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Analysis of Queueing Systems Management in Fuzzy Environment with Randomized Hexagonal Fuzzy Numbers

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Abstract
The primary goal of queuing analysis is to predict systems performance which studies how the lines form, how they function, and why there is a malfunction in turn help business decision making on how to construct more efficient and cost effective systems. When compared to crisp queues which are more commonly used, Fuzzy queues are much more realistic in every practical situation. Random numbers plays an important role where the outcome is unpredictable and cannot be reproduced. The aim of this paper, is to study $(FM/FM/1)$ fuzzy queueing systems with the randomized hexagonal fuzzy numbers are considered, the performance metrics using $\alpha-cut$ method are determined and is related with the original fuzzy queueing problem. This study is used to identify in how much variations in input values for a given variable will impact in the results for a mathematical model.

Keywords: Queueing Systems, Fuzzy Queues, Hexagonal Fuzzy Numbers, Performance Analysis.

1.Introducion

Gaining Customers loyalty, security, and improving quality of service, while putting an efficient queueing management systems will certainly help to automate the queueing process with reducing the possibility of people[1,2]. Queueing system helps to generate comprehensive, real time analytics and paid attentions from academicians and researchers[3,8]. In many situations, the uncertainties are due to fuzziness and in fuzzy set theory these cases in detail were established by[4]. In many practical applications the parameters arrival rate λ and service rate μ are fuzzy in nature and are exactly expressed[4,5]. This approach utilizes the advantage of to make the model less restrictive and more realistic[6]. Since the performance measures are always expressed by its membership functions rather than with their crisp values, and where the queueing systems in which some of the input is ambiguous these measures preserve the fuzziness[7]. Also in many situations, the input rate depends on the servers current state and different input rates of the queueing system will certainly affect the performance measures[8,9]. The waiting times and expected number of customers are computed by Luo et.al[10]. Defuzzification methods and their preference relation in comparing two fuzzy numbers are categorized by the fuzzy ranking methods[11,12]. These relations depicts the mathematical models with their intensity and is a better choice of preference that to be applied to the real world situations[13,14]. The graded mean integration procedure is used for defuzzification of the fuzzy characteristics[15]. Single Server $FM/FM/1$ queuing system with come first served discipline is considered and the inter arrival times and service times are described by its membership functions of the fuzzy sets. The basic idea is to transform a fuzzy queue by the $\alpha-cut$ approach and by the extension principle fuzzy input and service times, performance measures are derived[16]. Dong et. al. proposed the algorithm make use of inter-arrival times at different $\alpha-cut$ [17-19]. A new operation of hexagonal fuzzy numbers has been introduced with its basic member function followed by the properties of arithmetic operations of fuzzy numbers[20].

In this paper we study $FM/FM/1$ queuing system with randomized hexagonal fuzzy numbers are considered, the performance metrics using $\alpha-cut$ method are determined and is related with the original fuzzy queueing problem. Introduction and literature survey is given Section 1. In section 2, Preliminaries and algorithms are given. Later in section 3, optimal solution for a numerical example is given and compared using the proposed method with that of other methods. At last, conclusions are presented in section 4.

2. Definitions and Preliminaries

Definition - 1: The Fuzzy Set A on real line R with membership function $\mu_A(x): R \to [0,1]$ is a fuzzy number if

a) A is a normal and convex fuzzy set.
b) The support of A, must be bounded and
c) $\alpha . A$ is closed for each α in $[0,1]$.

...(1)

Definition – 2: Membership Function

The fuzzy number A is a fuzzy set, with its membership function $\mu_A(x)$ which satisfies the following

a) $\mu_A(x)$ is piecewise continuous.
b) $\mu_A(x)$ is convex.
c) $\mu_A(x)$ is normal i.e., $\mu_A(x) = 1$ ... (2)

Definition - 3: α - cut

An α - cut of a fuzzy set $\tilde{A}$, is a crisp set A_α, that contains all the elements of the universal set X that have a membership grade in A greater than or equal to the specified value of α. Thus

$$A_\alpha = \{ x \in X : \mu_{\tilde{A}}(x) \geq \alpha, \quad 0 \leq \alpha \leq 1 \}$$

... (3)

Definition-4: Hexagonal Fuzzy Number with Membership Function

A fuzzy number $\tilde{A} = (x_1, x_2, x_3, x_4, x_5, x_6)$ where $x_1 \geq x_2 \geq x_3 \geq x_4 \geq x_5 \geq x_6$ is said to be hexagonal fuzzy number if its membership function is given by

$$A_\alpha = \mu_{\tilde{A}}(x) = \begin{cases} 0, & for\ x < x_1 \\ \frac{1}{2}(\frac{x - x_1}{x_2 - x_1}), & for\ x_1 \leq x \leq x_2 \\ \frac{1}{2} + \frac{1}{2}(\frac{x - x_2}{x_3 - x_2}), & for\ x_2 \leq x \leq x_3 \\ 1, & for\ x_3 \leq x \leq x_4 \\ 1 - \frac{1}{2}(\frac{x - x_4}{x_5 - x_4}), & for\ x_4 \leq x \leq x_5 \\ \frac{1}{2}(\frac{x_6 - x}{x_6 - x_5}, & for\ x_5 \leq x \leq x_6 \\ 0, & for\ x > x_6 \end{cases}$$

... (4)

Definition-5 : Hexagonal Fuzzy Number with Maximum Membership Function

An Hexagonal fuzzy number denoted by A_H is defined as $A_W = (P_1(u), Q_1(v), Q_2(v), P_2(u))$ for $u \in [0,0.5]$ and $v \in [0.5, w]$ where,

a) $P_1(u)$ is a non-decreasing, left continuous and bounded function over $[0,0.5]$
b) $Q_1(v)$ is a non-decreasing, left continuous and bounded function over $[0.5, w]$
c) $Q_2(v)$ is a non-decreasing, left continuous and bounded function over $[w,0.5]$
d) $P_2(u)$ is a non-decreasing, left continuous and bounded function over $[w,0]$

... (5)

Remark 1:

If $w=1$, then the hexagonal fuzzy number is said to be a normal hexagonal fuzzy number. A_W, is a fuzzy number where w is the maximum membership value that a fuzzy number.

Remark 2:

An Hexagonal fuzzy number A_H is ordered Quadruple $(P_1(u), Q_1(v), Q_2(v), P_2(u))$ for $u \in [0,0.5]$ and $v \in [0.5, w]$ where,

$$P_1(u) = \frac{1}{2}\left(\frac{u - a_1}{a_2 - a_1}\right)$$

$$P_2(u) = \frac{1}{2}\left(\frac{a_6 - u}{a_6 - a_5}\right)$$

$$Q_1(v) = \frac{1}{2} + \frac{1}{2}\left(\frac{v - a_2}{a_3 - a_2}\right)$$

$$Q_2(v) = \frac{1}{2} - \frac{1}{2}\left(\frac{v - a_4}{a_5 - a_4}\right)$$

... (6)

Definition-6 : Alpha-Cut

The classical set A_α, called an Alpha-Cut set, is the set of elements whose degree of membership in $A_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ is not greater than α. It is defined as

$$A_\alpha = \{x \in X / \mu_{A_H}(x) \geq \alpha\} = \begin{cases} [P_1(\alpha), P_2(\alpha)] \ for\ \alpha \in [0,0.5) \\ [Q_1(\alpha), Q_2(\alpha)] \ for\ \alpha \in [0.5,0] \end{cases}$$

... (7)

Definition-7 : Alpha-Cut Operations:

For a crisp interval by α cut, an operation interval A_α is obtained for all $\alpha \in [0,1]$ as follows

If $Q_1(x)=\alpha$, then $\frac{1}{2}+\frac{1}{2}\left(\frac{x - a_2}{a_3 - a_2}\right)=\alpha$ implies $x = 2\alpha(a_3 - a_2) - a_3 + a_2$ that is $Q_1(\alpha) = 2\alpha(a_3 - a_2) - a_3 + a_2$ and

If $Q_2(x)=\alpha$, then $\frac{1}{2}-\frac{1}{2}(\frac{x - a_4}{a_5 - a_4})=\alpha$ implies $x = -2\alpha(a_5 - a_4) + 2a_5 - a_4$ that is $Q_2(\alpha) = -2\alpha(a_5 - a_4) + 2a_5 - a_4$

This implies $[Q_1(\alpha), Q_1(\alpha)] = [\, 2\alpha(a_3 - a_2) - a_3 + a_2, -2\alpha(a_5 - a_4) + 2a_5 - a_4 \,]$

If $P_1(x)=\alpha$, then $P_1(\alpha) = 2\alpha(a_2 - a_1) + a_1$ and

If $P_2(x)=\alpha$, then $P_2(\alpha) = -2\alpha(a_6 - a_5) + a_6$

This implies $[\, P_1(\alpha), P_2(\alpha)] = [2\alpha(a_2 - a_1) + a_1, -2\alpha(a_6 - a_5) + a_6]$

Hence $A_\alpha = \begin{cases} 2\alpha(a_2 - a_1) + a_1, -2\alpha(a_6 - a_5) + a_6 & for\ \alpha \in [0, 0.5) \\ 2\alpha(a_3 - a_2) - a_3 + a_2, -2\alpha(a_5 - a_4) + 2a_5 - a_4 & for\ \alpha \in [0.5, 1] \end{cases}$

... (9)

For a single server $FM/FM/1$ queuing system first come first served discipline, the inter arrival times A and service times S are given by following fuzzy sets

$$A = \{(a, \tilde{\mu}_A(a)) / a \in X\}$$
$$S = \{(s, \tilde{\mu}_S(s)) / s \in Y\}$$

... (10)

The α -cuts for inter arrival, and service times are represented as

$$A(\alpha) = \{(a \in X /, \tilde{\mu}_A(a)) \geq \alpha\}$$
$$S(\alpha) = \{(s \in Y /, \tilde{\mu}_S(S)) \geq \alpha\}$$

... (11)

Since the queue is first-come first served discipline and is an infinite source population, where both the arrival times and the service times follow Poisson and exponential distributions respectively with parameters λ and μ, are more realistic fuzzy variables rather than crisp values. The performance measures are given by

a) The mean number of customers in the queue $Lq = \frac{\lambda^2}{\mu(\mu-\lambda)}$

b) The mean number of customers in the system $Ls = \frac{\lambda}{(\mu-\lambda)}$

c) The mean waiting time in the queue $Wq = \frac{\lambda}{\mu(\mu-\lambda)}$

d) The mean waiting time in the system $Ws = \frac{1}{(\mu-\lambda)}$

... (12)

4. Numerical Examples

Numerical Example 1 : Consider a $FM/FM/1$ queue where both the arrival and service rates are randomly generated hexagonal fuzzy numbers represented by $\lambda = [3\,4\,5\,6\,8\,10]$ and $\mu = [11\ \ 13\ \ 14\ \ 15\ \ 16\ \ 19]$ per hour respectively

The membership functions of $Lq = \frac{x^2}{y(y-x)}$, $Ls = \frac{x}{(y-x)}$ and $Wq = \frac{x}{y(y-x)}$, $Ws = \frac{1}{(y-x)}$ are

$x = A_\alpha = [2\alpha + 3, 10 - 4\alpha]$ *and* $y = S_\alpha = [11 + 4\alpha, 19 - 6\alpha]$ *for* $\alpha \in [0, 0.5)$ and

$x = A_\alpha = [2\alpha + 3, 10 - 4\alpha]$ *and* $y = S_\alpha = [12 + 2\alpha, 17 - 2\alpha]$ *for* $\alpha \in [0.5, 1]$

By taking different values of α from $[0, 1]$, the results are depicted in Table -1 and Table - 2.

Table 1:The α Cuts of Lq and Ls

α	Lq	Lq	Ls	Ls
0	0.029605	9.090909	0.1875	10
0.1	0.036613	4.491228	0.210526	5.333333
0.2	0.0451	2.758801	0.236111	3.538462
0.3	0.055404	1.866924	0.264706	2.588235
0.4	0.067959	1.333333	0.296875	2
0.5	0.083333	0.984615	0.333333	1.6
0.6	0.096246	0.781385	0.362069	1.357143
0.7	0.110806	0.623977	0.392857	1.16129
0.8	0.127225	0.5	0.425926	1
0.9	0.145749	0.401097	0.461538	0.864865
1	0.166667	0.321429	0.5	0.75

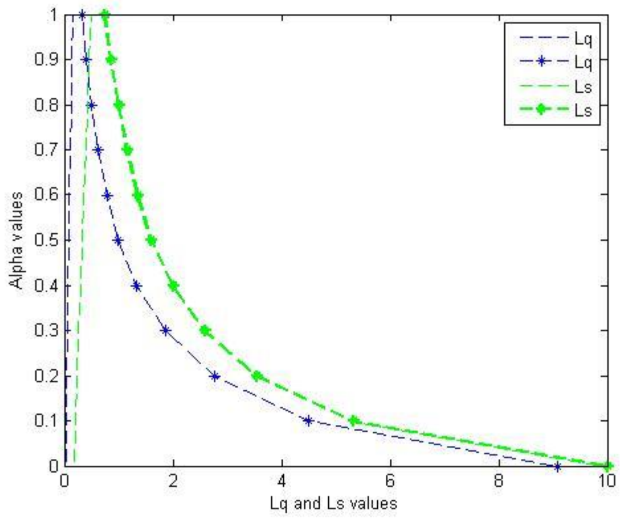

Figure 1.The α of L_q and L_s

Table 2:The α Cuts of Wq and Ws

α	Wq	Wq	Ws	Ws
0	0.009868	0.909091	0.0625	1
0.1	0.011442	0.467836	0.065789	0.555556
0.2	0.013265	0.29987	0.069444	0.384615
0.3	0.01539	0.21215	0.073529	0.294118
0.4	0.017884	0.15873	0.078125	0.238095
0.5	0.020833	0.123077	0.083333	0.2
0.6	0.022916	0.102814	0.086207	0.178571
0.7	0.025183	0.086663	0.089286	0.16129
0.8	0.027658	0.073529	0.092593	0.147059
0.9	0.030364	0.062671	0.096154	0.135135
1	0.033333	0.053571	0.1	0.125

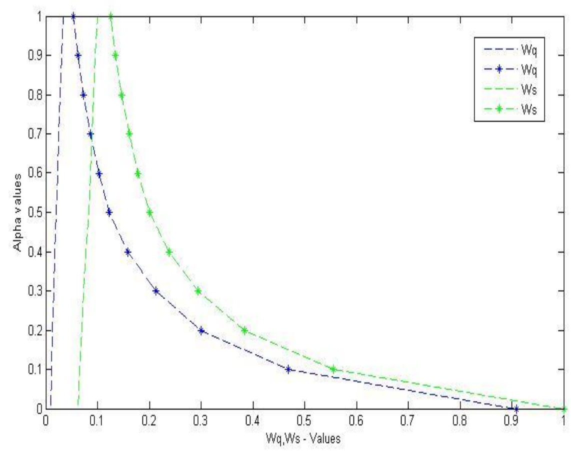

Figure 2.The α of W_q and W_s

Numerical Example 2: Consider a $FM/FM/1$ queue where both the arrival and service rates are randomly generated hexagonal fuzzy numbers represented by $\lambda = [111\ \ 124\ \ 137\ \ 139\ \ 178\ \ 190]$ and $\mu = [211\ \ 218\ \ 226\ \ 244\ \ 291\ \ 298]$

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The membership functions are given by

$$x = A_\alpha = [26\alpha + 111, 190 - 24\alpha] \quad and \ y = S_\alpha = \quad [211 + 14\alpha, 298 - 14\alpha] \quad for \ \alpha \in [0, 0.5)$$

$$x = A_\alpha = [26\alpha + 111, 217 - 78\alpha] \quad and \ y = S_\alpha = \quad [210 + 16\alpha, 338 - 94\alpha] \quad for \ \alpha \in [0.5, 1]$$

By taking different values of α from $[0, 1]$, the results are depicted in Table -3 and Table - 4.

Table 3:The α Cuts of Lq and Ls

α	Lq	Lq	Ls	Ls
0	0.22109967	8.14714511	0.59358289	9.04761905
0.1	0.23775761	6.68127696	0.62076503	7.56451613
0.2	0.25553057	5.60929435	0.64916201	6.47552448
0.3	0.27450044	4.79253293	0.67885714	5.64197531
0.4	0.29475684	4.15055376	0.70994152	4.98342541
0.5	0.31639813	3.63348624	0.74251497	4.45
0.6	0.36720033	2.67029859	0.81677419	3.44534413
0.7	0.42884551	2.02772755	0.9034965	2.76190476
0.8	0.5045848	1.5729663	1.00610687	2.26686217
0.9	0.59902502	1.23756363	1.12941176	1.89175258
1	0.71889842	0.9826569	1.28037383	1.59770115

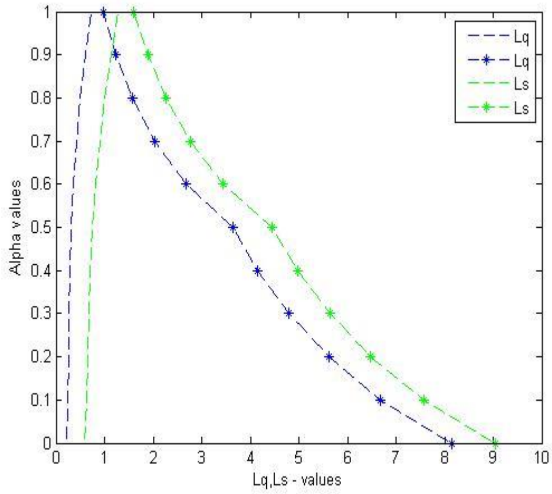

Figure 3. The α of L_q and L_s

Table 4:The α Cuts of Wq and Ws

α	Wq	Wq	Ws	Ws
0	0.00199189	0.04287971	0.00534759	0.04761905
0.1	0.00209294	0.03561448	0.00546448	0.04032258
0.2	0.00219906	0.03028777	0.00558659	0.03496503
0.3	0.00231061	0.02621736	0.00571429	0.0308642
0.4	0.00242798	0.0230075	0.00584795	0.02762431
0.5	0.0025516	0.02041284	0.00598802	0.025
0.6	0.00290048	0.01568918	0.00645161	0.02024291
0.7	0.00331924	0.01248601	0.00699301	0.0170068
0.8	0.00382841	0.01017443	0.00763359	0.01466276
0.9	0.00445703	0.00843027	0.00840336	0.0128866
1	0.00524743	0.00706947	0.00934579	0.01149425

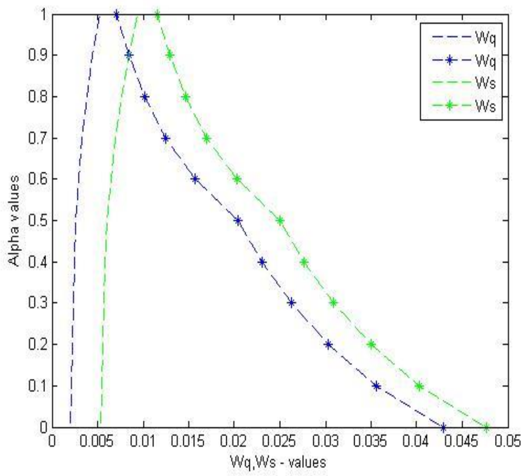

Figure 4.The α of W_q and W_s

Numerical Example 3 : Consider a $FM / FM / 1$ queue where both the arrival and service rates are randomly generated hexagonal fuzzy numbers represented by $\lambda = [1002 \ 1026 \ 1049 \ 1073 \ 1080 \ 1093]$ and $\mu = [1123 \ 1137 \ 1139 \ 1149 \ 1163 \ 1168]$

The membership functions are given by

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$x = A_\alpha = [48\alpha + 1002, 1093 - 26\alpha]$ *and* $y = S_\alpha =$ $[1123 + 28\alpha, 1168 - 10\alpha]$ *for* $\alpha \in [0, 0.5)$

$x = A_\alpha = [46\alpha + 1003, 1087 - 14\alpha]$ *and* $y = S_\alpha =$ $[1135 + 4\alpha, 1177 - 28\alpha]$ *for* $\alpha \in [0.5, 1]$

By taking different values of α from $[0, 1]$, the results are depicted in Table -5 and Table - 6.

Table 5:The α Cuts of Wq and Ws

α	Lq	Lq	Ls	Ls
0	5.178268	35.46005	6.036145	36.43333
0.1	5.421919	29.8337	6.284644	30.80226
0.2	5.684232	25.69792	6.551813	26.66176
0.3	5.967392	22.53001	6.839838	23.48918
0.4	6.273941	20.02611	7.151261	20.98062
0.5	6.60685	17.9975	7.489051	18.94737
0.6	7.063865	17.39523	7.95216	18.34354
0.7	7.57694	16.82884	8.471358	17.77558
0.8	8.15692	16.29521	9.057491	17.24038
0.9	8.81764	15.79159	9.724395	16.7352
1	9.577032	15.31552	10.49	16.25758

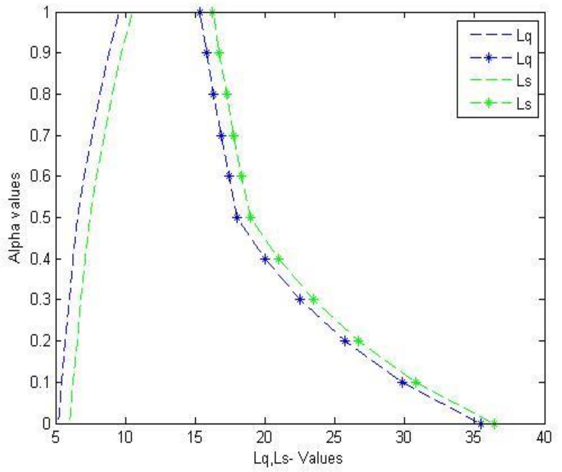

Figure 5.The α of L_q and L_s

Table 6:The α Cuts of Wq and Ws

α	Wq	Wq	Ws	Ws
0	0.005168	0.032443	0.006024	0.033333
0.1	0.005385	0.02736	0.006242	0.028249
0.2	0.005619	0.023624	0.006477	0.02451
0.3	0.005871	0.020761	0.006729	0.021645
0.4	0.006144	0.018498	0.007003	0.01938
0.5	0.006439	0.016664	0.007299	0.017544
0.6	0.006854	0.016128	0.007716	0.017007
0.7	0.007319	0.015623	0.008183	0.016502
0.8	0.007845	0.015147	0.008711	0.016026
0.9	0.008443	0.014698	0.009311	0.015576
1	0.00913	0.014274	0.01	0.015152

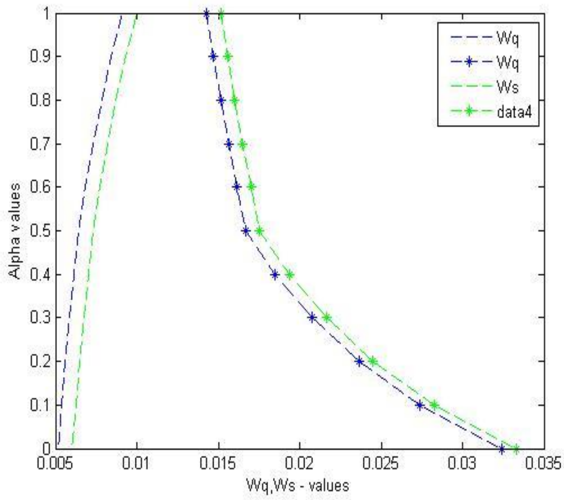

Figure 6.The α of W_q and W_s

5. Conclusions

In this paper, the performance measures of $FM / FM / 1$ Hexagonal fuzzy number with $\alpha-$ cut operations has been studied. The expected number of customers in the queue, system and the mean waiting time in the queue and system for Randomized Hexagonal fuzzy queue has been discussed with three differently randomly generated hexagonal fuzzy numbers with numerical example. The varying arrival and service rates are taken as a function of α in two intervals $[0, 0.5)$ and $[0.5, 1]$. It is found that with the randomly generated Hexagonal fuzzy numbers, the membership function is in between 0 and 1 and the waiting times decreases as the number of customers increase with a constant increase in arrival and service rates. The results shown by using Hexagonal fuzzy number are promising when compared with other fuzzy numbers, and are also easy to apply in the real life problems.

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