\bar{V}_{3,4}^2 = \frac{1}{2(1-\Omega)}\left\{\varepsilon \pm \sqrt{\varepsilon^2 - 4c_4^2}(1-\Omega)(\bar{c}_2^2 + \bar{c}_3^2)\right\} \quad (34)$$

where $\varepsilon = \bar{c}_4^2 + \bar{c}_2^2(1-\Omega) + \bar{c}_3^2(1-\Omega/2)$, they have also produced the equation (27) that is representation of a LD microrotational wave that propagates with the velocity

$$\bar{V}_5^2 = \bar{c}_4^2 + \bar{c}_5^2 + \frac{2\bar{\omega}_0^2}{\bar{\kappa}^2}.$$

Here, the refraction and reflection phenomena of LD wave at $(Z = 0)$ plane interface between MES and EMS half-spaces is discussed. The problem here is 2D *xz*-planes. Thus, *x*-axis & *z*-axis are considered along the interface and along the directional vertically downward respectively. Here, medium $M_1(Z > 0)$ represents the lower half-space for the MES half-space, and the upper half-space is represented by by medium $M_2(Z < 0)$for electro-microelastic solid half-space.

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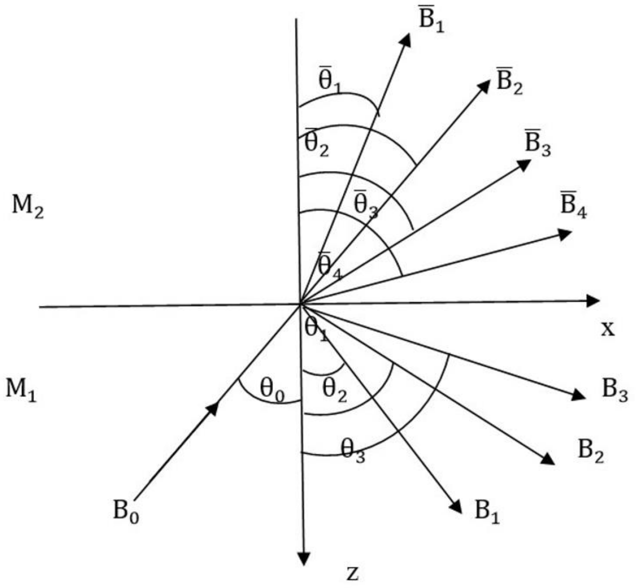


Figure 1: Geometry of the problem

*In medium $M_1$*

$$\phi = B_0 \exp\{ik_0 (x\sin\theta_0 - z\cos\theta_0) + i\omega_1 t\} + B_1 \exp\{ik_0 (x\sin\theta_1 + z\cos\theta_1) + i\omega_1 t\}, \tag{35}$$

$$\psi = B_2 \exp\{i\delta_1(x\sin\theta_2 + z\cos\theta_2) + i\omega_2 t\} + B_3 \exp\{i\delta_2(x\sin\theta_3 + z\cos\theta_3) + i\omega_3 t\}, \tag{36}$$

$$\Phi_2 = EB_2 \exp\{i\delta_1(x\sin\theta_2 + z\cos\theta_2) + i\omega_2 t\} + FB_3 \exp\{i\delta_2(x\sin\theta_3 + z\cos\theta_3) + i\omega_3 t\}, \tag{37}$$

where

$$E = \frac{\delta_1^2\left(\delta_1^2 - \frac{\omega^2}{(c_2{}^2 + c_3{}^2)} + pq\right)}{\text{deno.}}, \tag{38}$$

$$F = \frac{\delta_2^2\left(\delta_2^2 - \frac{\omega^2}{(c_2{}^2 + c_3{}^2)} + pq\right)}{\text{deno.}}, \tag{39}$$

and

$$\text{deno.} = p\left(2q - \frac{\omega^2}{c_4{}^2}\right), \quad \delta_1^2 = \lambda_1^2\omega^2, \ \ \delta_2^2 = \lambda_2^2\omega^2 . \tag{40}$$

where $B_0$ represents incident longitudinal wave's amplitudes, $B_1$ represents reflected LD wave, $B_2$ and $B_3$ represents reflected coupled transverse and micro-rotation waves respectively, and $\bar{B}_1, \bar{B}_2, \bar{B}_3, \bar{B}_4$ are respectively the amplitudes of refracted two coupled longitudinal waves, two sets of coupled transverse waves.

*In medium $M_2$*

For the two dimensional plane using

$$\bar{\mathbf{u}} = (\bar{u}_1, 0, \bar{u}_3), \ \bar{\boldsymbol{\Phi}} = (0, \bar{\Phi}_2, 0), \ \frac{\partial}{\partial y} \equiv 0. \tag{41}$$

Putting these into (22), obtained following expressions

$$\bar{u}_1 = \frac{\partial \bar{q}}{\partial x} - \frac{\partial \bar{U}_2}{\partial z}, \bar{u}_3 = \frac{\partial \bar{q}}{\partial z} + \frac{\partial \bar{U}_2}{\partial x}, \qquad \bar{\Phi}_2 = \frac{\partial \bar{\Pi}_3}{\partial x} - \frac{\partial \bar{\Pi}_1}{\partial z}$$

$\overline{U}_2$ Stands for y-component of $\overline{U}$, $\overline{\Pi}_1$ & $\overline{\Pi}_3$ are correspondingly the x & z-components of $\overline{\Pi}$. Now, the potentials of many reflected and refracted waves in medium $M_1$and medium $M_2$ respectively are represented as

$$\bar{q} = \sum_{p=1,2} \overline{B}_p\, exp\{i\bar{k}_p(\sin\bar{\theta}_p x - \cos\bar{\theta}_p z) - \bar{\omega}_p t\}, \quad (42)$$

$$\bar{\psi} = \sum_{p=1,2} \zeta_p \overline{B}_p\, exp\{i\bar{k}_p(\sin\bar{\theta}_p x - \cos\bar{\theta}_p z) - \bar{\omega}_p t\}, \quad (43)$$

$$\overline{U}_2 = \sum_{p=3,4} \overline{B}_p\, exp\{i\bar{k}_p(\sin\bar{\theta}_p x - \cos\bar{\theta}_p z) - \bar{\omega}_p t\}, \quad (44)$$

$$\overline{\Phi}_2 = \sum_{p=3,4} \eta_p \overline{B}_p\, exp\{i\bar{k}_p(\sin\bar{\theta}_p x - \cos\bar{\theta}_p z) - \bar{\omega}_p t\}, \quad (45)$$

Where the coupling parameters between $\bar{q}$ & $\bar{\psi}$ are $i = \sqrt{-1}$ and $\bar{\omega}_p = \bar{k}_p \overline{V}_p$ , $\zeta_{1,2}$, and the coupling parameters between $\overline{U}_2$ & $\overline{\Phi}_2$ are $\eta_{3,4}$. The expressions for $\zeta_i$ computed above by using equation (32) can be represented as

$$\zeta_{1,2} = \frac{\omega^2}{\bar{\lambda}_0}\left[\frac{\bar{c}_1^2 + \bar{c}_3^2}{\overline{V}_{1,2}^2} - 1\right]$$

And with the use of curl operator in equation (26) and after that using the equations (44) and (45) the expressions of $\eta_i$ can be computed. These expressions are defined as

$$\eta_{3,4} = \bar{\omega}_0^2\left[\overline{V}_{3,4}^2 - \frac{2\,\bar{\omega}_0^2}{\bar{k}_{3,4}^2} - \bar{c}_4^2\right]^{-1}$$

using the equations from (22) into equations (12)-(15), the required components of stress, microrotation, microstretch and displacements are represented as

$$\bar{t}_{zz} = (\bar{\lambda} + 2\bar{\mu} + \bar{\kappa})\bar{q}_{,zz} + (2\bar{\mu} + \bar{\kappa})\overline{U}_{2,xz} + \bar{\lambda}\bar{q}_{,xx} + \bar{\lambda}_0\bar{\psi},$$

$$\bar{t}_{zx} = (2\bar{\mu} + \bar{\kappa})\bar{q}_{,xz} + (\bar{\mu} + \bar{\kappa})\overline{U}_{2,zz} + \bar{\mu}\overline{U}_{2,xx} + \bar{\kappa}\overline{\Phi}_2,$$

$$\bar{m}_{zy} = \bar{\gamma}\overline{\Phi}_{2,z}, \qquad \bar{m}_z = \left(\alpha_0 - \frac{\bar{\lambda}_2^2}{1 + \chi^{\bar{E}}}\right)\bar{\psi}_{,z}$$

$$\bar{u}_1 = \bar{q}_{,x} - \overline{U}_{2,z}, \bar{u}_3 = \bar{q}_{,z} + \overline{U}_{2,x} \quad (46)$$

## 4.Boundary Conditions

The suitable boundary conditions for the considered model at the interface z=0 are described below

$$t_{zx} = \bar{t}_{zx}, t_{zz} = \bar{t}_{zz},\; m_{zy} = \bar{m}_{zy},\; m_z = \bar{m}_z,\; \Phi_2 = \overline{\Phi}_2, u_1 = \bar{u}_1,\; u_3 = \bar{u}_3,\; \psi = \bar{\psi}\;. \quad (47)$$

Using the equations (9) – (11) and (42) – (45), the boundary conditions represented in equation (47) are identically satisfied iff$k_i sin\theta_i = \bar{k}_i sin\bar{\theta}_i$ and$\omega_i = \bar{\omega}_i$, obtained the required result:

$$a_{11} = -\{\lambda + (2\mu + \kappa)\cos^2\theta_1\}, \quad a_{12} = -(2\mu + \kappa)\sin\theta_2 \cos\theta_2 \frac{\delta_1^2}{k_0^2},$$

$$a_{13} = -(2\mu + \kappa)\sin\theta_3 \cos\theta_3 \frac{\delta_2^2}{k_0^2}, \qquad a_{14} = \left\{\bar{\lambda} + (2\bar{\mu} + \bar{\kappa})\cos^2\bar{\theta}_1 - \frac{\bar{\lambda}_0\bar{\xi}_1}{\bar{k}_1^2}\right\}\frac{\bar{k}_1^2}{k_0^2},$$

$$a_{15} = \left\{\bar{\lambda} + (2\bar{\mu} + \bar{\kappa})\cos^2\bar{\theta}_2 - \frac{\bar{\lambda}_0\bar{\xi}_2}{\bar{k}_1^2}\right\}\frac{\bar{k}_1^2}{k_0^2}, \qquad a_{16} = -(2\bar{\mu} + \bar{\kappa})\sin\bar{\theta}_3\cos\bar{\theta}_3\frac{\bar{k}_3^2}{k_0^2},$$

$$a_{17} = -(2\bar{\mu} + \bar{\kappa})\sin\bar{\theta}_4\cos\bar{\theta}_4\frac{\bar{k}_4^2}{k_0^2}, \quad Y_1 = -a_{11}.$$

$a_{21} = \sin\theta_1 \cos\theta_1 , a_{22} = -\left\{\mu(1 - 2\sin^2\theta_2) + \kappa \cos^2\theta_2 - \frac{\kappa E}{\delta_1^2}\right\}\frac{\delta_1^2}{k_0^2} ,$

$a_{23} = -\left\{\mu(1 - 2\sin^2\theta_3) + \kappa \cos^2\theta_3 - \frac{\kappa F}{\delta_2^2}\right\}\frac{\delta_2^2}{k_0^2} , \quad a_{24} = (2\bar{\mu} + \bar{\kappa})\sin\bar{\theta}_1\cos\bar{\theta}_1\frac{\bar{k}_1^2}{k_0^2} ,$

$a_{25} = (2\bar{\mu} + \bar{\kappa})\sin\bar{\theta}_2\cos\bar{\theta}_2\frac{\bar{k}_2^2}{k_0^2} , \quad a_{26} = \left\{\frac{\bar{\mu}}{\bar{\kappa}}(\cos^2\bar{\theta}_3 - \sin^2\bar{\theta}_3) - \cos^2\bar{\theta}_3 - \frac{\eta_3}{\bar{k}_3^2}\right\}\frac{\bar{\kappa}\bar{k}_3^2}{k_0^2},$

$a_{27} = \left\{\frac{\bar{\mu}}{\bar{\kappa}}(\cos^2\bar{\theta}_4 - \sin^2\bar{\theta}_4) - \cos^2\bar{\theta}_4 - \frac{\eta_4}{\bar{k}_4^2}\right\}\frac{\bar{\kappa}\bar{k}_4^2}{k_0^2} , \quad Y_2 = a_{21} .$

$a_{31} = a_{34} = a_{35} = 0 , \quad a_{32} = \gamma\delta_1 E \cos\theta_2 , \ a_{33} = \gamma\delta_2 F \cos\theta_3 , \quad a_{36} = \bar{\gamma}\eta_3\bar{k}_3 \cos\bar{\theta}_3 ,$

$a_{37} = \bar{\gamma}\eta_4\bar{k}_4 \cos\bar{\theta}_4 , \quad Y_3 = a_{31} .$

$a_{41} = \sin\theta_1 , \quad a_{42} = -\cos\theta_2\frac{\delta_1}{k_0} , \quad a_{43} = -\cos\theta_3\frac{\delta_2}{k_0} , \quad a_{44} = -\sin\bar{\theta}_1\frac{\bar{k}_1}{k_0} ,$

$a_{45} = -\sin\bar{\theta}_2\frac{\bar{k}_2}{k_0} , \quad a_{46} = -\cos\bar{\theta}_3\frac{\bar{k}_3}{k_0} , a_{47} = -\cos\bar{\theta}_4\frac{\bar{k}_4}{k_0} , \quad Y_4 = -a_{41} .$

$a_{51} = \cos\theta_1 , \quad a_{52} = \sin\theta_2\frac{\delta_1}{k_0} , \quad a_{53} = \sin\theta_3\frac{\delta_2}{k_0} , \quad a_{54} = \cos\bar{\theta}_1\frac{\bar{k}_1}{k_0},$

$a_{55} = \cos\bar{\theta}_2\frac{\bar{k}_2}{k_0} , \quad a_{46} = -\sin\bar{\theta}_3\frac{\bar{k}_3}{k_0} , a_{47} = -\sin\bar{\theta}_4\frac{\bar{k}_4}{k_0} , \quad Y_5 = a_{51} .$

$a_{61} = a_{64} = a_{65} = 0 , \quad a_{62} = E , a_{63} = F , \quad a_{66} = -\eta_3 , \quad a_{67} = -\eta_4 , \ Y_6 = a_{61} .$

$a_{71} = a_{72} = a_{73} = a_{76} = a_{77} = 0 , \ a_{74} = \xi_1\cos\bar{\theta}_1\bar{k}_1 , \ a_{75} = \xi_2\cos\bar{\theta}_2\bar{k}_2 , \ Y_7 = a_{71} . \quad (48)$

**5.Discussion and numerical results**

To solve the equations of stresses, Microstretch, displacements and microrotation with the help of equations of displacements, potentials of various refracted and reflected waves, Snell's Lawand boundary conditions. After that, write these equations in the matrix form such that$[a_{ij}][Z_i] = [Y_i]$, where$[a_{ij}]_{7\times7}$,$[Z_i]_{7\times1}$and$[Y_i]_{7\times1}$ are matrices of respective order. Making a program using the coefficients$[a_{ij}]$ in the computer software MATLAB and execute.

Consequently, obtain the various graphs with respects to amplitude ratios$Z_i(i = 1,2,3,4,5,6,7)$. Following Gauthier [13], the constants for MES half-space's physical values are given as

$$\lambda = 7.85 \times 10^{11}\text{dyne/cm}^2, \quad \mu = 6.46 \times 10^{11}\text{dyne/cm}^2, \kappa = 0.0139 \times 10^{11}\text{dyne/cm}^2,$$

$$\rho = 1.9\text{gm/cm}^3, \quad \gamma = 0.0365 \times 10^{11} \text{ dyne} , \ j = 0.0212 \text{ cm}^2, \ \frac{\omega^2}{{\omega_0}^2} = 20. \quad (49)$$

the physical constants for EMS half-space are given as

$$\bar{\lambda} = 7.59 \times 10^{11}\text{dyne/cm}^2, \quad \bar{\mu} = 1.89 \times 10^{11}\text{dyne/cm}^2, \ \bar{\kappa} = 0.0149 \times 10^{11}\text{dyne/cm}^2,$$

$$\bar{\rho} = 2.2\text{gm/cm}^3, \alpha_0 = 0.095 \times 10^{11}\text{dyne} , \quad \bar{\lambda}_0 = 0.032 \times 10^{11}\text{dyne/cm}^2,$$

$$\bar{\lambda}_1 = 0.030 \times 10^{11}\text{dyne/cm}^2, \quad \bar{\lambda}_2 = 0.3364 \times 10^{11}\text{dyne}, \ \bar{j}_0 = 0.0196\text{cm}^2,$$

$$\bar{\gamma} = 0.0345 \times 10^{11} \text{ dyne} , \ \chi^{\bar{E}} = 298, \ \bar{\omega}/\bar{\omega}_0 = 10. \quad (50)$$

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Below in figures from (2) to (7), the respectively changes in the ratios of amplitudes of reflected & refracted waves is represented by the solid lines, when the incident wave is LD wave.

New figures

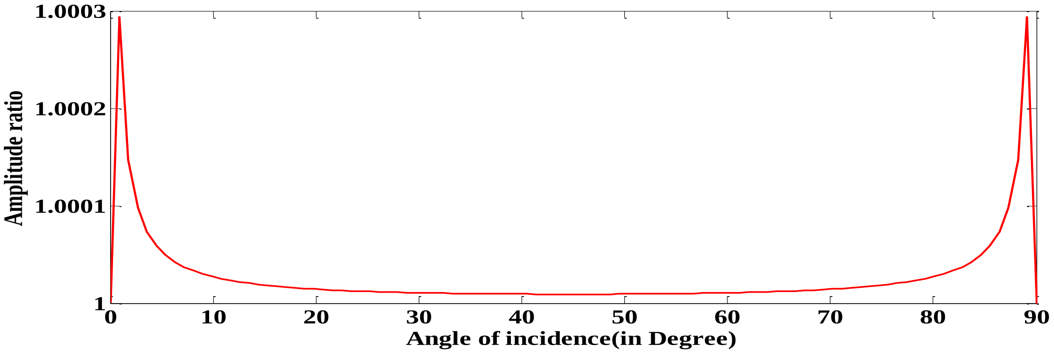


Figure 2: Variation in amplitude ratio $|Z_1|$ w.r.t. angle of incidence of incident wave

Fig. (2), represents that the amplitude ratio's minimum values $Z_1$ at angles $0^o$ and $90^o$, maximum value attains approximately at angles $2^o$ and $88^o$. The values of $Z_1$ are speedily increasing from the beginning at angles $0^o$ and $2^o$ and after that decreasing from the angles $2^o$ to $24^o$ and similarly increasing and decreasing from the angles $66^o$ to $88^o$and minor changes in its values likes up and down from the angles $25^o$ to $65^o$.

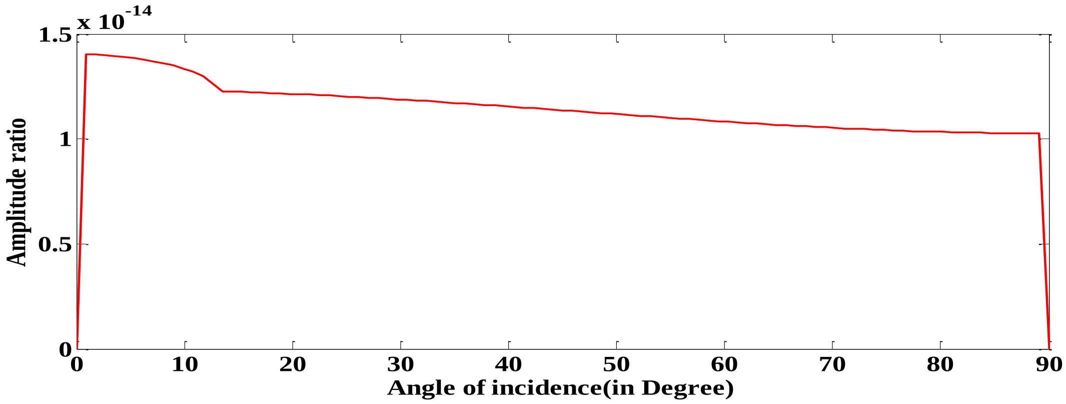


Figure 3: Variation in amplitude ratio $|Z_2|$ w.r.t. angle of incidence of incident wave

Minimum values of amplitude ratio $Z_2$ in the figure (3) are behaving alike the figures (2) and maximum value attains approximately at angle $2^o$. After that the values of $Z_2$ are decreasing from the angles $2^o$to$14^o$and stop at an angle $14^o$for a moment and again decreases very slowly from the angles$14^o$to $89^o$, speedily decreasing from the angles $89^o$ to $90^o$, which are shown in the figure (3).

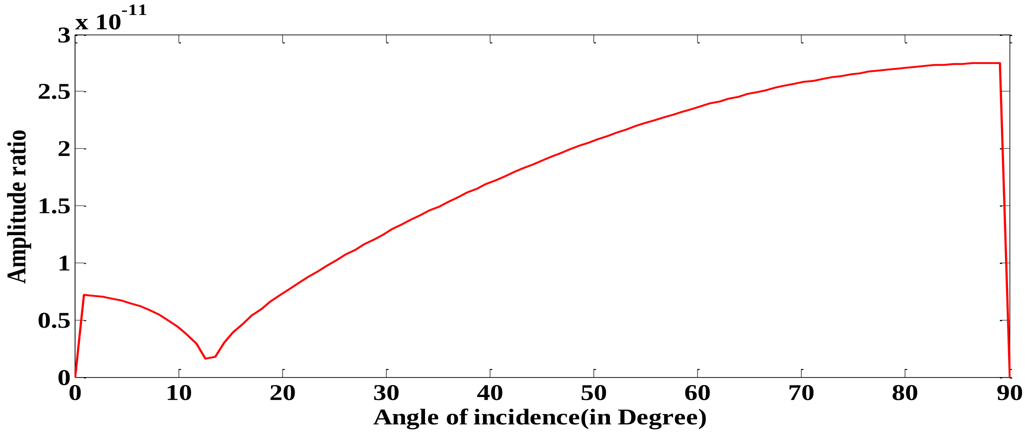


Figure 4: Variation in amplitude ratio $|Z_3|$ w.r.t. angle of incidence of incident wave

In the figure (4), the minimum values $Z_3$ of amplitude ratios are behaving alike the fig. (2) and (3). Now the values of $Z_3$ suddenly increasing from the angles $0^o$ to$1^o$, decreasing slowly from the angles $1^o$ to $14^o$and again increasing from the angles $14^o$ and $89^o$, and after that speedily decreasing from the angles $89^o$ to $90^o$.

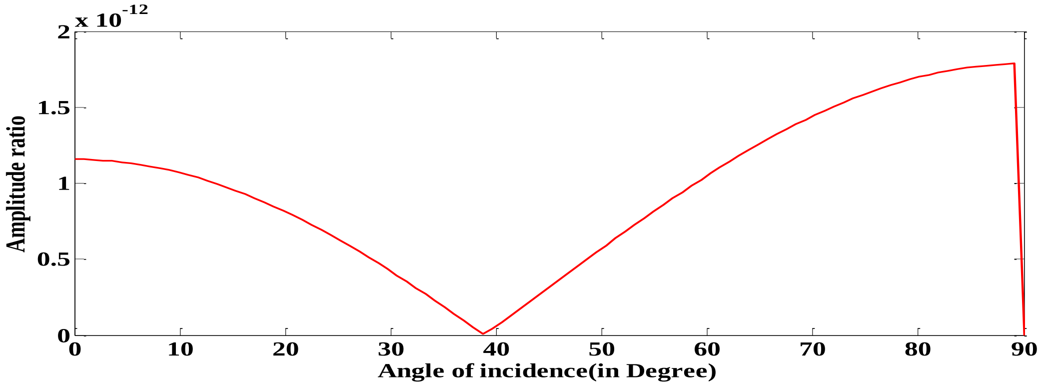


Figure 5: Variation in amplitude ratio $|Z_4|$ w.r.t. angle of incidence of incident wave

In the figure (5), minimum values of amplitude ratio $Z_4$ lies approximately at the angles $39^o$ and $90^o$. Now, the values are decreasing from the angles $0^o$ and $39^o$ and after that values are increasing from the angles $39^o$ and $89^o$. The values from the angles $89^o$ to $90^o$are behaving alike the figure (4).

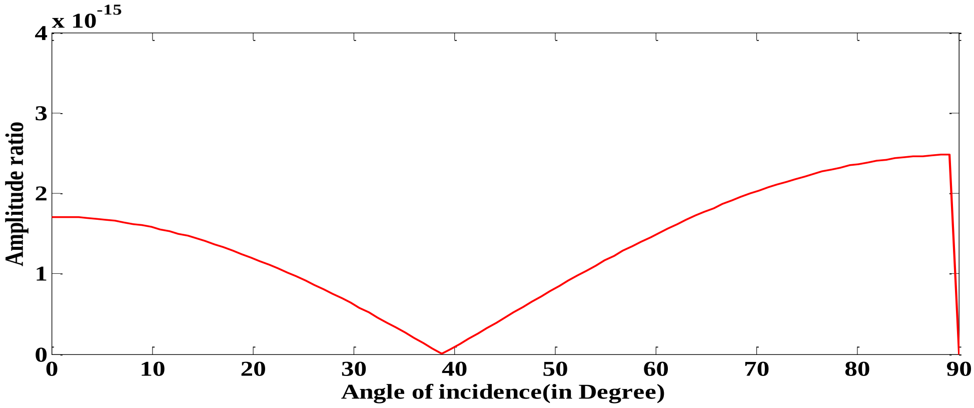


Figure 6: Variation in amplitude ratio $|Z_5|$ w.r.t. angle of incidence of incident wave

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The amplitude ratio's values $Z_5$ in the figure (6) are approximately same.

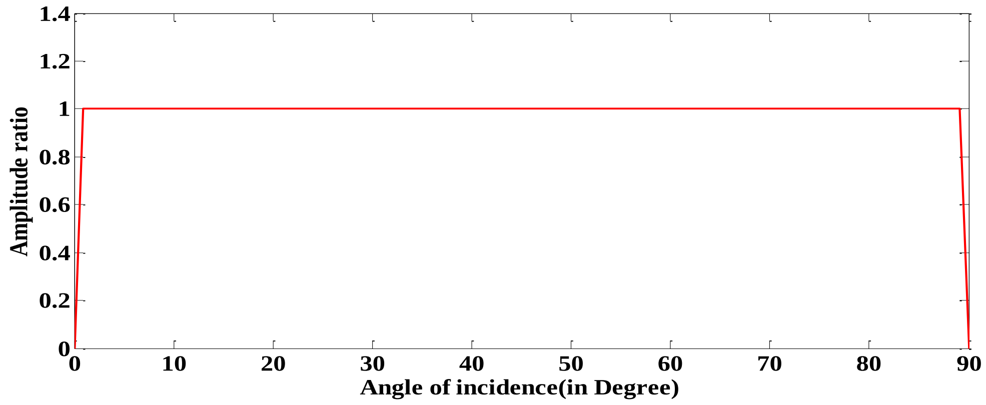


Figure 7: Variation in amplitude ratio $|Z_6|$ w.r.t. angle of incidence of incident wave

In the figure (7), the minimum values of amplitude ratio $Z_6$ are lies at the angles $0^o$ and $90^o$. Now, the values are increasing from the angles $0^o$ and $1^o$ and after that values are having negligible difference from the angles $1^o$ and $89^o$. The values from the angles $89^o$ to $90^o$are behaving alike the figure (4).

The figure for the amplitude ratio $Z_7$ is not shown here because the values are same for the values of the amplitude ratio $Z_6$.

## 6.Conclusion

In this paper, a mathematical discussion of refraction & reflection coefficients at interfaces separating electro-microelastic solids (EMS) and micropolar elastic solids (MES) half-spaces has been given when a longitudinal wave is in incident nature. From the graphical and numerical results, it is observed that

1. The modulus of amplitude ratios of the different types of refracted and reflected waves depends on the material properties of half spaces and the angle of incidence of the incident wave.
2. The values of amplitudes ratios $Z_i$($i$=1, 2, 3) various reflected values are different at corresponding angles.
3. The amplitude ratio's values $Z_i$ ($i$=4, 6) various refracted values are different at corresponding angles. But the values of $Z_i$ ($i$=5, 7) are approximately same to the values of $Z_i$ ($i$=4, 6).

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