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A New Algorithm to Solve Pentagonal Fuzzy Transportation Problem

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ABSTRACT:
Optimization is very useful in the problems like transportation problems which is a special case of linear programming problems. It is an extremely handy tool for demand and supply chains to minimize time or cost and maximize profit. In this paper, we presented a new ranking method and a new algorithm to solve pentagonal FTP. We compared the solution with the proposed algorithm with the solution of existing methods. Here we transform FTP into a crisp problem and by using the proposed ranking technique we solved numerical examples to demonstrate the method.
Keywords: Ranking Function, Fuzzy Transportation Problem, Optimum Solution, Pentagonal Fuzzy Numbers, Interquartile range.
Abbreviations: FN – Fuzzy numbers, TP – Transportation Problem, FTP – Fuzzy Transportation Problem, PFN – Pentagonal Fuzzy Number, BCM – Best Candidate Method, ZSM – Zero Suffix Method.

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1. Introduction

Most real-world problems of transportation, which is a special type of linear programming problem, are used to decide the transportation schedule that reduces the entire transportation cost and time. A FTP is a problem in which all the quantities are fuzzy quantities such as transportation cost, supply and demand. These quantities are unknown due to many unmanageable factors. To manage inexact details in building decisions [10] established the concept of fuzziness.

[1] presented a method to solve FTP working on PFN. Here, the range technique in Statistics is used for ranking FN which is very easy to apply and also attain a minimum cost. The three existing methods are considered to find the solution to TP in [2]. The best optimality condition has been checked using various methods. Also [3] put forth a comparative study of different FN using the centroid ranking method, robust ranking method and BCM to minimize the cost. Adopting a robust ranking method and ZSM [3] finalized that the solution of FTP is acquired more productively. [4] used the centroid method to rank hexagonal and octagonal FN. Also, they explained the result using the ZSM. BCM is operated to infer the

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FTP by [6, 7]. They worked out illustrations to confirm the finding. A ranking method is adopted to rank octagonal FN by [8]. They managed to explain the balanced and unbalanced FTP. An improved ZSM is tried by [9] to solve FTP. They consumed a robust ranking method.
In statistics, the IQR is used to estimate the distance of the middle half of given data. Use the IQR to examine the variable where most of the value lies. Larger values indicate that the middle part of data spread out further smaller values show that the middle part collects more tightly. Here, IQR is used to rank PFN. Also, a new algorithm is presented to solve FTP. We obtain the least value for the optimum solution.

2. Preliminaries

2.1 Definition (Fuzzy set)
Let X be a nonempty set. A fuzzy set A of X is defined as A= {< x, μA (x)>/ x ϵ X} where (x) is called membership function , which maps each element of X to a value between 0 and 1.

2.2 Definition (Fuzzy Number)
A fuzzy number is a generalization of a regular real number and which does not refer to a single value but rather to a connected a set of possible values, where each possible values has its weight between 0 and 1. The weight is called the membership function.
A fuzzy number A is a convex normalized fuzzy set on the number line R such that there exists at least one x ϵ R with μA (x) = 1. μA (x) is piecewise continuous.
2.3 Definition (Triangular Fuzzy (Number) [TFN]
A fuzzy number A is said to a triangular fuzzy number, which is named as ($s_1, s_2, s_3,$) if membership function
μS (x) has the following properties:

$$\mu S(x) = \begin{cases} \frac{x-s1}{s2-s1}, & \text{if } s_1 \leq x < s_2 \\ 1, & \text{if } x = s_2 \\ \frac{s3-x}{s3-s2}, & \text{if } s_2 < x \leq s_3 \\ 0, & \text{if otherwise} \end{cases}$$

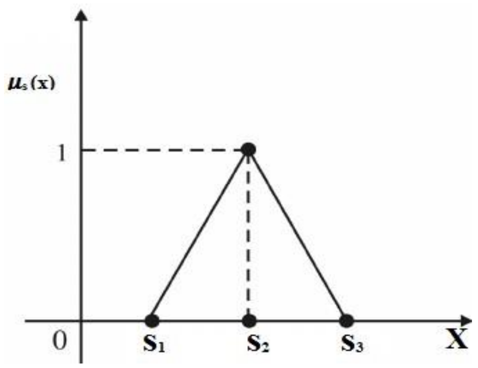

Fig 1. Graphical representation of Triangular FN

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2.4 Definition (Trapezoidal Fuzzy Number) [TrFN]

A fuzzy number $\tilde{A}$ is said a trapezoidal fuzzy number, which is named as (s_1, s_2, s_3, s_4) whose membership function μS (x) is given by

$$\mu A(x) = \begin{cases} 0, \text{ if } x < s_1 \\ \frac{x-s1}{s2-s1}, \text{ if } s_1 \leq x < s_2 \\ 1, \text{ if } s_2 < x \leq s_3 \\ \frac{s4-x}{s4-s3} \text{ if } s_3 < x \leq s_4 \\ 0, \text{ if } x > s_4 \end{cases}$$

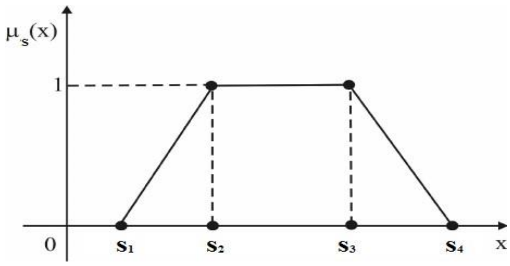

Fig 2. Graphical representation of Trapezoidal FN

2.4 Definition (Pentagonal Fuzzy Number)[PFN]

A fuzzy number $\hat{A}$ is said a pentagonal fuzzy number, which is named as (s_1, s_2, s_3, s_4, s_5) whose membership function μS (x) is given by

$$\mu A(x) = \begin{cases} 0, \text{ if } x < s_1 \\ u_1 \left(\frac{x-s1}{s2-s1}\right), \text{ if } s_1 \leq x < s_2 \\ 1-(1 - u_1)\left(\frac{x-s2}{s3-s2}\right), \text{ if } s_2 \leq x < s_3 \\ 1, \text{ if } x = s_3 \\ 1-(1-u_2)\left(\frac{s4-x}{s4-s3}\right), \text{ if } s_3 \leq x < s_4 \\ u_2\left(\frac{s5-x}{s5-s4}\right), \text{ if } s_4 \leq x < s_5 \\ 0, \text{ if } x > s_5 \end{cases}$$

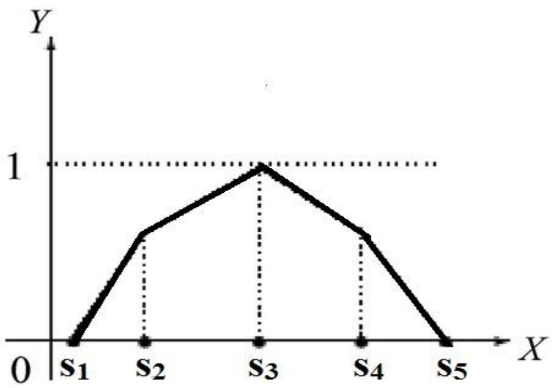

Fig 3. Graphical representation of PFN

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3. GORHE & GHADLE ALGORITHM

The ranking of FN is an extremely crucial chore to reduce numbers. Several ranking techniques are usable for FN. Here, we provide an effortless technique to convert PFN into crisp values. Also, we suggested a new algorithm to solve pentagonal FTP which is very easy.

3.1 Proposed Ranking Techniques:

To calculate the IQR we need to follow the following steps:

1. Arrange the numbers in increasing order.
2. Find the Median. The median is the 'centre 'of the numbers. If the numbers are odd, then the median is the centre most number.
3. Place the brackets around the numbers before and after the median. It makes Q_1 and Q_3 easier to spot. Calculate the median of both upper (Q_3) and lower (Q_1) data. We have the even numbers so the median is the average of the middle two numbers. Calculate the median of both upper (Q_3) and lower (Q_1) data.
4. Observe the difference between Q_3 and Q_1. IQR = $Q_3 - Q_1$.

3.2 Proposed Method.

1. Set up the transportation problem table and examine whether it is balanced or not. Ifit is not balanced then add a dummy row/column.
2. Using the proposed ranking method we convert the FTP into crisp values.
3. For each row we take the sum of row maximum and just less than the maximum and divide it by the number of columns.
4. For each column we take the sum of the column maximum and just less than the maximum and divide it by the number of rows.
5. We choose the greatest of the resultant value. After that, we choose the least value of the cost and make the allocation. If we get multiple greatest values, we can choose any value.
6. Repeat steps 3 to 5 to complete the allocations and the optimum solution.

4.NUMERICAL EXAMPLES

4.1 Consider a balanced FTP.

	A	B	C	D	Supply
E	(1,3,5,7,9)	(0,1,2,3,4)	(2,4,7,8,9)	(0,3,4,5,8)	50
F	(2,4,5,6,8)	(0,3,4,6,7)	(3,5,6,8,9)	(0,2,3,4,5)	40
G	(2,4,5,6,9)	(0,2,4,6,8)	(1,2,5,7,10)	(2,3,4,5,6)	60
H	(0,1,3,5,6)	(1,2,4,5,6)	(0,1,2,3,5)	(2,3,5,7,9)	30
Demand	40	45	45	50	

Solution:

Find IQR for (1, 3, 5, 7, 9).

Given numbers are in increasing order 1, 3, 5, 7, 9.

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Its middle value i.e. median is 5. We place the bracket to find Q_1 and Q_3. (1,3) 5 (7, 9)
We have two (even) numbers so the median is the average of the middle two numbers.

$$Q_1 = \frac{1+3}{2} = 2 \quad Q_3 = \frac{7+9}{2} = 8$$

$$IQR = Q_3 - Q_1 = 8 - 2 = 6$$

By using proposed ranking technique, we transform given PFN into crisp values.

	A	B	C	D	Supply
E	6	3	5.5	5	50
F	4	5	4.5	3.5	40
G	4.5	6	7	3	60
H	5	4	3.5	5.5	30
Demand	40	45	45	50	

Presently we can use the proposed method to obtain the resultant values by the formula

$$\frac{Max + just\ less\ than\ max}{Number\ of\ Columns/Number\ of\ rows}$$

For rows take number of columns and for columns take number of rows.

	A	B	C	D		
E	6	3	5.5	5	50	11.5/4
F	4	5	4.5	3.5	40	9.5/4
G	4.5	6	7	3	60	13/4
H	5	4	3.5	5.5	30	10.5/4
Demand	40	45	45	50		
	11/4	11/4	12.5/4	10.5/4		

Now, we find the greatest resultant value. We look at the least cost value and allocate that cost. Now, three columns and four rows have remained. So the resultant values will be changed.

	A	B	C	D	Supply	
E	6	3	5.5	5	50	11.5/3
F	4	5	4.5	3.5	40	9.5/3
G	4.5	6	7	3 **50**	60	13/3
H	5	5	3.5	5.5	30	10.5/3
Demand	40	45	45	50		
	11/4	11/4	12.5/4	10.5/4		

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We use the same technique as above repeatedly until we get the final allocation.

	A		B		C		D		Supply
E	6	**5**	3	**45**	5.5		5		50
F	4	**25**	5		4.5	**15**	3.5		40
G	4.5	**10**	6		7		3	**50**	60
H	5		4		3.5	**30**	5.5		30
Demand	40		45		45		50		

The transportation cost Z = 3 *50 + 4.5 * 10 + 3 * 45 + 6 * 5 + 4 * 25 + 4.5 * 15 + 3.5 * 30

Z = 632.5

Comparison with other techniques

The below table involves the comparison of the solution by the proposed technique along with other existing methods. It is expressed that the proposed technique provides the optimum solution.

Techniques	**Optimum Solution**
North-West Corner Method	972.5
VAM Method	635
LCM Method	647.5
Zero suffix Method	635
Best Candidate Method	647.5
Proposed Technique	**632.5**

2) Consider an unbalanced FTP.

	A	B	C	D	Supply
E	(1,3,5,7,9)	(0,1,2,3,4)	(2,4,7,8,9)	(0,3,4,5,8)	50
F	(2,4,5,6,8)	(0,3,4,6,7)	(3,5,6,8,9)	(0,2,3,4,5)	40
G	(2,4,5,6,9)	(0,2,4,6,8)	(1,2,5,7,10)	(2,3,4,5,6)	60
H	(0,1,3,5,6)	(1,2,4,5,6)	(0,1,2,3,5)	(2,3,5,7,9)	30
Demand	40	25	35	45	

Solution By using proposed ranking technique, we transform given PFN into crisp values.

	A	B	C	D	Supply
E	6	3	5.5	5	50
F	4	5	4.5	3.5	40
G	4.5	6	7	3	60
H	5	4	3.5	5.5	30
Demand	40	25	35	45	

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The stated Problem is unbalanced so we add 0 columns to make it balanced.

	A	B	C	D	Z	Supply
E	6	3	5.5	5	0	50
F	4	5	4.5	3.5	0	40
G	4.5	6	7	3	0	60
H	5	4	3.5	5.5	0	30
Demand	40	25	35	45	35	

Presently we can use the proposed method to obtain the resultant values by the above formula.

	A	B	C	D	Z	Supply	
E	6	3	5.5	5	0	50	11.5/5
F	4	5	4.5	3.5	0	40	9.5/5
G	4.5	6	7	3	0	60	13/5
H	5	4	3.5	5.5	0	30	10.5/5
Demand	40	25	35	45	35		
	11/4	11/4	12.5/4	10.5/4			

Now, we find the greatest resultant value. We look at the least cost value and allocate that cost. Now, three rows and five columns have remained. So the resultant values will be changed.

	A	B	C	D	Z	Supply	
E	6	3	5.5	5	0	50	11.5/5
F	4	5	4.5	3.5	0	40	9.5/5
G	4.5	6	7	3	0	60	13/5
H	5	4	3.5 **30**	5.5	0	30	10.5/5
Demand	40	25	35	45	35		
	10.5/3	11/3	12.5/3	8.5/3	0/3		

We use the same technique as above repeatedly until we get the final allocation.

	A	B	C	D	Z	Supply
E	6	3 **25**	5.5	5	0 **25**	50
F	4 **35**	5	4.5 **5**	3.5	0	40
G	4.5 **5**	6	7	3 **45**	0 **10**	60
H	5	4	3.5 **30**	5.5	0	30
Demand	40	25	35	45	35	

The transportation cost Z = 3*25 +0*25 + 4*35 + 4.5*5 +4.5*5 + 3*45 + 0*10 + 3.5*30
Z = 500

Comparison with other techniques

In the below table the analogy of the proposed technique along with the other techniques are given. It is evidently presented that the proposed technique gives optimum solution.

Techniques	**Optimum Solution**
North-West Corner Method	662.5
VAM Method	560
LCM Method	540
Zero suffix Method	500
Best Candidate Method	510
Proposed Technique	**500**

5. CONCLUSIONS

There are many efficient ranking techniques to rank FN. This work is distinct in the sense that it is a new attempt to rank PFN in statistical way. The IQR is very easy to apply. Also, a new algorithm is applied to solve pentagonal FTP. The balanced and unbalanced pentagonal FTP has been explained. It is believed that we obtain an optimum solution using this method.

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