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On Odd Prime Labelings of Snake Related Graphs

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Abstract
For a graph *G* mapping *f* is called an odd prime labeling , if *f* is a bijection from *V* to {1, 3, 5, ..., 2|*V* | − 1} satisfying the condition that for each line uv in G the greatest common divisor of the labels of end points $f(u), f(v)$ is one. Investigated in this paper the odd prime labeling of some new graphs and we prove that some snake related graphs such as quadrilateral snake $D(Q_n)$, Triangular snake $S(T_n)$, Double Triangular snake $D(T_n)$, Alternate Triangular snake (AT_n), Triangular ladder TL_n, Open Triangular ladder $O(TL_n)$ are odd prime graphs.

1. Introduction

In this paper by a graph G = (V (G),E(G)) for graph theoretical notations we refer Bondy.J.A&Murthy.U.S.R [1]. For entire survey of graph labeling we refer [4]. The concept of prime labeling was Roger Etringer introduced prime labelings and then it was investigated by Tout et al [2, 8] Deretsky and Meena.S and Kavitha.P [5]. The concept of odd prime labeling was introduced by Prajapati.U.M&Shah.K.P [7] and then studied by many researchers. Meena.S and Kavitha.P and Gajalakshmi.G [6].

In this paper we prove some snake related graphs are odd prime graphs.

Definition 1.1. *Let* $G = \langle V(G), E(G)\rangle$ *be a graph. A bijection f : V (G) →* $O_{|V|}$ *is called an odd prime labeling if for each line uv∈ E, greatest common divisor* $\langle f(u), f(v)\rangle$ *is one. A graph is called an odd prime graph if which admits odd prime labeling.*

Here $O_{|V|} = \{1, 3, 5, ...2|V| - n\}$

Definition 1.2. *A subdivision graph S(G) is got from G by splitting every line of G exactly once.*

Definition 1.3. *A graph got from a path* $r_1, r_2, ..., r_n$ *by joining* r_k *and* r_{k+1} *to two points* v_k *and* w_k*,* $1 \le k \le n - 1$ *respectively and then joining* v_k *and* w_k *is know as quadrilateral snake* Q_n.

Definition 1.4. *A graph got from a path by replacing each line by a triangle is called Triangular snake* T_n.

Definition 1.5. *A graph got from the path* $r_1, r_2, ..., r_n$ *by joining* r_k *and* r_{k+1} *with two new points* v_k *and* w_k*,* $1 \le k \le n - 1$.

Definition 1.6. *An Alternate Triangular snake* $A(T_n)$*, is got from a path* P_n *by replacing each alternate line of* P_n *by a cycle* C_3.

Definition 1.7. *The Ladder* $L_n = P_2 \times P_n$.

Definition 1.8. *A triangular ladder* TL_n*,* $n \ge 2$ *is a graph got from* L_n *by adding the lines* u_kv_{k+1}*,* $1 \le k \le n-1$*. The vertices of* L_n *are* u_k *and* v_k*.* u_k *and* v_k *are the two paths in the graph* L_n.

Definition 1.9. *An open triangular ladder*

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$O(TL_n)$, $n \geq 2$ is a graph got from an open ladder $O(L_n)$ by adding the edges u_kv_{k+1}, $1 \leq k \leq n - 1$.

2. Main Results

Theorem 2.1. *The subdivision graph of a quadrilateral snake $S(QS_n)$ $(n \geq 3)$ is an odd prime graph.*

Proof. Let G = $S(QS_n)$ be the Subdivision graph of a quadrilateral snake

$V(G) = \{u_k, v_k, x_k, y_k, w_k, z_k, s_k / 1 \leq k \leq n\}$

$E(G) = \{u_kv_k, u_kw_k, w_kx_k, x_ky_k, y_kz_k, z_ks_k / 1 \leq k \leq n\}$

$\cup\{s_ku_{k+1}, v_ku_{k+1} / 1 \leq k \leq - 1\}$

Here $|V(G)| = 7n - 6$ and $|E(G)| = 7n - 2$

Define a mapping f from V (G) to O_{7n} as follows

$f(u_k) = 14k-13$ for $1 \leq k \leq n$

$f(v_k) = 14k-1$ for $1 \leq k \leq n - 1$ $k \not\equiv 2(\text{mod}3)$

$f(v_k) = 14k-5$ for $1 \leq k \leq n - 1$ $k \equiv 2(\text{mod}3)$

$f(w_k) = 14k-11$ for $1 \leq k \leq n - 1$

$f(x_k) = 14k-9$ for $1 \leq k \leq n - 1$

$f(y_k) = 14k-7$ for $1 \leq k \leq n - 1$

$f(z_k) = 14k-5$ for $1 \leq k \leq n - 1$ $k \not\equiv 2(\text{mod}3)$

$f(z_k) = 14k-3$ for $1 \leq k \leq n - 1$ $k \equiv 2(\text{mod}3)$

$f(s_k) = 14k-3$ for $1 \leq k \leq n - 1$ $k \not\equiv 2(\text{mod}3)$

$f(s_k) = 14k-1$ for $1 \leq k \leq n - 1$ $k \equiv 2(\text{mod}3)$

Clearly the point labels are distinct with this labeling for each line $e \in E$.

greatest common divisor (f(u), f(v)) = 1.

(i)

$e = u_kv_k, \gcd(f(u_k), f(v_k)) = \gcd(14k-13, 14k-1) = 1$

for $1 \leq k \leq n$, $k \not\equiv 2(\text{mod}3)$;

(ii)

$e = u_kv_k, \gcd(f(u_k), f(v_k)) = \gcd(14k-13, 14k-5) = 1$

for $1 \leq k \leq n - 1$, $k \equiv 2(\text{mod}3)$

(iii)

$e = u_kw_k, \gcd(f(u_k), f(w_k)) = \gcd(14k-13, 14k-11) = 1$

for $1 \leq k \leq n - 1$

(iv)

$e = w_kx_k, \gcd(f(w_k), f(x_k)) = \gcd(14k-11, 14k-9) = 1$

for $1 \leq k \leq n - 1$

(v)

$e = x_ky_k, \gcd(f(x_k), f(y_k)) = \gcd(14k-9, 14k-7) = 1$

for $1 \leq k \leq n - 1$

(vi)

$e = y_kz_k, \gcd(f(y_k), f(z_k)) = \gcd(14k-7, 14k-5) = 1$

for $1 \leq k \leq n - 1$, $k \not\equiv 2(\text{mod}3)$;

(vii)

$e = y_kz_k, \gcd(f(y_k), f(z_k)) = \gcd(14k-7, 14k-3) = 1$

for $1 \leq k \leq n - 1$, $k \equiv 2(\text{mod}3)$

(viii)

$e = z_ks_k, \gcd(f(z_k), f(s_k)) = \gcd(14k-5, 14k-3) = 1$

for $1 \leq k \leq n-1$, $k \not\equiv 2(\text{mod}3)$;

(ix)

$e = z_ks_k, \gcd(f(z_k), f(s_k)) = \gcd(14k-3, 14k-1) = 1$

for $1 \leq k \leq n - 1$, $k \equiv 2(\text{mod}3)$;

(x)

$e = u_{k+1}s_k, \gcd(f(u_{k+1}), f(s_k)) = \gcd(14k+1, 14k-3) = 1$

for $1 \leq k \leq n - 1$, $k \not\equiv 2(\text{mod}3)$;

(xi)

$e = v_ku_{k+1}, \gcd(f(v_k), f(u_{k+1})) = \gcd(14k-1, 14k+1) = 1$

for $1 \leq k \leq n - 1$, $k \equiv 2(\text{mod}3)$;

(xii)

$e = u_{k+1}s_k, \gcd(f(u_{k+1}), f(s_k)) = \gcd(14k+1, 14k-1) = 1$

for $1 \leq k \leq n - 1$, $k \not\equiv 2(\text{mod}3)$

(xiii)

$e = v_ku_{k+1}, \gcd(f(v_k), f(u_{k+1})) = \gcd(14k-5, 14k-1) = 1$

for $1 \leq k \leq n - 1$, $k \equiv 2(\text{mod}3)$;

Thus $S(QS_n)$ is an odd prime graph.

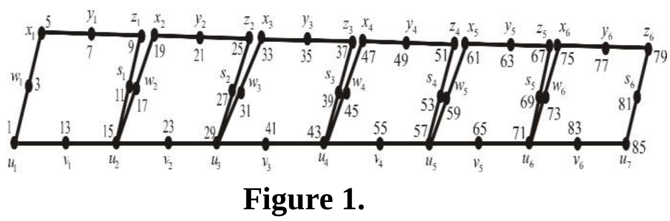

Figure 1.

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***Theorem* 2.2.** *The Subdivision graph of a triang* $(n \geq 1)$ *is an odd prime graph.*

Proof. Let ST_n be the subdivision graph of a triangular snake T_n.

$V(ST_n) = \{u_k, v_k, x_k, y_k, w_k / 1 \leq k \leq n-1\} \cup \{u_n\}$

$E(ST_n) = \{u_kx_k, x_ky_k, y_kw_k, u_kv_k / 1 \leq k \leq n\} \cup \{v_ku_{k+1}, w_ku_{k+1} / 1 \leq k \leq n-1\}$

Here $|V(ST_n)| = 5n-4$ and $|E(ST_n)| = 6n-6$

Define a mapping f from V (G) to O_{5n} as follows

$f(u_k) = 10k-9$ for $1 \leq k \leq n$

$f(v_k) = 10k-1$ for $1 \leq k \leq n$

$f(x_k) = 10k-7$ for $1 \leq k \leq n-1$

$f(y_k) = 10k-5$ for $1 \leq k \leq n-1$

$f(w_k) = 10k-3$ for $1 \leq k \leq n-1$

Clearly the point labels are distinct with this labeling for each line e ∈ E.

greatest common divisor (f(u), f(v)) = 1.

(i)

$e = u_kx_k, \gcd(f(u_k), f(x_k)) = \gcd(10k-9, 10k-7) = 1$

for $1 \leq k \leq n-1$

(ii)

$e = x_ky_k, \gcd(f(x_k), f(y_k)) = \gcd(10k-7, 10k-5) = 1$

for $1 \leq k \leq n-1$

(iii)

$e = y_kw_k, \gcd(f(y_k), f(w_k)) = \gcd(10k-5, 10k-3) = 1$

for $1 \leq k \leq n-1$

(iv)

$e = u_kv_k, \gcd(f(u_k), f(v_k)) = \gcd(10k-9, 10k-1) = 1$

for $1 \leq k \leq n$

(v)

$e = v_kv_{k+1}, \gcd(f(v_k), f(v_{k+1})) = \gcd(10k-1, 10k+1) = 1$

for $1 \leq k \leq n-1$

(vi)

$e = w_ku_{k+1}, \gcd(f(w_k), f(u_{k+1})) = \gcd(10k-3, 10k+1) = 1$

for $1 \leq k \leq n-1$

Thus $S(T_n)$ is an odd prime graph.

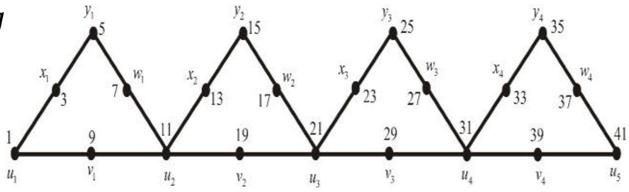

Figure 2.

Theorem 2.3. *The subdivision graph S(D(Tn)) is an odd prime graph.*

Proof. Let DT_n be the subdivision graph of a double triangular snake DT_n.

$V(DT_n) = \{u_k, v_k, w_k, s_k, t_k, p_k, q_k, r_k / 1 \leq k \leq n\}$

$E(DT_n) = \{u_kv_k, u_ks_k, s_kw_k, w_kt_k, u_kq_k, q_kr_k, r_kp_k / 1 \leq k \leq n\} \cup \{t_ku_{k+1}, p_ku_{k+1} / 1 \leq k \leq n-1\}$

Here $|V(DT_n)| = 7n-6$ and $|E(DT_n)| = 7n-2$

Define $f : V(G) \rightarrow O_{8n}$ as follows

$f(u_k) = 16k-15$ for $1 \leq k \leq n$

$f(v_k) = 16k-1$ for $1 \leq k \leq n$

$f(w_k) = 16k-11$ for $1 \leq k \leq n$ $k \not\equiv 4 \pmod{10}$

$f(w_k) = 16k-9$ for $1 \leq k \leq n$ $k \equiv 4 \pmod{10}$

$f(s_k) = 16k-13$ for $1 \leq k \leq n$

$f(t_k) = 16k-9$ for $1 \leq k \leq n$ $k \not\equiv 4 \pmod{10}$

$f(t_k) = 16k-11$ for $1 \leq k \leq n$ $k \equiv 4 \pmod{10}$

$f(p_k) = 16k-11$ for $1 \leq k \leq n$

$f(q_k) = 16k-9$ for $1 \leq k \leq n$

$f(r_k) = 16k-13$ for $1 \leq k \leq n$

Clearly the point labels are distinct with this labeling for each line e ∈ E.

greatest common divisor (f(u), f(v)) = 1.

(i)

$e = u_kv_k, \gcd(f(u_k), f(v_k)) = \gcd(16k-15, 16k-1) = 1$

for $1 \leq k \leq n$

(ii)

$e = u_kq_k, \gcd(f(u_k), f(q_k)) = \gcd(16k-15, 16k-7) = 1$

for $1 \leq k \leq n$

(iii)

$e = u_ks_k, \gcd(f(u_k), f(s_k)) = \gcd(16k-15, 16k-13) = 1$

for $1 \leq k \leq n$

(iv)

$e = s_kw_k, \gcd(f(s_k), f(w_k)) = \gcd(16k-13, 16k-9) = 1$

for $1 \leq k \leq n$, $k \not\equiv 4 \pmod{10}$

(v)

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$e = s_k w_k, \gcd(f(s_k), f(w_k)) = \gcd(16k-13, 16k-11) = 1$
for $1 \le k \le n$, $k \equiv 4 \pmod{10}$
(vi)
$e = w_k t_k, \gcd(f(w_k), f(t_k)) = \gcd(16k-11, 16k-9) = 1$
for $1 \le k \le n$, $k \not\equiv 4 \pmod{10}$
(vii)
$e = w_k t_k, \gcd(f(w_k), f(t_k)) = \gcd(16k-9, 16k-11) = 1$
for $1 \le k \le n$, $k \equiv 4 \pmod{10}$
(viii)
$e = q_k r_k, \gcd(f(q_k), f(r_k)) = \gcd(16k-7, 16k-5) = 1$
for $1 \le k \le n$
(ix)
$e = p_k r_k, \gcd(f(p_k), f(r_k)) = \gcd(16k-3, 16k-5) = 1$
for $1 \le k \le n$
(x)
$e = t_k u_{k+1}, \gcd(f(t_k), f(u_{k+1})) = \gcd(16k-9, 16k-15) = 1$
for $1 \le k \le n-1$
(xi)
$e = p_k u_{k+1}, \gcd(f(p_k), f(u_{k+1})) = \gcd(16k-3, 16k-15) = 1$
for $1 \le k \le n-1$
(xii)
$e = v_k u_{k+1}, \gcd(f(v_k), f(u_{k+1})) = \gcd(16k-1, 16k-15) = 1$
for $1 \le k \le n-1$
Thus $D(T_n)$ is an odd prime graph.

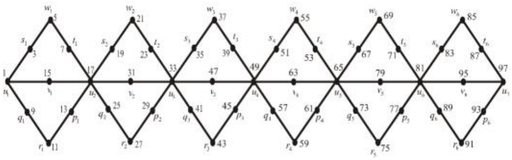

Figure 3.

Theorem 2.4. *The Alternate Triangular snake $A(T_n)$ admits odd prime labeling.*

Proof. $V(G) = \{u_k, v_k, x_k / 1 \le k \le n\}$
$E(G) = \{u_k v_k, u_k x_k, x_k v_k / 1 \le k \le n\} \cup \{v_k u_{k+1} / 1 \le k \le n-1\}$
Define a function f from V (G) to O_{3n} as follows
$f(u_k) = 6k-5$ for $1 \le k \le n$
$f(v_k) = 6k-1$ for $1 \le k \le n$
$f(x_k) = 6k-3$ for $1 \le k \le n$
Clearly the point labels are distinct with this labeling for each line $e \in E$.
greatest common divisor $(f(u), f(v)) = 1$.
(i)
$e = u_k v_k, \gcd(f(u_k), f(v_k)) = \gcd(6k-5, 6k-1) = 1$
for $1 \le k \le n$
(ii)
$e = u_k x_k, \gcd(f(u_k), f(x_k)) = \gcd(6k-5, 6k-3) = 1$
for $1 \le k \le n$
(iii)
$e = x_k v_k, \gcd(f(x_k), f(v_k)) = \gcd(6k-3, 6k-1) = 1$
for $1 \le k \le n$
(iv)
$e = v_k u_{k+1}, \gcd(f(v_k), f(u_{k+1})) = \gcd(6k-1, 6k+1) = 1$
for $1 \le k \le n-1$
Thus $A(T_n)$ is an odd prime graph.

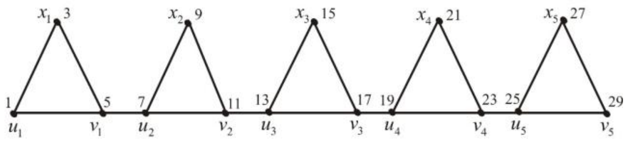

Figure 4.

Theorem 2.5. *The odd prime labeling number of a triangular ladder TL_n, $n \ge 2$.*

Proof. Let $G = TL_n$ be any triangular graph with
$V(G) = \{u_k, v_k / 1 \le k \le n\}$
$E(G) = \{u_k v_k / 1 \le k \le n\} \cup \{u_k u_{k+1}, v_k u_{k+1}, v_k v_{k+1} / 1 \le k \le n-1\}$
Define $f : V \to O_{2n}$ as follows
$f(u_k) = 4k-3$ for $1 \le k \le n$
$f(v_k) = 4k-1$ for $1 \le k \le n$
Clearly point labels are distinct for each line $e \in E$, $\gcd(f(u), f(v)) = 1$.
(i)
$e = u_k v_k, \gcd(f(u_k), f(v_k)) = \gcd(4k-3, 4k-1) = 1$
for $1 \le k \le n$
(ii)
$e = u_k u_{k+1}, \gcd(f(u_k), f(u_{k+1})) = \gcd(4k-3, 4k+1) = 1$
for $1 \le k \le n-1$
(iii)
$e = v_k v_{k+1}, \gcd(f(v_k), f(v_{k+1})) = \gcd(4k-1, 4k+3) = 1$
for $1 \le k \le n-1$
(iv)
$e = v_k u_{k+1}, \gcd(f(v_k), f(u_{k+1})) = \gcd(4k-1, 4k+1) = 1$
for $1 \le k \le n-1$

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ThusTL_n is an odd prime graph.

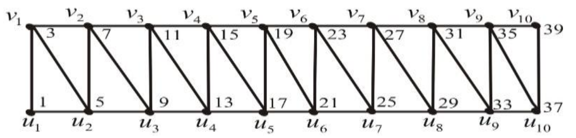

Figure 5.

Theorem 2.6. *Open triangular ladder $O(TL_n)$ is an odd prime graph $n \geq 2$.*

Proof. Let G = $O(TL_n)$ be any triangular ladder graph on 2n vertices with

V (G) = $\{u_k, v_k / 1 \leq k \leq n\}$

E(G) = $\{u_k, v_k / 1 \leq i \leq n\} \cup \{u_k u_{k+1}, u_k v_{k+1}, v_k v_{k+1} / 1 \leq k \leq n-1\}$

Define f : V (G) → O_{2n} as follows

$f(u_k) = 4k-1$ for $1 \leq k \leq n$

$f(v_k) = 4k-3$ for $1 \leq k \leq n$

Clearly the point labels are distinct with this labeling for each line e ∈ E.

greatest common divisor (f(u), f(v)) = 1.

(i)

$e = u_k v_k, \gcd(f(u_k), f(v_k)) = \gcd(4k-1, 4k-3) = 1$

for $2 \leq k \leq n-1$

(ii)

$e = u_k u_{k+1}, \gcd(f(u_k), f(u_{k+1})) = \gcd(4k-1, 4k+3) = 1$

for $1 \leq k \leq n-1$

(iii)

$e = v_k v_{k+1}, \gcd(f(v_k), f(v_{k+1})) = \gcd(4k-3, 4k+1) = 1$

for $1 \leq k \leq n-1$

(iv)

$e = v_k u_{k+1}, \gcd(f(v_k), f(u_{k+1})) = \gcd(4k-1, 4k+1) = 1$

for $1 \leq k \leq n-1$

Thus $O(TL_n)$ is an odd prime graph.

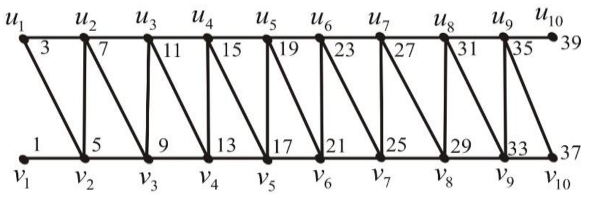

Figure 6.

Conclusion

Odd Prime labelings of various classes of graphs such as quadrilateral snake $D(Q_n)$, Triangular snake $S(T_n)$, Double Triangular snake $D(T_n)$, Alternate Triangular snake $A(T_n)$, Triangular ladder TL_n, Open Triangular ladder $O(TL_n)$ graphs are investigated. To investigate similar theorems for other graph families is an open area of research.

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