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Using Dirichlet Kernel to Analyze Long Memory Time Series

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Abstract:
In this paper, we propose a new spectral density estimator based on Dirichlet -Kernel homogeneity in the cycle diagram. Our methodology is nonparametric and can be applied to processes that do not necessarily follow a normal distribution. Dirichlet kernel is an asymmetric density function of a shape that varies according to the frequency at which the spectrum is estimated. Dirichlet kernel smoothing was introduced because Dirichlet kernel diverges at zero when its bandwidth shrinks; it becomes smoother and more attractive than the cycle diagram when the process is a long memory. It automatically adjusts to a time series range. If the process is a short memory, the resulting estimation of the spectral density is automatically constrained, while the estimator diverges at the origin when applied to the certified long-term data. Kernel smoothing or kernel density estimation is a well-known methodology for non-parametric characterization of the probability density function of a random variable or random vector. It can be considered as a proxy of packages for histograms, and is particularly useful in multivariate cases; density estimation for non-parametric alternatives for regression and classification can be used to represent how the conditional probabilities of a categorical variable depend on quantitative variables. Our main goal in this paper is to find a non-parametric estimation for a long memory time series, to reconsider the Dirichlet Kernel estimator and to study its asymptotic properties in detail. Our main contribution to this paper is to find asymptotic expressions for point bias, point variance, mean squared error (MSE) and mean integrated square error (MISE). These results generalize to those of the beta kernel. Choosing the optimal bandwidth for parameter b (bandwidth parameters b).
Key Words: Dirichlet distribution, Kernel function, Bandwidth, time series.

1. Introduction

Estimating spectral density often requires knowing whether fixed time series are short or long memory. The short memory time series is known as the auto-covariance function which rapidly decreases with increasing time lag. In the case of long memory time series, there is a much stronger dependence between values at different times, and the decay of the spontaneous covariance function is slow. Long-term memory or long-term dependence has an infinite spectral density at frequency zero, therefore, the choice of the optimal non-parametric estimator will be different if the spectral density is limited or not. Our goal is to go beyond this limit and propose an estimator applicable to any long-term fixed time series data.

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Brown and Chen were the first to theoretically study the beta kernel (d=1) in the context of smoothing regression curves with equally spaced fixed design points. Approaches to point bias, integral variance and MISE were found for the regression function estimator. These results were extended to beta-kernel estimates in some parts of the results.

2. Dirichlet distribution

The Dirichlet distribution is a continuous distribution and is considered a multivariate generalization of the beta distribution. Dirichlet distribution has many applications in different areas. Important in probability and statistics, the Dirichlet distribution is the most normal distribution of synthetic data and proportional modeling measures in Bayesian statistics, the Dirichlet distribution is a pre-common coupling of the polynomial distribution. The Dirichlet Distribution is used to derive the distribution function for system statistics in biology, that the Dirichlet distribution can be used to calculate forensic matching probabilities of several distinct populations. The Dirichlet distribution can be used to model a player's abilities in a league, and the Dirichlet distribution can be used to model consumer buying behavior.

The pooled Dirichlet distribution and the nested Dirichlet distribution can be used for statistical analysis of incomplete categorical data. Also, there are some distributions related to this distribution, such as the generalized Dirichlet distribution, the hyper-distribution, the Dirichlet-Multinomial distribution, the gradient Dirichlet distribution and the mixed distribution. The Dirichlet distribution can also be derived from the gamma distribution.[1]

Let $y = (y_1 \; \dots . y_k)^T$ is a positive random vector K×1 such that $(y_1 + \cdots + y_k) = 1$ and $yi \in (0,1)$for each $i \in (1, \dots, k)$. The random vector y follows a Dirichlet distribution with positive parameters $\alpha = (\alpha_1, \dots \dots, \alpha_k)^T$. It means y~Dir(α), if the probability density function is :[2]

$f(y|\alpha) = \frac{1}{\beta(\alpha)} \prod_{i=1}^{k} yi^{\alpha i - 1}$ ……. (1)

So, the polynomial beta function that acts as a normalizing constant is:

$\beta(\alpha) = \prod_{i=1}^{k} \Gamma(\alpha_i) / \Gamma(\sum_{i=1}^{k} \alpha_i)$ ……..(2)

Γ is Euler's gamma function, Dirichlet distribution, which is a multivariate generalization of the beta distribution where $(y_1 + \cdots + y_k) = 1$.

Moreover $= (\alpha_1 + \cdots \dots + \alpha_k)$ can be interpreted as the inverse scale modulus or concentration coefficient, and the expectation of each element is $E(y_{it}) = \alpha_{it}/\tau_t$,

The variance is $var(y_{it}) = \{\alpha_{it}(\tau_t - \alpha_{it})\}/\{\tau_t^2(\tau_t + 1)\}$

And the covariance i≠j is $cov(y_i, y_j) = (-\alpha_i \alpha_j)/\{\tau^2(\tau + 1)\}$

We assume that the vector $y_t | y_{t-1}, y_{t-2}, \dots, y_1$follows a Dirichlet distribution with positive parameters:

$\alpha_t = (\alpha_{1t}, \alpha_{2t}, \dots \dots, \alpha_{kt}) : y_t | y_{t-1}, y_{t-2}, \dots, y_1 \; {}_{\sim Dir(\alpha_t)}$ …….(3)

In order to associate $\alpha_t$with $y_{t-1}, y_{t-2}, \dots, y_1$ we assume that $ln(\alpha_{jt}/\alpha_{kt}) = \eta_{jt}$ when $\eta_{jt}$is defined:

$$\eta_{jt} = \alpha_{j0} + \sum_{p=1}^{P} \alpha_{jp} \,.\, \ln\left(\frac{y_{jt-p}}{y_{kt-p}}\right) + \sum_{p=1}^{P} b_{jp} \,.\, ln \frac{\left(\prod_{S=1\, s \neq j}^{K-1} y_{st-p}\right)^{1/K-2}}{y_{Kt-p}} \, , \qquad j = 1,2, \dots, K-1$$

…… (4)

By adding the log-ratio ratio and through the Taylor transformation of the first order, we can determine the following:

$E\left(\ln\left(\frac{y_{jt}}{y_{kt}}\right)\right) = E\left(alr(y_{jt})\right) \approx alr\left(E(y_{jt})\right) = alr\left(\frac{\alpha_{jt}}{\tau}\right) = \ln\left(\frac{\alpha_{jt}}{\alpha_{Kt}}\right)$ ……(5)

Using the expression $ln(\alpha_{jt}/\alpha_{kt}) = \eta_{jt}$, α_{jt} and α_{kt} can be determined by:

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From this one can calculate the expectation $E(y_{it})$ and calculate the variance $var(y_{it})$.

Deriving the Dirichlet Distribution

Let X_i be a random variable that follows a gamma distribution; G(α_i ,1) i=1,2,…..,k and assuming that $X_1, \ldots\ldots, X_k$ is independent, then the pdf probability density function for $(X_1, \ldots\ldots, X_k)$ is:

$$f(x_1, \ldots \ldots x_k) = \begin{cases} \prod_{i=1}^{k} \frac{1}{\Gamma(\alpha_i)} x_i^{\alpha i-1} \quad e^{-x_i} & if\ 0 < x_i < \infty \\ 0 \qquad otherwise \end{cases} \quad \ldots\ldots..(6)$$

Let:

$$Y_i = \frac{X_i}{X_1+X_2+\cdots\ldots+X_k} \quad ; i = 1,2, \ldots\ldots, k-1 \ ; \ and \ \ Z_k = X_1 + X_2 + \cdots \ldots . +X_k \quad \ldots.(7)$$

And by transforming using the technique of changing variables (transformation maps):

$M = \{(x_1, \ldots\ldots, x_k): 0 < x_i < \infty , i = 1,2, \ldots\ldots, k \ \}$ onto

N $= \{(y_1, \ldots., y_{k-1}, Z_k: \quad y_i > 0 , i = 1, \ldots.. k-1 , 0 < Z_k < \infty , y_1 + \cdots \ldots + y_{k-1} < 1\}$

The Jacobin matrix is:

$$J = \begin{vmatrix} z_k & 0 & \cdots & 0 & y_1 \\ 0 & z_k & \cdots & 0 & y_2 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & z_k & y_{k-1} \\ -z_k & -z_k & \cdots & -z_k & 1 - y_1 - \cdots - y_{k-1} \end{vmatrix} = z_k^{k-1} \quad \ldots..(8)$$

The joint probability density function joint pdf for $(Y_1, \ldots., Y_{k-1}, Z_k\ )$ is:

$$f(y_1, \ldots., y_{k-1}, Z_k) = \frac{y_1^{\alpha 1-1} \cdots y_{k-1}^{\alpha_{k-1}-1} (1-y_1-\cdots-y_{k-1})^{\alpha k-1}}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_k)} e^{-zk} \ z_k^{\alpha 1+\cdots+\alpha k-1} \quad \ldots.. (9)$$

By integrating z_k, the joint PDF of $(Y_1, \ldots., Y_{k-1}, Z_k)$ is:

$$f(y_1, \ldots., y_{k-1}, Z_k) = \frac{\alpha_1+\cdots+\alpha_k}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_k)} y_1^{\alpha 1-1} \cdots y_{k-1}^{\alpha_{k-1}-1} ((1 - y_1 - \cdots - y_{k-1})^{\alpha k-1} \quad \ldots (10)$$

Where:

$y_i > 0 , y_1 + \cdots + y_{k-1} < 1 , i = 1, \ldots, k-1$

$Z_k \sim G(\sum_{i=1}^{k} \alpha_i \ , 1)$

In a special case if $k = 2$ ، $f(y1, y2)$denotes a beta distribution with the parameters α_1, α_2.[3]

mean

$$E(Y_i) = \frac{\alpha_i}{\alpha_0} , i = 1,2, \ldots, k$$

$$E(Y_1) = \int \cdots \int y_1 \frac{\Gamma(\alpha_0)}{\prod_{i=1}^{k} \Gamma(\alpha_i)} \prod_{i=1}^{k} y_i^{\alpha_i-1} dy1 \cdots dyk$$

$$= \int \cdots \int \ \frac{\Gamma(\alpha_0)}{\prod_{i=1}^{k} \Gamma(\alpha_i)} y_1 \, y_1^{\alpha_1-1} \prod_{i=1}^{k} y_i^{\alpha_i-1} (1 - \sum_{i=1}^{k-1} y_i)^{\alpha_k-1} \, dy1 \cdots dyk-1$$

$$= \frac{\Gamma(\alpha_0)}{\Gamma(\alpha_1) \prod_{i=2}^{k} \Gamma(\alpha_i)} \ \frac{\Gamma(\alpha_1+1) \prod_{i=2}^{k} \Gamma(\alpha_i)}{\Gamma(\alpha_0+1)}$$

$$= \frac{\Gamma(\alpha_0)}{\Gamma(\alpha_0+1)} \ \frac{\Gamma(\alpha_0+1)}{\Gamma(\alpha_1)}$$

$$= \frac{(\alpha_0)}{(\alpha_1)} \quad \ldots..(11)$$

Variance

$$\boldsymbol{var(Y_i) = \frac{\alpha_i(\alpha_0-\alpha_i)}{\alpha_0^2(\alpha_0+1)} , i = 1, 2, \ldots., k} \quad \ldots..(12)$$

We showed previously $E(Y_i)$ and now we calculate $E(Y_i^2)$ as follows:[4]

$$E(Y_i^2) = \frac{\Gamma(\alpha_0)}{\Gamma(\alpha_0+2)} \ \frac{\Gamma(\alpha_i+2)}{\Gamma(\alpha_i)}$$

$$= \frac{(\alpha_i+1)\alpha_i}{(\alpha_0+1)\alpha_0} \quad .....(13)$$

$$\boldsymbol{var(Y_i)} = E(Y_i^2) - E(Y_i)^2$$

$$= \frac{(\alpha_i+1)\alpha_i}{(\alpha_0+1)\alpha_0} - (\frac{\alpha_i}{\alpha_0})^2$$

$$= \frac{\alpha_i(\alpha_0-\alpha_i)}{\alpha_0^2(\alpha_0+1)} \quad .....(14)$$

Covariance

$$Cov(Y_i, Y_j) = \frac{-\alpha_i\alpha_j}{\alpha_0^2(\alpha_0+1)} \ , i = 1,2, \dots, k \,; j = 1,2, \dots, k \ and \ i \neq j$$

$$E(Y_i, Y_j) = \frac{\Gamma(\alpha_0)}{\Gamma(\alpha_0+2)} \frac{\Gamma(\alpha_i+1)}{\Gamma(\alpha_i)} \frac{\Gamma(\alpha_j+1)}{\Gamma(\alpha_j)}$$

$$= \frac{\alpha_i\alpha_j}{\alpha_0(\alpha_0+1)} \ , \ i \neq j \quad .....(15)$$

$$Cov(Y_i, Y_j) = E(Y_i, Y_j) - E(Y_i)E(Y_j)$$

$$= \frac{\alpha_i\alpha_j}{\alpha_0(\alpha_0+1)} - \frac{\alpha_i}{\alpha_0}\frac{\alpha_j}{\alpha_0}$$

$$= \frac{-\alpha_i\alpha_j}{\alpha_0^2\,(\alpha_0+1)} \ , \ i \neq j \quad .....(16)$$

The marginal distribution Y_i

$Beta(\alpha_i, \sum_{j=1}^{k} \alpha_i - \alpha_i), \ i = 1,2, \dots, k$

$Z_k - X_i \sim G(\sum_{j=1}^{k} \alpha_i - \alpha_i \ , 1)$

Y_i يتبع توزيع بيتا

$$Y_i = \frac{X_i}{Z_k}$$

$$= \frac{X_i}{X_i + (Z_K - X_i)} \quad .....(17)$$

marginal distribution of $Y_i \sim Beta(\alpha_i, \sum_{j=1}^{k} \alpha_i - \alpha_i)$

The logarithm of the spectral density of the three FARIMA generation models used in the simulation. Estimates were calculated based on 1000 Monte Carlo model simulations, for sample sizes N = (400, 600, 1000). Since the semi-parameter estimator is a local estimator around the pole, we did not compare it with the beta-core estimator at all frequencies but only in the vicinity of frequency zero. [5]

The logarithm of the three FARIMA spectral intensities used in the simulations calculates the estimation error at frequencies[6]

$$\omega_j \ \epsilon(\omega_{j0}, \omega_{j1}) \ where \ j0 = [\sqrt[5]{T}] \ and \ j1 = [\sqrt{T}] \quad .....(18)$$

with [α] the integer part of α . The computed experimental error is the mean of the relative absolute deviation

$$RMAD_{[j0,j1]} = \frac{1}{j_1 - j_0 + 1} \sum_{j=j0}^{j1} \frac{\left|\hat{f}_S(\frac{j}{T}) - f(\frac{j}{T})\right|}{f(\frac{j}{T})} \quad .....(19)$$

As $\hat{f}_s$ is either a kernel beta estimator or a Robinson sem-parametric estimator. The mean absolute relative deviation was taken instead of the mean absolute deviation driven by the cutoff f at frequency zero. Since j0 and j1 depend on T, the range of frequencies at which the error is calculated varies with different sample sizes. Therefore, the RMAD values presented in the pilot study below are only comparable for a given sample size. The FARIMA time series were generated via the "fracdiff" library in R. To avoid dependent our conclusions on bandwidth selection, we computed the RMAD for a set of bandwidths which is [0.01, 0.05], the m range in the semi-parametric estimate is $[T^{1/2} \ , \ T^{4/5}]$.[7]

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3. Dirichlet kernels

Dirichlet distribution density (α, β) is:[8]

$$K_{\alpha,\beta}(s)=\frac{\Gamma(\|\alpha\|_1+\beta)}{\Gamma(\beta)\prod_{i=1}^{d}\Gamma(\alpha_i)}\cdot(1-\|s\|_1)^{\beta-1}\prod_{i=1}^{d}s_i^{\alpha i-1}\quad,\ s\in S_d\qquad\ldots\ldots(20)$$

To add the bandwidth of parameter b>0 to the sample observations X_1,……..,X_n for the F distribution with density f supported on S_d, the Dirichlet kernel estimator is defined by:[9][10]

$$\hat{f}_{n,b}(s)=\frac{1}{n}\sum_{i=1}^{n}K_s/b+1\,,(1-\|s\|_1)/b+1\,(X_i)\,,\ s\in S_d\qquad\ldots\ldots..(21)$$

The shape of the nucleus changes with the s position slightly; Unlike traditional estimators where the kernel is the same for each point. This variable smoothing allows Dirichlet kernel estimators (and more generally, asymmetric kernel estimators) to avoid the boundary bias problem of traditional kernel estimators.

4. Simulation study:

Estimating spectral density often requires knowing whether fixed time series are short or long memory. Short memory time series is known as the autocorrelation function which decreases rapidly with increasing time lag. In the case of long memory time series, the decay of the autocorrelation function is slow. , because it has a strong dependence between values at different times.

Long memory has an unlimited spectral density at frequency zero, so choosing an optimal non-parametric estimator will be different if the spectral density is limited or not.

Our goal is to go beyond this limit and propose an estimator applicable to any stable time series with long memory.

We present a proposal for a new nonparametric estimator for spectral density obtained by the cycle plot and smoothing the graph using kernel density (kernel). Our methodology is non-parametric and can be applied to data that are not subject to a normal distribution.

The Dirichlet Kernel estimator of the spectral density.

We will compare the results using the RMAD criterion with the Beta kernel estimator of the spectral density.

Long-term time series were generated using the R programming language based on the “fracdiff” software package. The RMAD standard was calculated to compare the capabilities and its formula is:

$$RMAD_{[j0,j1]}=\frac{1}{j_1-j_0+1}\sum_{j=j0}^{j1}\frac{\left|\hat{f}_s\left(\frac{j}{T}\right)-f\left(\frac{j}{T}\right)\right|}{f(\frac{j}{T})}\qquad\ldots.\ (22)$$

Several values were used for the Hurst coefficient (H), which determines the value of the fractional difference (d_0), and these values are (H=0.1, H=0.4, H=0.6, H=0.9), which results in d values through the relationship that links Hearst's coefficient H with the degree of fractional integration d: d=H-0.5 We also adopted a set of sample sizes (N=500, N=750, N=1000) for the purpose of knowing the behavior of the estimators used in this thesis. In addition, the spectrum function estimator was calculated according to the previously mentioned methods with the proposed methods, and their estimations were drawn as in Figure (1,2). The simulation experiment was repeated 1000 times for the purpose of arriving at stable and reliable estimates in the comparison process, and the results were as in the following tables:

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Table (1) shows RMAD and S.D values when H=0.1

N =500			N =750			N =1000		
Beta Kernel			Beta Kernel			Beta Kernel		
b	RMAD	s.d	b	RMAD	s.d	b	RMAD	s.d
0.005	0.71235	0.04547	0.005	0.72915	0.04496	0.005	0.74818	0.04183
0.01	0.78601	0.04428	0.01	0.80833	0.04297	0.01	0.82842	0.03828
0.05	0.94421	0.01799	0.05	0.95659	0.01486	0.05	0.965	0.01035
0.08	0.96538	0.01152	0.08	0.97392	0.00917	0.08	0.97948	0.00622
Dirichlet Kernel			Dirichlet Kernel			Dirichlet Kernel		
b	RMAD	s.d	b	RMAD	s.d	b	RMAD	s.d
0.005	0.68017	0.19115	0.005	0.66345	0.11325	0.005	0.65896	0.08828
0.01	0.66009	0.12377	0.01	0.6508	0.09737	0.01	0.64919	0.08034
0.05	0.64334	0.1023	0.05	0.63924	0.08283	0.05	0.64005	0.07642
0.08	0.64126	0.10024	0.08	0.63816	0.08179	0.08	0.63944	0.0773

Table (1) Monte Carlo simulation results for FARIMA(1,H=0.1,0) with α_1=0.6 with RMAD Relative Mean Absolute Deviation. It is clear from the above table that the Dirichlet Kernel estimator had the lowest values in the RMAD criterion, regardless of the sample size difference, as we notice at b = 0.08 and N = 500, which gave the best results.

Table (2) shows RMAD and S.D values when H=0.4

N =500			N =750			N =1000		
Beta Kernel			Beta Kernel			Beta Kernel		
b	RMAD	s.d	b	RMAD	s.d	b	RMAD	s.d
0.005	0.71562	0.04695	0.005	0.73536	0.04151	0.005	0.74822	0.04042
0.01	0.78758	0.04836	0.01	0.81302	0.03927	0.01	0.82991	0.03796
0.05	0.9371	0.02222	0.05	0.95428	0.01297	0.05	0.96179	0.01226
0.08	0.95884	0.01478	0.08	0.97114	0.00832	0.08	0.97633	0.00774
Dirichlet Kernel			Dirichlet Kernel			Dirichlet Kernel		
b	RMAD	s.d	b	RMAD	s.d	b	RMAD	s.d
0.005	0.67626	0.18158	0.005	0.65572	0.09628	0.005	0.64873	0.08629
0.01	0.65696	0.13207	0.01	0.64577	0.08505	0.01	0.64198	0.07842
0.05	0.63792	0.09389	0.05	0.63505	0.07699	0.05	0.6335	0.07108
0.08	0.6351	0.0901	0.08	0.63342	0.07635	0.08	0.63233	0.07086

Table (2) The same process except that H = 0.4, that is, the time series is still dependent on a long range, Monte Carlo simulation results for FARIMA model(1,H=0.4,0) with α_1=0.6 with mean absolute relative deviation (RMAD) Relative Mean Absolute Deviation. It is clear from the above table that the Dirichlet Kernel estimator had the lowest values in the RMAD criterion regardless of the difference in the sample size, as we notice at b = 0.08 and N = 750, which gave the best.

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Table (3) shows RMAD and S.D values when H=0.9

N =500				N =750				N =1000		
Beta Kernel				Beta Kernel				Beta Kernel		
b	RMAD	s.d		b	RMAD	s.d		b	RMAD	s.d
0.005	0.68067	0.04504		0.005	0.69501	0.04208		0.005	0.70992	0.03805
0.01	0.72764	0.04379		0.01	0.74817	0.04295		0.01	0.76614	0.03738
0.05	0.84907	0.04032		0.05	0.87796	0.03277		0.05	0.89367	0.02707
0.08	0.8771	0.0376		0.08	0.90445	0.02823		0.08	0.9178	0.02302
Dirichlet Kernel				Dirichlet Kernel				Dirichlet Kernel		
b	RMAD	s.d		b	RMAD	s.d		b	RMAD	s.d
0.005	0.69838	0.10851		0.005	0.68329	0.08687		0.005	0.6735	0.07887
0.01	0.69386	0.10537		0.01	0.6788	0.08608		0.01	0.6709	0.07841
0.05	0.68475	0.10052		0.05	0.67006	0.08393		0.05	0.66489	0.07656
0.08	0.68138	0.09939		0.08	0.66679	0.08317		0.08	0.66196	0.07552

Table (3) Results of Monte Carlo simulation of FARIMA(1,H=0.9,0) model with α_1=0.6 with Relative Mean Absolute Deviation (RMAD). It is clear from the above table that the Dirichlet Kernel had the lowest values in the RMAD criterion, regardless of the difference in the sample size, as we notice at b = 0.05 and N = 500, which gave the best results.

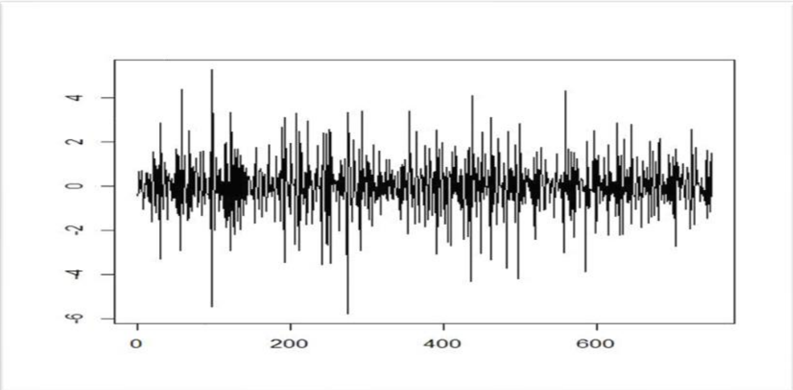

Figure (1) Generated time series according to ARFIMA model(1,H=0.6,0)

The autocorrelation functions ACF and partial autocorrelation PACF were also found and plotted, as in the figure below, which shows the behavior of the generated model according to ARFIMA(1,H=0.6,0)

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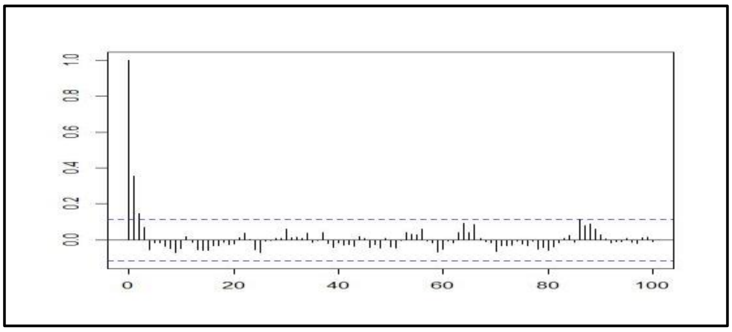

Figure (2) ACF empirical autocorrelation function

The ACF diagram of the data generated for the ARFIMA(1,H,0) model shows that the time series is characterized by the long memory feature, and this is evident by the very slow decreasing of the ACF function over the long time gaps.

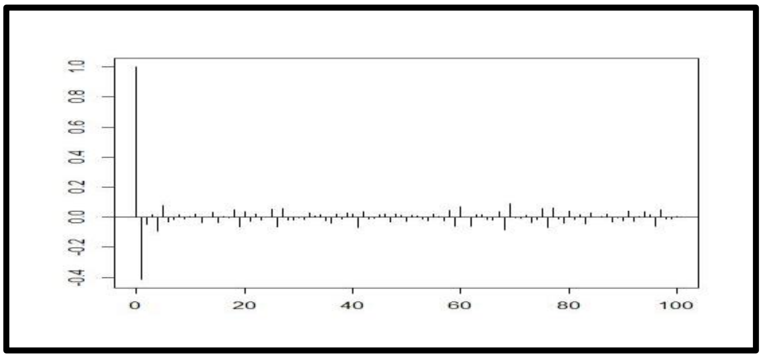

Figure (3) Absolute empirical autocorrelation function

In addition, the values of the spectrum logarithm of the time series estimated according to the beta kernel estimator were found using a set of bandwidths in order to know its behavior, as shown in the following figure:

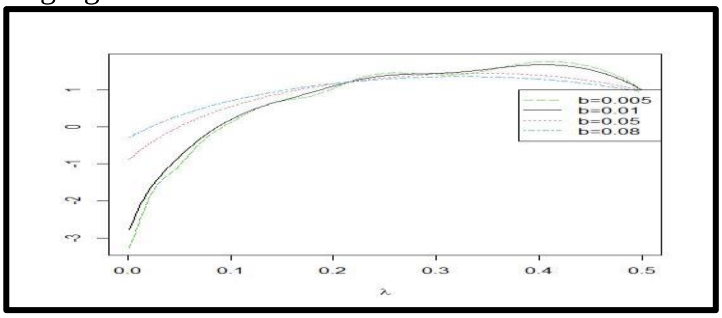

Figure (4) Logarithm values of the spectrum logarithm of the logarithm series of data estimated using a beta kernel estimator with different bandwidth values

From the above figure it is clear that the frequency of the spectrum varies with the different values of the bandwidth, as the spectrum function increased with the increase in the frequencies values λ and for all the values of the bandwidth, it is also clear that at the bandwidth $b = 0.08$ the spectral function estimators were higher than the rest of the estimators and these values increased with the height of the frequency λ . And the least values of the spectrum function were at the bandwidth $b = 0.005$. In general, we note that the

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spectrum function estimator for the beta kernel was somewhat similar between the different bandwidth values.

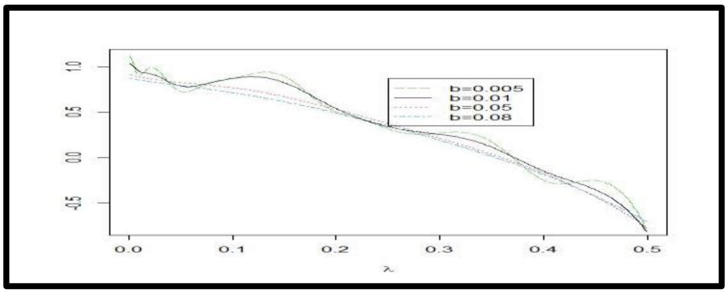

Figure (5) Logarithm values of the logarithm spectrum of the logarithm series of absolute data estimated using a beta kernel estimator with different bandwidth values.

As for the Figure (5) it shows the values of the spectrum estimators for the absolute values of the logarithm of the time series, and it turns out that the bandwidth b = 0.05 was higher than the rest of the values. In addition, the spectrum function discovered high frequencies at λ = (0.1, 0.15), which indicates the existence of a hidden behavior in data at this frequency. It should be noted that the estimated spectrum function values decrease with the increase in λ .values

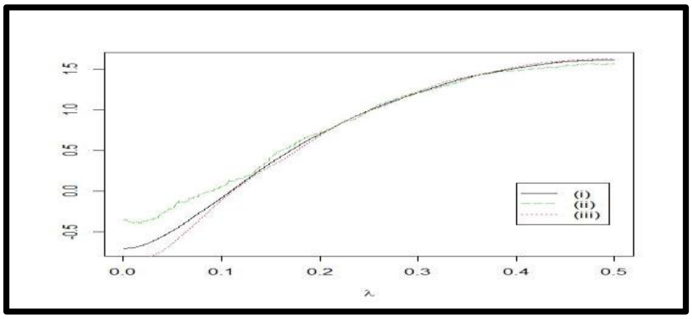

Figure (6) The logarithm values of the spectrum for logarithm series of data estimated using a semiparametric estimator and vary according to estimation methods) i) Symmetric Daniell kernel; (ii) Rectangular kernel (iii) Asymmetric triangular kernel.

From the figure (6) it is clear that the spectrum frequency varies according to the different methods of estimation, as the spectrum function decreased when the frequencies λ decreased and for all estimation methods. The lowest values of the spectrum function were at the asymmetric triangular kernel estimation.

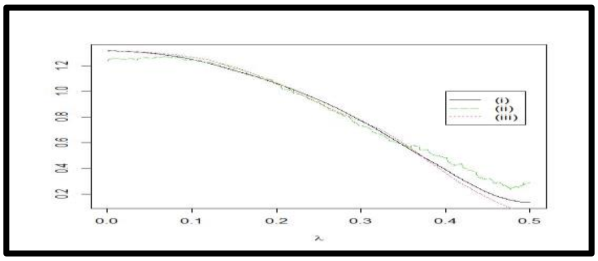

Figure (7) The logarithm values of the spectrum logarithm of the logarithm series of absolute data estimated using the Semiparametric estimator and vary according to estimation methods (i) Symmetric Daniell kernel; (ii) Rectangular kernel (iii) Asymmetric triangular kernel.

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As for the figure (7) it shows the values of the spectrum estimators for the absolute values of the logarithm of the time series, and it turns out that the Symmetric Daniell kernel estimation method was higher than the rest of the values. In addition, the spectrum function discovered high frequencies at λ = (0.1, 0.15), which indicates a hidden behavior in data at this frequency. It should be noted that the estimated spectrum function values decrease with the increase in λ values.

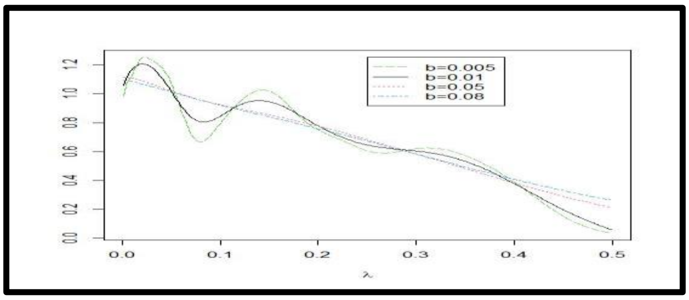

Figure (8) (Log spectrum logarithm values of the logarithm series of data estimated using a Dirichlet Kernel estimator with different bandwidth values

Figure (8) shows that the spectrum frequency varies with the different values of the bandwidth, as the spectrum function rose at bandwidth b = 0.005, then went back down at frequency λ = 0.1, then went back up at λ = 0.15, then continued to decline for high frequencies, as well as at bandwidth 0.01, but At a lower drop level, the rest of the band values started decreasing one pace at higher frequencies.

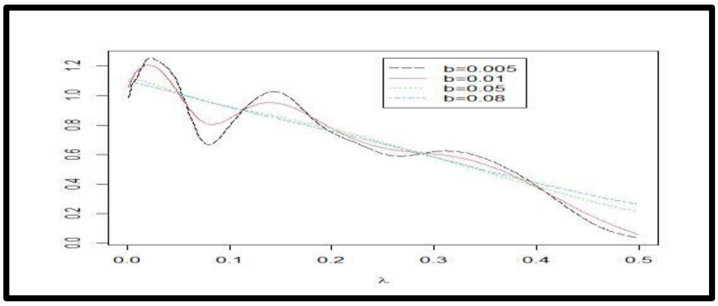

Figure (9) Logarithm values of the logarithm of the absolute data series estimated using a Dirichlet Kernel estimator with different bandwidth values.

As for the figure (9), which relates to the spectrum frequency of the logarithm series of absolute data, it also varies according to the different values of the bandwidth. We note that the spectrum function at bandwidth b = 0.005 and b = 0.01 rose at λ = 0, then went back down at frequency λ = 0.1, and then went back up. At λ = 0.15, then it continued to decline for high frequencies, as well as at 0.01 bandwidth, but with a lower level of decrease. As for the rest of the band values, they began to decline one pace at high frequencies until they reached the lowest level at bandwidth b = 0.005 and λ = 0.5.

5. The practical side:

Test hypothesis: Is the studied series have a long memory?

H_0: There is a short memory in the time series, which means that H=0 is accepted at the level of significance
(0.05).

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H_1: There is a long memory in the time series if the null hypothesis is rejected.

Table (4) long memory test

Statistics test	p. value
Simple R/S Hurst estimation	0.5744465
Corrected R over S Hurst exponent	0.5947502
Empirical Hurst exponent	0.5619452
Corrected empirical Hurst exponent	0.5324144
Theoretical Hurst exponent	0.5443251

Theoretical Hurst exponent is 0.5443251
Statistical tests were adopted to determine the type of time series and it was found that it is a stable time series with long memory. It is clear from the previous table that the value of p. value is greater than 0.5 .

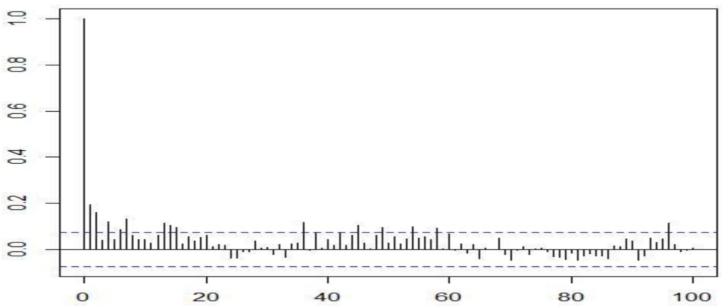
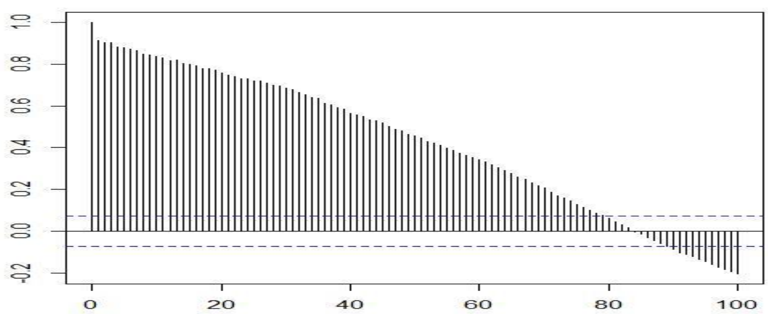


Figure (10) autocorrelation function

The ACF diagram of the data generated for the ARFIMA(1,H,0) model shows that the time series is characterized by the long memory feature, and this is evident by the very slow decreasing of the ACF function over the long time gaps.



Figure(11) absolute autocorrelation function

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Figure() of the absolute autocorrelation function

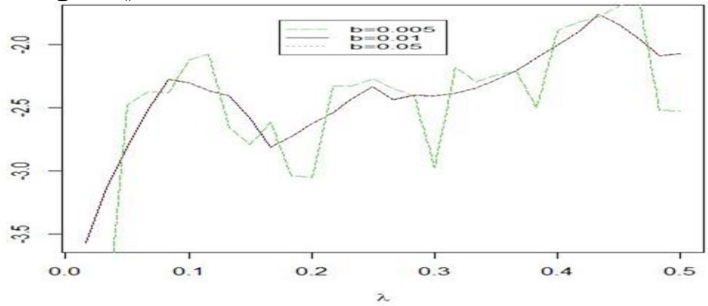

Figure (12) Logarithm values of the spectrum logarithm of the logarithm series of data estimated using a Lumax Kernel estimator with different bandwidth values.

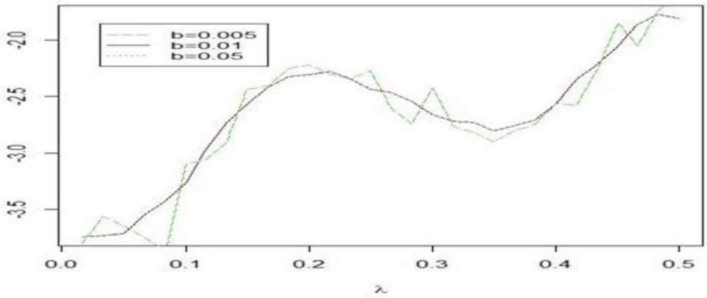

Figure (13) Logarithm values of the spectrum logarithm of the logarithm series of absolute data estimated using a Lumax Kernel estimator with different bandwidth values.

We note by drawing the logarithm of the spectrum function for the absolute values of the logarithm of the estimated values according to the kernel with different bandwidth values, as it was shown that the spectrum function differs with the values of the bandwidth. When b = (0.005, 0.05) the values of the spectrum estimators were as low as possible at frequency λ = (0, 0.1) and begin to rise in the values of the spectrum function estimated by the rise in frequency values λ, with an increase in the value of the spectrum function to reach the highest peak at high frequencies $\lambda = 0.5$.

6. Discuss the results:

It was found from the RMAD values that the best estimator is the Lumax Kernel estimator compared to the Beta Kernel estimator. The Draget Kernel estimator is second only to the Lumax estimator. Increasing the sample size increases the RMAD values for all estimators except for the Reciprocal inverse Gaussian Kernel. The RMAD values decrease with the increase in the sample size. In a beta kernel estimator, the value of RMAD increases with increasing bandwidth b. As for the rest of the capabilities, the value of RMAD decreases with increasing the bandwidth. The estimation of the spectrum function was clearly demonstrated by drawing the spectrum function to the presence of the hidden components of the long-memory time series behavior. It was shown by drawing the autocorrelation functions and through the long memory tests that the studied time series is characterized by the characteristic of long-term dependence (long memory). The estimation of the fractal parameter showed that the time series has a long memory and is characterized by stability. By comparing the estimators, it was found that the estimator of Lumax or less, RMAD, and thus it is considered the most accurate among the estimators, and that it is an estimator that

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achieves an optimal convergence rate in the mean squares of errors and approaches the real values faster than the rest of the estimations under study. By estimating the spectral density, it was possible to clarify the hidden compounds that characterize the time series and know their characteristics (duration of 3 months and repeated every 8 months) by knowing the values of frequencies λ.

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