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Estimation of the Error Performance of Waiting Time on Fuzzy Queueing System

S.Andal[1], Bharathi Ramesh kumar[2] and S.Balamurugan[3]
[1]Assistant Professor, PG and Research Department of Mathematics,
Mannar Thirumalai Naicker College, Madurai, Tamil Nadu, India
E.Mail:andalmtnc@gmail.com
[2]Associate Professor, Department of Mathematics,
Vel Tech Rangarajan Dr.Sagunthala R&D Institute of Science and Technology, Avadi, Chennai Tamil Nadu, India.
E.Mail:brameshkumar@veltech.edu.in
[3]Assistant Professor, Department of Mathematics,
Velammal College of Engineering and Technology (Autonomous), Madurai- 625009, Tamil Nadu, India
E.Mail:balasudalai@yahoo.com

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Abstract
We analyze the system performance in Erlang queueing model and all the parameters are considered Fuzzy numbers [4] has analyzed the system performance in fuzzy environment. In this paper we extended the work [4] and analyzed the purity level of system performance with the help of MLE method. In this case first constructed the model and derived the non-parametric equation. Finally, Using Alpha cut method in order to obtain uncertain data's and also analyzed the error performance (estimation matrix) through proposed algorithm.
Keywords: Membership function: Linear Programming: Erlang Distribution, Fuzzy set

1. Introduction

Any decision making problem without the contribution of statistical methods does not possibly analyze the system performance level because it is easy to find the inaccuracy node. Queueing theory has developed largely an expressive theory and it is concerned with the structure of probabilistic model and the behavior of the system in steady state and transient state level. The relation of Queueing system has been done on regulatory theory dealing with MLE, control of queueing system and etc.,. Recently, many researchers has proposed many result and also evaluated the system models. For particularly, [1, 3] analyzed the MLE of queueing parameter in single server queueing model. [2, 9] obtained the queue length in steady state service time. If the queuing parameter values exist, it can easily be found at the system performance level. Due to uncertain situation the parameters may not be estimated as precisely. In this case, with the help of Fuzzy set theory, it will solve all uncertain problems. The basic principle of [11] extension is to help to convert the Fuzzy model into crisp model.

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[8, 10] proposed the Fuzzy model of multichannel queueing system and estimated the parameter values in statistical manner, [7] has investigated to simulate the two variables in general queuing systems. [6] The investigated the queue waiting time in uncertain stage but they did not conclude the system performance. [4] It has investigated performance in general queueing with statistical approach. So, this paper investigate to analyze the system performance in Erlang queueing system and all the parameters that are considered Fuzzy numbers and it has extended that work and analyzed the purity level of system performance with the help of MLE method.

2. Erlang single service k-phase arrival model:

Consider the mean inter-arrival rate $(1/k\lambda)$, arrival and service time distribution as $\frac{k\lambda(k\lambda t)^{k-1}e^{-k\lambda t}}{(k-1)!}$, & $\mu e^{-\mu t}$ respectively.

Probability of n customer in the system is defined as $\sum_{j=nk}^{nk+k-1} P_j^{(p)}$, j taken as a phase service.

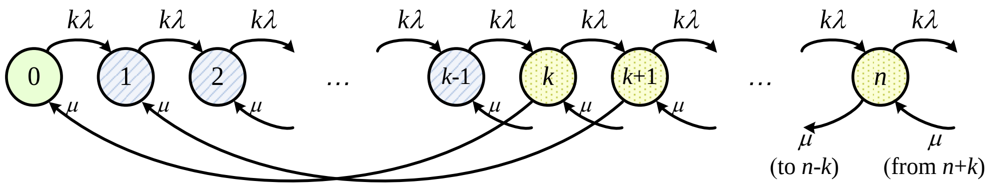

Fig. 1 The arrival form of the system

The equation is:

$$\begin{cases} -k\lambda p_0^{(P)} + \mu p_K^{(P)} = 0 & \\ -k\lambda p_n^{(P)} + k\lambda p_{n-1}^{(P)} + \mu p_{n+k}^{(P)} = 0 & ,n \le k\text{-}1 \\ -(k\lambda+\mu) p_n^{(P)} + k\lambda p_{n-1}^{(P)} + \mu p_{n+k}^{(P)} = 0 & ,n \ge k \end{cases}$$

$$P_j^{(p)} = \rho(1-r_0)r_0^{j-k},\ j \ge k,\ \rho = \frac{\lambda}{\mu}$$

Where $p_0^{(P)} = \frac{1-r_0}{k}, \rho = \frac{\lambda}{\mu}$, and r_0 is the characteristic equation is $\mu r^{k+1} - (k\lambda+\mu)r + k\lambda = 0$. Hence for n≥1,

$$P_n = \sum_{j=nk}^{nk+k-1} P_j^{(P)} = \rho(1-r_0)\left[\frac{1}{r_0^k} + \frac{1}{r_0^{k-1}} + \ldots\ldots \frac{1}{r_0}\right]\left(r_0^k\right)^n = \rho(1-r_0^k)\left(r_0^k\right)^{n-1}$$

The "nth" customer queue length in the system $q_n = (1-r_0^k)\, r_0^{nk}$ …….. (*)

$w_q(t) = q_0 + \sum_{n=1}^{\infty} q_n \int_0^t \frac{\mu(\mu x)^{n-1}}{(n-1)!} e^{-\mu x} dx$, From q_n, $q_0 = 1-r_0^k$ So, $w_q(t) = 1 - r_0^k e^{-\mu(1-r_0^k)t}$

If t = 0, then $W_q(0) = q_0 = 1 - r_0^k$

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$L=\sum_{n=1}^{\infty} n P_n=\sum_{n=1}^{\infty} \rho(1-r_0^k)\left(r_0^k\right)^{n-1}.n=\frac{\rho}{\left(1-r_0^k\right)}$, using little formula obtained $W_q=\frac{L_q}{\lambda}$,

$L_q=L_s-\frac{\lambda}{\mu}$, $W_s=\frac{L_s}{\lambda}$

3. Fuzzy Queue:

Consider the arrival and service parameter value is triangular fuzzy number and it defined as $\bar{\lambda}=\{(x,\mu_{\bar{\lambda}}(x))/x\in X\}$, $\bar{\mu}=\{(y,\mu_{\bar{\lambda}}(y))/y\in Y\}$ and membership function value represented as $\mu_{\bar{\lambda}}(x)\, and\, \mu_{\bar{\mu}}(y)$. P(x, y) denotes the measuring value of system performance and membership function: $\mu_{\bar{p}(\bar{\lambda},\bar{\mu})}(z)=$ min $\{\mu_{\bar{\lambda}}(x)\ ,\mu_{\bar{\mu}}(y)/z=p(x,y)\}---(1)$ From the derivation (sec 2) of L_q and W_q, we have $w_q\lambda=L_q$ and $w_q=\frac{\rho}{\lambda(1-r_0^k)}-\frac{1}{\mu}$

$$\mu_{\hat{W}_q}(z)=\min\left\{\mu_{\bar{\lambda}}(x)\ ,\mu_{\bar{\mu}}(y)/z=\frac{\rho}{\lambda(1-r_0^k)}-\frac{1}{\mu}\right\} \text{ ----- (ii)}$$

$$\mu_{\hat{L}_q}(z)=\min\left\{\mu_{\bar{\lambda}}(x)\ ,\mu_{\bar{\mu}}(y)/z=\frac{\rho}{(1-r_0^k)}-\frac{\lambda}{\mu}\right\} \text{ ------- (iii)}$$

3.1 Method to construct the performance measure of two variables using Zadeh's Extension Principle:

Step1: Suppose that x and y are two fuzzy real variables (fuzzy subset of real numbers) represented by the fuzzy sets (fuzzy triangular numbers) $\bar{\lambda}$ and $\bar{\mu}$ $\bar{\lambda}=\{(x,\eta_{\bar{\lambda}}(x)/x\in R)\},\bar{\mu}=\{(y,\eta_{\bar{\mu}}(y)/y\in R)\}$ where R is the set of all real numbers and $\eta_{\bar{\lambda}}(x)\ and\ \eta_{\bar{\mu}}(y)$ are the corresponding membership functions.

Step2: Find the α-cuts of $\bar{\lambda}\ and\ \bar{\mu}$: $\lambda(\alpha)=\{x\in R/\eta_{\bar{\lambda}}(x)\geq\alpha\}$ and $\mu(\alpha)=\{y\in R/\eta_{\bar{\mu}}(y)\geq\alpha\}$ where $\lambda(\alpha)\ and\ \mu(\alpha)$ are crisp sets.

Step 3: Take the confidence intervals of the fuzzy sets $\bar{\lambda}$ as $[l_{\lambda(\alpha)},u_{\lambda(\alpha)}]$ and the confidence interval of $\bar{\mu}\ \ as\ \ [l_{\mu(\alpha)},u_{\mu(\alpha)}]$.

Step 4: P(x, y) follow as: $\eta_{P(\bar{\lambda},\bar{\mu})}(Z)=\{\min(\eta_{\bar{\lambda}}(x),\eta_{\bar{\mu}}(y)):z=P(x,y)\}$ by Zadeh's extension principle,.

Step 5: By the Mixed Integer Non linear technique, the confidence interval taken as $[l_{p(\alpha)},u_{p(\alpha)}]$ where, $l_{P(\alpha)}=\inf P(x,y) and\ u_{P(\alpha)}=\sup P(x,y)$ such that $l_{\lambda(\alpha)}\leq x\leq u_{\lambda(\alpha)}\ and\ l_{\mu(\alpha)}\leq y\leq u_{\mu(\alpha)}$.

Step 6:

$$\eta_{P(\bar{\lambda},\bar{\mu})}(z)=\left\{\begin{array}{lll} L(z) & , & z_1\leq z\leq z_2 \\ 1 & , & z=z_2 \\ R(z) & , & z_2\leq z\leq z_3) \end{array}\right\} where\ z_1\leq z_2\leq z_3\ \ and\ \ L(Z_1)=R(Z_3)=0.$$

If any one of $l_{p(\alpha)}$ *and* $u_{P(\alpha)}$ is not invertible, then we find an invertible function by selecting a unique pre-image z in the proper real domain for a givenα.

Step 7: Giving 11 values of α in [0, 1] in $l_{p(\alpha)}$ *and* $u_{P(\alpha)}$, we tabulate the values. From the table values applying statistical analysis, we obtain the optimum solution of the performance measure.

Step 8: Using ranking function technique, we find the crisp values corresponding to the degree of uncertainty of the parameters. Comparing the crisp optimum and the fuzzy optimum, we see that fuzzy optimum is better than crisp optimum.

4. Statistical Methods

The system performance can be analyzed with the help of statistical method and its follows: Consider the two cases, the time axis it may be decaying the sequence it means that system is busy $\begin{Bmatrix} n & if & t>0 \\ 0 & if & t=0 \end{Bmatrix}$ and the interval of the time parameter is independent for the 'm' number of service channels. f the period is busy , before that we can take $n(0)=\tau$. Maximum likelihood function L as follows:

(i) If $m<\tau$ then Z_m is the Maximum value.

(ii) Independent time frequency is $\tau = (1-\rho)\,\rho^k \;\; k = 0,1,2....$

(Iii) fixed the piecewise interval for the length τ into m+1.

(iv) The number of arrival depends on the time is $\frac{[k\lambda(T-x_{m-\tau})]^{n-m+\tau}}{(n-m+\tau)!} e^{-k\lambda(\tau-z_{m-\tau})}$, $n-m+\tau = 0,1,2...$.

(v) If $m>\tau$, the likelihood function defined as $L=(1-\rho)e^{-r(k\mu+\lambda)}\,(k\mu)^{m-r}\,\lambda^{n+r}\,\theta$

(vi) The new parameter in MLE is $\lambda^1 = \begin{Bmatrix} \left(k\mu^1-\lambda^1\right)\left(n+\tau-\lambda^1 T\right) & when\ \lambda>\mu \\ \left(\lambda^1-k\mu^1\right)\left(n+\tau-k\mu^1 T\right) & when\ \mu>\lambda \end{Bmatrix}$

(vii) If $f(0)=(n+\tau)r>0$, $f(1)=-\tau-T<0$, then unique solution is obtained.

(viii) The range value is $0<\rho_1-\rho^1<\frac{2\rho_1^1}{(1-\rho_1)(m-\tau)}$

4. Numerical Example:

By considering the arrival pattern is kept at different phase and each phases having some service channels and size of the 3-phases Erlang distribution. The arrival and service pattern are consists of the Poisson and Erlang distribution as follows $\lambda=[1,5,7]$ and $\mu=[9,10,11]$ per minute. Find the Purity level and analyze the system performance of the Erlang queueing system.

The above problem satisfied the steady state $\rho=\frac{\lambda}{\mu}<1$.Using Alpha cut membership function obtained $\overline{L_q}=[L^L_{q_\alpha}, L^U_{q_\alpha}]$= [2.089$\alpha$ +0.045, 8.338-6.204α] &$\overline{W_q}=[W^L_{q_\alpha}, W^U_{q_\alpha}]$ = [0.4866α +0.0474, 1.668-1.134α],

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Using MLE method obtained the purity level of the system level and also obtained the range of intervals table as follows:

Table: 1 Range of Uncertainty Value of collected data table

Range of α	L_q	W_q
0	4.1915	0.0474
0.1	3.9857	0.0961
0.2	3.7800	0.1447
0.3	3.5742	0.1934
0.4	3.3685	0.2420
0.5	3.1627	0.2907
0.6	2.957	0.3394
0.7	2.7512	0.3880
0.8	2.5455	0.4367
0.9	2.3397	0.4853
1	2.13	0.534

Table: 2 Statistical value of collected data table

Variable	Minimum	Maximum	Mean	S.D
λ^1	0.0474	0.5340	0.2907	0.1614
μ^1	2.1300	4.1915	3.1624	0.6830

Table: 5 Estimation Matrix

Confusion matrix for the estimation sample:												
$\alpha \setminus \mu$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	Total	% correct
0.1	1	0	0	0	0	0	0	0	0	0	1	100.00%
0.2	0	1	0	0	0	0	0	0	0	0	1	100.00%
0.3	0	0	1	0	0	0	0	0	0	0	1	100.00%
0.4	0	0	1	0	0	0	0	0	0	0	1	0.00%
0.5	0	0	1	0	0	0	0	0	0	0	1	0.00%
0.6	0	0	0	0	0	1	0	0	0	0	1	100.00%
0.7	0	0	0	0	0	1	0	0	0	0	1	0.00%
0.8	0	0	0	0	0	1	0	0	0	0	1	0.00%
0.9	0	0	0	0	0	1	0	0	0	0	1	0.00%
1	0	0	0	0	0	1	0	0	0	0	1	0.00%
Total	1	1	3	0	0	5	0	0	0	0	10	40.00%

5. Conclusion:

The entire system can be considered for the queue parameter rates (assumption) as imprecise. Using proposed model first derived the non parametric linear equation and the possibility data's are derived with the help of Alpha cut method in obtained equation, analyzed the system performance of the above data with the help of MLE method to obtain the Purity level and estimation matrix. Because the tree chart and estimation matrix is easily gives the error value. Using proposed algorithm, we can conclude that the node 0.1 to 0.3 and 0.6 only reached the 100 % purity level remaining node does not reached the perfect level. Finally, concluded the overall system performance can reached only 40% level. So it must simulate the system because it is expected to reach the good performance level.

References:

1. Basawa.I.V (1981) "Estimation in Single Server Queues" *Naval Resear Logistics Quarterly*, vol.28 (3), pp: 475 -487.
2. Bhat.U.N and Rao.S.S (1987) "Statistical analysis of queueing systems", *Queueing Systems*, vol.1,pp: 217–247.
3. Clarke.A.B (1957) "Maximum likelihood estimates in a simple queue", *Ann. Math. Statist.* vol.28, pp: 1036 -1040, 1957.
4. Geetha et.al(2016) "Evaluation of the performance analysis in Fuzzy Queueing theory" *ICCTIDE*, pp:1-15, doi:10.1109/ICCTIDE.2016.7725348
5. Goyal, T.L. and C.M. Harris(1972) "Maximum Likelihood Estimation for Queues with State-Dependent Service," *Sankhya*, A.34, pp: 65-80.
6. Li,R.J and Lee.E.S(1989)"Analysis of fuzzy queues", *Computers and Mathematical Applications*, Vol.17, PP:1143–1147.
7. Negi,D.S and Lee.E.S(1992)"Analysis and simulation of fuzzy queues", *Fuzzy Sets and Systems*, Vol.46, pp:321–330.
8. Stanford.R.E(1982)"The set of limiting distributions for a Markov chain with fuzzy transition probabilities", *Fuzzy Sets and Systems*, Vol.7, pp: 71–78.
9. Subba Rao.S(1956) "Statistical Analysis of Queueing System", *Queueing system*, Vol.1, pp: 217 -247.
10. Wolff, R.W (1965) "Problems of Statistical Inference for Birth and Death Queueing Models",*Operations Research*, vol.13, pp: 343-357.
11. Zadeh.L.A(1978)"Fuzzy sets as a basis for a theory of possibility", *Fuzzy Sets and Systems*, Vol.1, pp: 3–28.
12. S. Bhuvaneswari; B. Rameshkumar; S. Murugesan 2017 "Interval optimality analysis in Erlang bulk arrival model in fuzzy environment", *International Journal of Fuzzy Computation and Modelling (IJFCM)* vol. 2, (3), pp: 240-251.
13. S. Bhuvaneeswari, B. Ramesh Kumar, R. Solaiappan, S. Murugesan 2016 Optimality Analysis in K-Policy -- Generalized Erlang Service Queueing Model, IRA Intern Journal of Technology & Engineering, vol. 4(1), pp:53-63.