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Effective Coloring in Single-Valued Neuromorphic Graph

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ABSTRACT

The SVNG $G(X,Y)$, an effective colouring function is a mapping $C: X(G) \to \{1,2,3,...n\}$ such that $C(u) \neq C(v)$ if uv is an effective edge in $G(X,Y)$. In this paper, the concept of an effective colouring is introduced. Further investigate the an effective colouring in operation of SVNG like, join, strong product, semi product, composition etc.

Keyword: Single valued neuromorphic graph (SVNG), colouring, effective colouring.

1. INTRODUCTION:

Neutrosophic set proposed by Smarandache is an excessive tool to deal with unsatisfactory, unspecified and untrustworthy evidence in real world. It is an over simplification of the theory of fuzzy set , intuitionistic fuzzy sets ,interval-valued fuzzy sets and interval-valued intuitionistic fuzzy sets, then the neutrosophic set is categorized by a truth-membership degree (T), an indeterminacy-membership degree (I) and a falsity-membership degree (F) independently, which are within the real typical or nonstandard unit interval [0,1].

A SVNG with underlying set V is defined to be a pair $G=(A,B)$ where, the functions $t_1: V \to [0,1]$, $i_1: V \to [0,1]$ and $f_1: V \to [0,1]$ denote the truth membership, indeterminacy membership and falsity membership of the vertex $v_i \in V$, respectively and $0 \le t_1(v_i) + i_1(v_i) + f_1(v_i) \le 3$. The functions $t_2: E \subseteq V \times V \to [0,1]$, $i_2: E \subseteq V \times V \to [0,1]$ and $f_2: E \subseteq V \times V \to [0,1]$ are defined by

$$t_2(a_i a_j) \le t_1(a_i) \wedge t_1(a_j)$$
$$i_2(a_i a_j) \ge i_1(a_i) \vee i_1(a_j)$$
$$f_2(a_i a_j) \ge f_1(a_i) \vee f_1(a_j)$$

Denote the truth membership, indeterminacy and falsity membership of the edge $(a_i a_j) \in E$ respectively, where $0 \le t_2(a_i a_j) + i_2(a_i a_j) + f_2(a_i a_j) \le 3$.

Let $G=(A,B)$ SVNG of $G^* = (V,E)$. An edge $(a_i a_j) \in E$ is said to be an effective edge, if

$$t_2(a_i a_j) = t_1(a_i) \wedge t_1(a_j)$$
$$i_2(a_i a_j) = i_1(a_i) \vee i_1(a_j)$$
$$f_2(a_i a_j) = f_1(a_i) \vee f_1(a_j)$$

Let $G=(A,B)$ SVNG is said to be complete SVNG if their an effective edge between every pair of vertices

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$t_2(a_ia_j) = t_1(a_i) \wedge t_1(a_j) \quad \forall\ a_i, a_j \in V$

$i_2(a_ia_j) = i_1(a_i) \vee i_1(a_j) \quad \forall\ a_i, a_j \in V$

$f_2(a_ia_j) = f_1(a_i) \vee f_1(a_j) \quad \forall\ a_i, a_j \in V$

Let $G = (A, B)$ SVNG graph is said to be strong SVNG if their an effective edge between every pair of vertices

$t_2(a_ia_j) = t_1(a_i) \wedge t_1(a_j) \quad \forall\ a_ia_j \in E$

$i_2(a_ia_j) = i_1(a_i) \vee i_1(a_j) \quad \forall\ a_ia_j \in E$

$f_2(a_ia_j) = f_1(a_i) \vee f_1(a_j) \quad \forall\ a_ia_j \in E$

In this paper, the concept of effective colouring is introduced. Further investigate an effective colouring in operation of SVNG like, join, strong product, semi product, composition etc.

2. EFFECTIVE COLORING IN SVNG.

In this paper, the concept of an effective colouring is introduced. Further investigate an effective colouring in operation of SVNG like, join, strong product, semi product, composition etc.

Definition 2.1: The SVNG $G(X,Y)$, an effective colouring function is a mapping $C : X(G) \to \{1,2,3,...n\}$ such that $C(u) \neq C(v)$ if uv is an effective edge in $G(X,Y)$.

Definition 2.2: Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are two SVNG. The join of G_1 and G_2 is defined by

$(t_{11}+t_{21})(a) = \begin{cases} t_{11}(a), if\ a \in V_1 \\ t_{21}(a), if\ a \in V_2 \end{cases}$, $(i_{11}+i_{21})(a) = \begin{cases} i_{11}(a), if\ a \in V_1 \\ i_{21}(a), if\ a \in V_2 \end{cases}$

$(f_{11}+f_{21})(a) = \begin{cases} f_{11}(a), if\ u \in V_1 \\ f_{21}(a), if\ u \in V_2 \end{cases}$ and the edge set E is defined by

$$(t_{12}+t_{22})(ab) = \begin{cases} t_{12}(ab), if\ ab \in E_1 \\ t_{22}(ab), if\ ab \in E_2 \\ t_{11}(ab) \wedge t_{21}(ab), if\ a \in V_1 \,\&\, b \in V_2 \end{cases}$$

$$(i_{12}+i_{22})(ab) = \begin{cases} i_{12}(ab), if\ uv \in E_1 \\ i_{22}(ab), if\ uv \in E_2 \\ i_{11}(ab) \vee I_{21}(ab), if\ a \in V_1 \,\&\, b \in V_2 \end{cases}$$

$$(f_{12}+f_{22})(ab) = \begin{cases} f_{12}(ab), if\ ab \in E_1 \\ f_{22}(ab), if\ ab \in E_2 \\ f_{11}(ab) \vee I_{21}(ab), if\ a \in V_1 \,\&\, b \in V_2 \end{cases}$$

Theorem 2.1. Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $k_1\, and\, k_2$ coloring single valued neutrosophic graphs respectively. Then $\chi_v(G_1 + G_2) = (k_1 + k_2)$.

Proof: Let $G_1(V_1, E_1)$, $G_2(V_2, E_2)$ are $k_1\, and\, k_2$ coloring single valued neutrosophic graphs respectively.

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Case(i): If every vertices $a \in V_1$ in $G_1 + G_2$. This implies every vertices $a \in V_1$ are k_1 colourable .

Case(ii): If every vertices $a \in V_2$ in $G_1 + G_2$. This implies every vertices $a \in V_2$ are k_2 colourable .

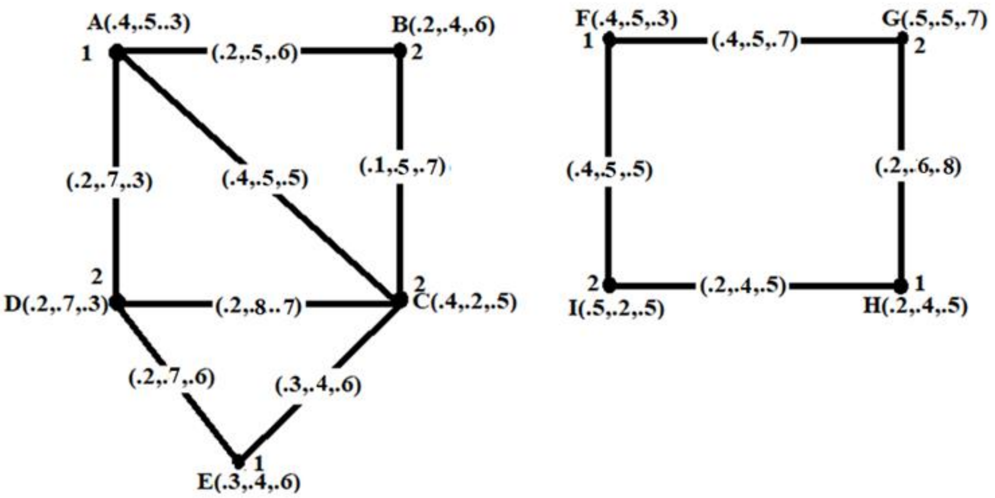
Case(iii): If $a \in V_1 \& b \in V_2$ such that

$$t_2(ab) = t_1(a) \wedge t_1(b)$$
$$i_2(ab) = i_1(a) \vee i_1(b)$$
$$f_2(ab) = f_1(a) \vee f_1(b)$$

Therefore there is an effective edge between $a \& b$ in $G_1 + G_2$. Note that these vertices $a \& b$ not same color in $G_1 + G_2$. This implies $G_1 + G_2$ not colored by $k_1 \, or \, k_2$. this implies $G_1 + G_2$ is colored by $(k_1 + k_2)$ color.

From case (i), (ii) and (iii) $G_1 + G_2$ is colored by $(k_1 + k_2)$ color . Hence $\chi_v(G_1 + G_2) = (k_1 + k_2)$.

Example 2.1.



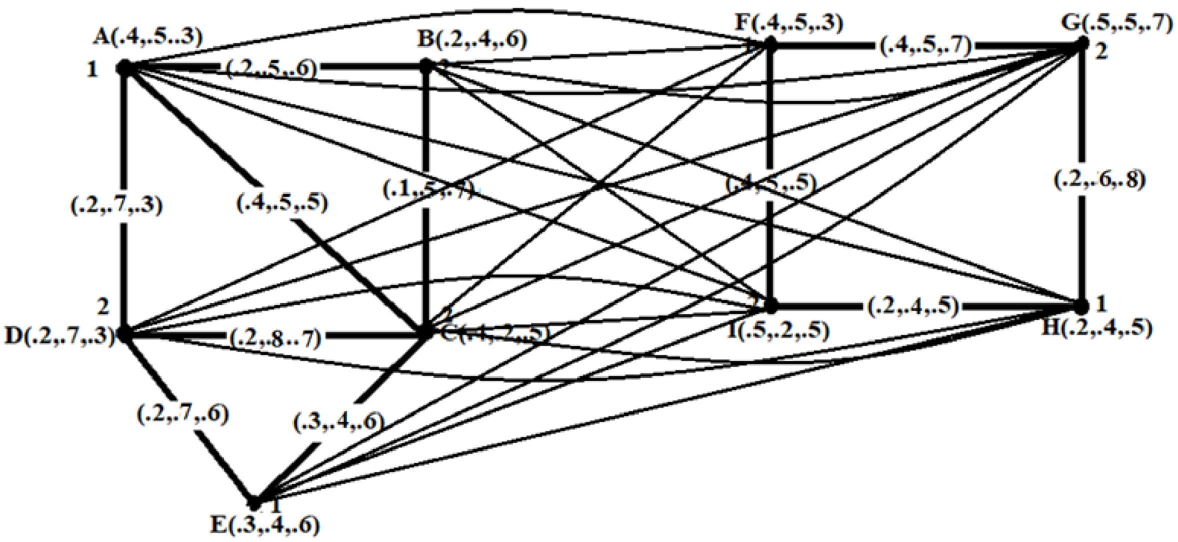


Figure 2.1: The join graph $G_1 + G_2$

Edge	t_2	i_2	f_2	**Edge**	t_2	i_2	f_2	**Edge**	t_2	i_2	f_2
AF	.4	.5	.3	CF	.4	.5	.5	EF	.3	.5	.6
AG	.4	.5	.7	CG	.4	.5	.7	EG	.3	.5	.7
AH	.2	.5	.5	CH	.2	.4	.5	EH	.2	.4	.6
AI	.4	.5	.5	CI	.4	.2	.5	EI	.3	.4	.6
BF	.2	.5	.6	DF	.2	.7	.3	BI	.2	.4	.6
BG	.2	.5	.7	DG	.2	.7	.7	DI	.2	.7	.5
BH	.2	.4	.6	DH	.2	.7	.5				

In the above example, the effective coloring number of $G_1(A_1, B_1)$ and $G_2(A_2, B_2)$ are $\chi_v(G_1) = 2$ and $\chi_v(G_2) = 2$ recpectively .The effective coloring number of $G_1 + G_2$ is $\chi_v(G_1 + G_2) = 4$.

Definition 2.3. Let $G_1(A_1, B_1)$ and $G_2(A_2, B_2)$ be a SVNG of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ respectively. The direct product $G_1 * G_2$ is defined as SVNG $G(A, B)$ of $G^*(A, B)$ such that,

$(t_{11} * t_{21})(a_1 a_2) = t_{11}(a_1) \wedge t_{21}(a_2)$

$(i_{11} * i_{21})(a_1 a_2) = i_{11}(a_1) \vee i_{21}(a_2)$

$(f_{11} * f_{21})(a_1 a_2) = f_{11}(a_1) \vee f_{21}(a_2)$

For every $a_1 a_2 \in V = V_1 \times V_2$, and

$(t_{12} * t_{21})((a_1 a_2)(b_1 b_2)) = t_{12}(a_1 b_1) \wedge t_{22}(a_2 b_2)$

$(i_{12} * i_{21})((a_1 a_2)(b_1 b_2)) = i_{12}(a_1 b_1) \vee i_{22}(a_2 b_2)$

$(f_{12} * f_{21})((a_1 a_2)(b_1 b_2)) = f_{12}(a_1 b_1) \vee f_{22}(a_2 b_2)$

For every $a_1 b_1 \in E_1$, and $a_2 b_2 \in E_2$.

Theorem 2.2. Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are k_1 *and* k_2 coloring single valued neutrosophic graphs respectively. Then $\chi_v(G_1 * G_2) = \min\{k_1, k_2\}$.

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Proof: Let $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ are $k_1\ and\ k_2$ coloring single valued neutrosophic graphs respectively.

Case(i): If the vertices a_1,b_1 are different color in $G_1(V_1,E_1)$ and the vertices a_2,b_2 are different color in $G_2(V_2,E_2)$. Therefore the edges a_1b_1 and a_2b_2 are an effective edges in $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ respectively . This implies the edge $(a_1a_2)(b_1b_2)\in G_1*G_2$ such that

$$(t_{12}*t_{22})((a_1a_2)(b_1b_2))\le t_{12}(a_1b_1)\wedge t_{22}(a_2b_2)$$
$$=t_{11}(a_1)\wedge t_{11}(b_1)\wedge t_{21}(a_2)\wedge t_{21}(b_2)$$
$$=t_{11}(a_1)\wedge t_{21}(a_2)\wedge t_{11}(b_1)\wedge t_{21}(b_2)$$
$$(t_{12}*t_{22})((a_1a_2)(b_1b_2))=(t_{12}*t_{22})(a_1a_2)\wedge(t_{12}*t_{22})(b_1b_2)$$
$$(i_{12}*i_{22})((a_1a_2)(b_1b_2))\ge i_{12}(a_1b_1)\vee i_{22}(a_2b_2)$$
$$=i_{11}(a_1)\vee i_{11}(b_1)\vee i_{21}(a_2)\vee i_{21}(b_2)$$
$$=i_{11}(a_1)\vee i_{21}(a_2)\vee i_{11}(b_1)\vee i_{21}(b_2)$$
$$(i_{12}*i_{22})((a_1a_2)(b_1b_2))=(i_{12}*i_{22})(a_1a_2)\vee(i_{12}*i_{22})(b_1b_2)$$
$$(f_{12}*f_{22})((a_1a_2)(b_1b_2))\ge f_{12}(a_1b_1)\vee f_{22}(a_2b_2)$$
$$=f_{11}(a_1)\vee f_{11}(b_1)\vee f_{21}(a_2)\vee f_{21}(b_2)$$
$$=f_{11}(a_1)\vee f_{21}(a_2)\vee f_{11}(b_1)\vee f_{21}(b_2)$$
$$(f_{12}*f_{22})((a_1a_2)(b_1b_2))=(f_{12}*f_{22})(a_1a_2)\vee(f_{12}*f_{22})(b_1b_2)$$

Therefore the edges $(a_1a_2)(b_1b_2)\in G_1*G_2$ is an effective edge . This implies the vertices $(a_1a_2)\,\&\,(b_1b_2)$ are different color in G_1*G_2.

Case(ii): If the vertices u_1,v_1 are same color in $G_1(V_1,E_1)$ and the vertices a_1,b_1 are different color in $G_2(V_2,E_2)$. Therefore the edges a_1b_1 is not an effective edge $G_1(V_1,E_1)$ and a_2b_2 is an effective edges in $G_2(V_2,E_2)$. This implies the edge $(a_1a_2)(b_1b_2)\in G_1*G_2$ such that

$$(t_{12}*t_{22})((a_1a_2)(b_1b_2))\le t_{12}(a_1b_1)\wedge t_{22}(a_2b_2)$$
$$<t_{11}(a_1)\wedge t_{11}(b_1)\wedge t_{21}(a_2)\wedge t_{21}(b_2)$$
$$<t_{11}(a_1)\wedge t_{21}(a_2)\wedge t_{11}(b_1)\wedge t_{21}(b_2)$$
$$(t_{12}*t_{22})((a_1a_2)(b_1b_2))<(t_{12}*t_{22})(a_1a_2)\wedge(t_{12}*t_{22})(b_1b_2)$$
$$(i_{12}*i_{22})((a_1a_2)(b_1b_2))\ge i_{12}(a_1b_1)\vee i_{22}(a_2b_2)$$
$$>i_{11}(a_1)\vee i_{11}(b_1)\vee i_{21}(a_2)\vee i_{21}(b_2)$$
$$>i_{11}(a_1)\vee i_{21}(a_2)\wedge i_{11}(b_1)\vee i_{21}(b_2)$$
$$(i_{12}*i_{22})((a_1a_2)(b_1b_2))>(i_{12}*i_{22})(a_1a_2)\vee(i_{12}*i_{22})(b_1b_2)$$
$$(f_{12}*f_{22})((a_1a_2)(b_1b_2))\ge f_{12}(a_1b_1)\vee f_{22}(a_2b_2)$$
$$>f_{11}(a_1)\vee f_{11}(b_1)\vee f_{21}(a_2)\vee f_{21}(b_2)$$
$$>f_{11}(a_1)\vee f_{21}(a_2)\vee f_{11}(b_1)\vee f_{21}(b_2)$$
$$(f_{12}*f_{22})((a_1a_2)(b_1b_2))>(f_{12}*f_{22})(a_1a_2)\vee(f_{12}*f_{22})(b_1b_2)$$

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Therefore the edges $(a_1a_2)(b_1b_2) \in G_1 * G_2$ is not an effective edge . This implies the vertices $(a_1a_2)\,\&\,(b_1b_2)$ are same color in $G_1 * G_2$.

Simillarly If the vertices a_1, b_1 are different color in $G_1(V_1, E_1)$ and the vertices a_1, b_1 are same color in $G_2(V_2, E_2)$. Therefore the edges a_1b_1 is an effective edge $G_1(V_1, E_1)$ and a_2b_2 is not an effective edges in $G_2(V_2, E_2)$. Therefore the edges $(a_1a_2)(b_1b_2) \in G_1 * G_2$ is not an effective edge . This implies the vertices $(a_1a_2)\,\&\,(b_1b_2)$ are same color in $G_1 * G_2$

From case (i) and (ii) $G_1 * G_2$ is k_1 *or* k_2 color graph. Hence $\chi_v(G_1 * G_2) = \min\{k_1, k_2\}$.

Example 2.2.

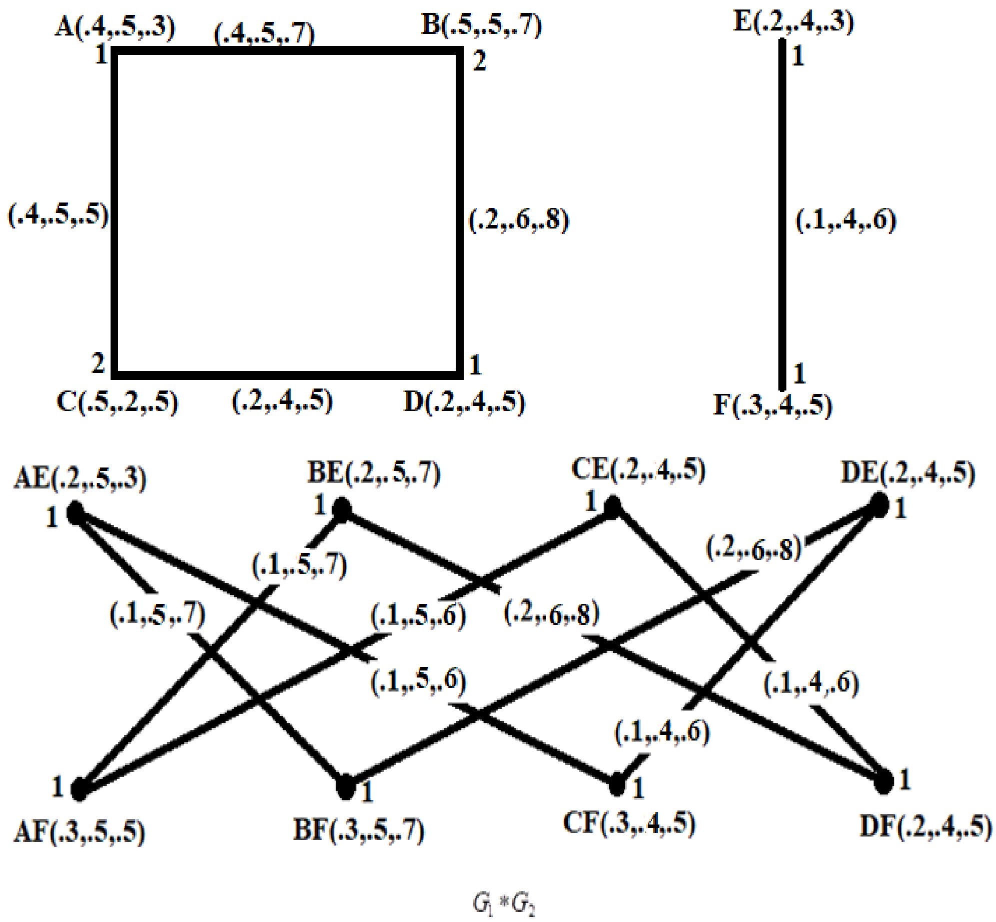

Figure 2.2: The direct product graph $G_1 * G_2$

In the above example, the effective coloring number of $G_1(A_1, B_1)$ and $G_2(A_2, B_2)$ are $\chi_v(G_1) = 2$ and $\chi_v(G_2) = 1$ recpectively .The effective coloring number of $G_1 * G_2$ is $\chi_v(G_1 * G_2) = 1$.

Definition 2.4. Let $G_1(A_1, B_1)$ and $G_2(A_2, B_2)$ be a SVNG of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ respectively. The semi strong product $G_1 \bullet G_2$ is defined as SVNG $G(A, B)$ of $G^*(A, B)$ such that,

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$(t_{11} \bullet t_{21})(a_1 a_2) = t_{11}(a_1) \wedge t_{21}(a_2)$
$(i_{11} \bullet i_{21})(a_1 a_2) = i_{11}(a_1) \vee i_{21}(a_2)$
$(f_{11} \bullet f_{21})(a_1 a_2) = f_{11}(a_1) \vee f_{21}(a_2)$
For every $a_1 a_2 \in V_1 \times V_2$, and
$(t_{12} \bullet t_{21})((a_1 a_2)(b_1 b_2)) = t_{11}(a_1) \wedge t_{22}(a_2 b_2)$
$(i_{12} \bullet i_{21})((a_1 a_2)(b_1 b_2)) = i_{11}(a_1) \vee i_{22}(a_2 b_2)$
$(f_{12} \bullet f_{21})((a_1 a_2)(b_1 b_2)) = f_{11}(a_1) \vee f_{22}(a_2 b_2)$
For every $a_2 b_2 \in E_2$ and $a_1 = b_1$.
$(t_{12} \bullet t_{21})((a_1 a_2)(b_1 b_2)) = t_{12}(a_1 b_1) \wedge t_{22}(a_2 b_2)$
$(i_{12} \bullet i_{21})((a_1 a_2)(b_1 b_2)) = i_{12}(a_1 b_1) \vee i_{22}(a_2 b_2)$
$(f_{12} \bullet f_{21})((a_1 a_2)(b_1 b_2)) = f_{12}(a_1 b_1) \vee f_{22}(a_2 b_2)$
For every $a_1 b_1 \in E_1$, and $a_2 b_2 \in E_2$.

Theorem 2.3. Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $k_1\, and\, k_2$ coloring single valued neutrosophic graphs respectively. Then $\chi_v(G_1 \bullet G_2) = \min\{k_1, k_2\}$.

Proof: Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $k_1\, and\, k_2$ coloring single valued neutrosophic graphs respectively.

Case(i): If the vertices a_1, b_1 are different color in $G_1(V_1, E_1)$ and the vertices a_2, b_2 are different color in $G_2(V_2, E_2)$. Therefore the edges $a_1 b_1$ and $a_2 b_2$ are effective edges in $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively . This implies the edge $(a_1 a_2)(b_1 b_2) \in G_1 * G_2$ such that

$$
\begin{aligned}
(t_{12} \bullet t_{22})((a_1 a_2)(b_1 b_2)) &\le t_{12}(a_1 b_1) \wedge t_{22}(a_2 b_2) \\
&= t_{11}(a_1) \wedge t_{11}(b_1) \wedge t_{21}(a_2) \wedge t_{21}(b_2) \\
&= t_{11}(a_1) \wedge t_{21}(a_2) \wedge t_{11}(b_1) \wedge t_{21}(b_2) \\
(t_{12} \bullet t_{22})((a_1 a_2)(b_1 b_2)) &= (t_{12} \bullet t_{22})(a_1 a_2) \wedge (t_{12} \bullet t_{22})(b_1 b_2) \\
(i_{12} \bullet i_{22})((a_1 a_2)(b_1 b_2)) &\ge i_{12}(a_1 b_1) \vee i_{22}(a_2 b_2) \\
&= i_{11}(a_1) \vee i_{11}(b_1) \vee i_{21}(a_2) \vee t_{21}(b_2) \\
&= i_{11}(a_1) \vee i_{21}(a_2) \vee i_{11}(b_1) \vee i_{21}(b_2) \\
(i_{12} \bullet i_{22})((a_1 a_2)(b_1 b_2)) &= (i_{12} \bullet i_{22})(a_1 a_2) \vee (i_{12} \bullet i_{22})(b_1 b_2) \\
(f_{12} \bullet f_{22})((a_1 a_2)(b_1 b_2)) &\ge f_{12}(a_1 b_1) \vee f_{22}(a_2 b_2) \\
&= f_{11}(a_1) \vee f_{11}(b_1) \vee f_{21}(a_2) \vee f_{21}(b_2) \\
&= f_{11}(a_1) \vee f_{21}(a_2) \vee f_{11}(b_1) \vee f_{21}(b_2) \\
(f_{12} \bullet f_{22})((a_1 a_2)(b_1 b_2)) &= (f_{12} \bullet f_{22})(a_1 a_2) \vee (f_{12} \bullet f_{22})(b_1 b_2)
\end{aligned}
$$

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Therefore the edges $(a_1a_2)(b_1b_2) \in G_1 * G_2$ is an effective edge . This implies the vertices $(a_1a_2)\,\&\,(b_1b_2)$ are different color in $G_1 * G_2$.

Case(ii): If the vertices a_2, b_2 are same color in $G_2(V_2, E_2)$ and the vertices $a_1 = b_1$ in $G_1(V_1, E_1)$. Therefore the edges a_2b_2 is an effective edges in $G_2(V_2, E_2)$. This implies the edge $(a_1a_2)(b_1b_2) \in G_1 \bullet G_2$ such that

$$
\begin{aligned}
(t_{12} \bullet t_{22})((a_1a_2)(b_1b_2)) &\le t_{12}(a_1b_1) \wedge t_{22}(a_2b_2)\\
&= t_{11}(a_1) \wedge t_{21}(a_2) \wedge t_{21}(b_2)\\
&= t_{11}(a_1) \wedge t_{21}(a_2) \wedge t_{11}(a_1) \wedge t_{21}(b_2)\\
(t_{12} \bullet t_{22})((a_1a_2)(b_1b_2)) &= (t_{12} \bullet t_{22})(a_1a_2) \wedge (t_{12} \bullet t_{22})(b_1b_2)\\
(i_{12} \bullet i_{22})((a_1a_2)(b_1b_2)) &\ge i_{12}(a_1b_1) \vee i_{22}(a_2b_2)\\
&= i_{11}(a_1) \vee i_{21}(a_2) \vee i_{21}(b_2)\\
&= i_{11}(a_1) \vee i_{21}(a_2) \vee i_{11}(a_1) \vee i_{21}(b_2)\\
(i_{12} \bullet i_{22})((a_1a_2)(b_1b_2)) &= (i_{12} \bullet i_{22})(a_1a_2) \vee (i_{12} \bullet i_{22})(b_1b_2)\\
(f_{12} \bullet f_{22})((a_1a_2)(b_1b_2)) &\ge f_{12}(a_1b_1) \vee f_{22}(a_2b_2)\\
&= f_{11}(a_1) \vee f_{21}(a_2) \vee f_{21}(b_2)\\
&= f_{11}(a_1) \vee f_{21}(a_2) \vee f_{11}(a_1) \vee f_{21}(b_2)\\
(f_{12} \bullet f_{22})((a_1a_2)(b_1b_2)) &= (f_{12} \bullet f_{22})(a_1a_2) \vee (f_{12} \bullet f_{22})(b_1b_2)
\end{aligned}
$$

Therefore the edges $(a_1a_2)(b_1b_2) \in G_1 \bullet G_2$ is an effective edge . This implies the vertices $(a_1a_2)\,\&\,(b_1b_2)$ are diffrent color in $G_1 \bullet G_2$.

Case(iii): If the vertices u_2, v_2 are diffrent color in $G_2(V_2, E_2)$ and the vertices $a_1 = b_1$ in $G_1(V_1, E_1)$. Therefore the edges u_2v_2 is not an effective edges in $G_2(V_2, E_2)$. This implies the edge $(a_1a_2)(b_1b_2) \in G_1 \bullet G_2$ such that

$$
\begin{aligned}
(t_{12} \bullet t_{22})((a_1a_2)(b_1b_2)) &\le t_{12}(a_1b_1) \wedge t_{22}(a_2b_2)\\
&< t_{11}(a_1) \wedge t_{21}(a_2) \wedge t_{21}(b_2)\\
&< t_{11}(a_1) \wedge t_{21}(a_2) \wedge t_{11}(a_1) \wedge t_{21}(b_2)\\
(t_{12} \bullet t_{22})((a_1a_2)(b_1b_2)) &< (t_{12} \bullet t_{22})(a_1a_2) \wedge (t_{12} \bullet t_{22})(b_1b_2)\\
(i_{12} \bullet i_{22})((a_1a_2)(b_1b_2)) &\ge i_{12}(a_1b_1) \vee i_{22}(a_2b_2)\\
&> i_{11}(a_1) \vee i_{21}(a_2) \vee i_{21}(b_2)\\
&> i_{11}(a_1) \vee i_{21}(a_2) \vee i_{11}(a_1) \vee i_{21}(b_2)\\
(i_{12} \bullet i_{22})((a_1a_2)(b_1b_2)) &> (i_{12} \bullet i_{22})(a_1a_2) \wedge (i_{12} \bullet i_{22})(b_1b_2)\\
(f_{12} \bullet f_{22})((a_1a_2)(b_1b_2)) &\ge f_{12}(a_1b_1) \vee f_{22}(a_2b_2)\\
&> f_{11}(a_1) \vee f_{21}(a_2) \vee f_{21}(b_2)\\
&> f_{11}(a_1) \vee f_{21}(a_2) \vee f_{11}(a_1) \vee f_{21}(b_2)\\
(f_{12} \bullet f_{22})((a_1a_2)(b_1b_2)) &< (f_{12} \bullet f_{22})(a_1a_2) \vee (f_{12} \bullet f_{22})(b_1b_2)
\end{aligned}
$$

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Therefore the edges $(u_1u_2)(u_1v_2) \in G_1 \bullet G_2$ is not an effective edge . This implies the vertices $(u_1u_2)\,\&\,(u_1v_2)$ are same color in $G_1 \bullet G_2$.

Case(iv): If the vertices u_1, v_1 are same color in $G_1(V_1, E_1)$ and the vertices u_1, v_1 are different color in $G_2(V_2, E_2)$. Therefore the edge u_1v_1 is not an effective edge $G_1(V_1, E_1)$ and u_2v_2 is an effective edges in $G_2(V_2, E_2)$. This implies the edge $(u_1u_2)(v_1v_2) \in G_1 * G_2$ such that

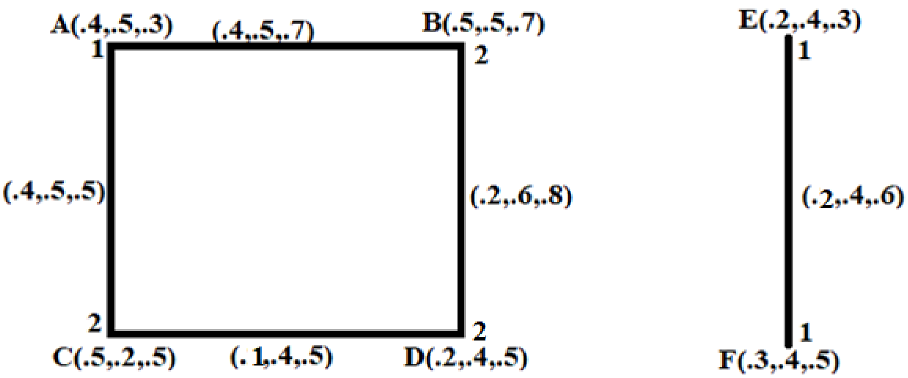
$$\begin{aligned}
(t_{12} \bullet t_{22})((a_1a_2)(b_1b_2)) &\le t_{12}(a_1b_1) \wedge t_{22}(a_2b_2) \\
&< t_{11}(a_1) \wedge t_{11}(b_1) \wedge t_{21}(a_2) \wedge t_{21}(b_2) \\
&< t_{11}(a_1) \wedge t_{21}(a_2) \wedge t_{11}(b_1) \wedge t_{21}(b_2) \\
(t_{12} \bullet t_{22})((a_1a_2)(b_1b_2)) &< (t_{12} \bullet t_{22})(a_1a_2) \wedge (t_{12} \bullet t_{22})(b_1b_2) \\
(i_{12} \bullet i_{22})((a_1a_2)(b_1b_2)) &\ge i_{12}(a_1b_1) \vee i_{22}(a_2b_2) \\
&> i_{11}(a_1) \vee i_{11}(b_1) \vee i_{21}(a_2) \vee i_{21}(b_2) \\
&> i_{11}(a_1) \vee i_{21}(a_2) \vee i_{11}(b_1) \vee i_{21}(b_2) \\
(i_{12} \bullet i_{22})((a_1a_2)(b_1b_2)) &> (i_{12} \bullet i_{22})(a_1a_2) \vee (i_{12} \bullet i_{22})(b_1b_2) \\
(f_{12} \bullet f_{22})((a_1a_2)(b_1b_2)) &\ge f_{12}(a_1b_1) \vee f_{22}(a_2b_2) \\
&> f_{11}(a_1) \vee f_{11}(b_1) \vee f_{21}(a_2) \vee f_{21}(b_2) \\
&> f_{11}(a_1) \vee f_{21}(a_2) \vee f_{11}(b_1) \vee f_{21}(b_2) \\
(f_{12} \bullet f_{22})((a_1a_2)(b_1b_2)) &> (f_{12} \bullet f_{22})(a_1a_2) \vee (f_{12} \bullet f_{22})(b_1b_2)
\end{aligned}$$

Therefore the edges $(a_1a_2)(b_1b_2) \in G_1 * G_2$ is not an effective edge . This implies the vertices $(a_1a_2)\,\&\,(b_1b_2)$ are same color in $G_1 * G_2$.

Simillarly If the vertices u_1, v_1 are different color in $G_1(V_1, E_1)$ and the vertices a_1, b_1 are same color in $G_2(V_2, E_2)$. Therefore the edge u_1v_1 is an effective edge $G_1(V_1, E_1)$ and a_2b_2 is not an effective edges in $G_2(V_2, E_2)$. Therefore the edges $(a_1a_2)(b_1b_2) \in G_1 \bullet G_2$ is not an effective edge . This implies the vertices $(a_1a_2)\,\&\,(b_1b_2)$ are same color in $G_1 \bullet G_2$.

From case (i) and (ii) $G_1 \bullet G_2$ is $k_1 \; or \; k_2$ color graph. Hence $\chi_v(G_1 \bullet G_2) = \min\{k_1, k_2\}$.

Example 2.3.



$G_1(V_1, E_1)$ $\qquad\qquad$ $G_2(V_2, E_2)$

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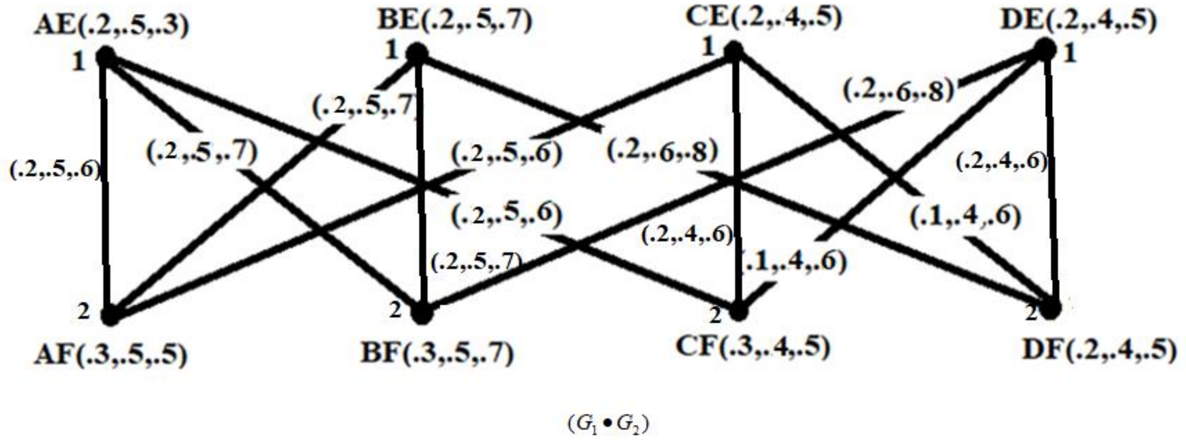

Figure 2.3: The semi strong product graph $G_1 \bullet G_2$

In the above example, the effective coloring number of $G_1(A_1, B_1)$ and $G_2(A_2, B_2)$ are $\chi_v(G_1) = 2$ and $\chi_v(G_2) = 2$ recpectively .The effective coloring number of $G_1 \bullet G_2$ is $\chi_v(G_1 \bullet G_2) = 2$.

Definition 2.5. Let $G_1(A_1, B_1)$ and $G_2(A_2, B_2)$ be a SVNG of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ respectively. The Cartesian product $G_1 \times G_2$ is defined as SVNG $G(A, B)$ of $G^*(A, B)$ such that,

$(t_{11} \times t_{21})(a_1a_2) = t_{11}(a_1) \wedge t_{21}(a_2)$
$(i_{11} \times i_{21})(a_1a_2) = i_{11}(a_1) \wedge i_{21}(a_2)$
$(f_{11} \times f_{21})(a_1a_2) = f_{11}(a_1) \vee f_{21}(a_2)$

For every $a_1a_2 \in V = V_1 \times V_2$, and

$(t_{12} \times t_{21})((a_1a_2)(b_1b_2)) = t_{11}(a_1) \wedge t_{22}(a_2b_2)$
$(t_{12} \times t_{21})((a_1a_2)(b_1b_2)) = i_{11}(a_1) \vee i_{22}(a_2b_2)$
$(f_{12} \times f_{21})((a_1a_2)(b_1b_2)) = f_{11}(a_1) \vee f_{22}(a_2b_2)$

For every $a_2b_2 \in E_2$ and $a_1 = b_1$.

$(t_{12} \times t_{21})((a_1a_2)(b_1b_2)) = t_{12}(a_1b_1) \wedge t_{21}(b_2)$
$(i_{12} \times i_{21})((a_1a_2)(b_1b_2)) = i_{12}(a_1b) \vee i_{21}(b_2)$
$(f_{12} \times f_{21})((a_1a_2)(b_1b_2)) = f_{12}(a_1b) \vee f_{21}(b_2)$

For every $a_1b_1 \in E_1$ and $a_2 = b_2$.

Theorem 2.4. Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are k_1 *and* k_2 coloring single valued neutrosophic graphs respectively. Then $\chi_v(G_1 \times G_2) = \min\{k_1, k_2\}$.

Proof: Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are k_1 *and* k_2 coloring single valued neutrosophic graphs respectively.

Case(i): If the vertices a_2, b_2 are same color in $G_2(V_2, E_2)$ and the vertices $a_1 = b_1$ in $G_1(V_1, E_1)$. Therefore the edges a_2b_2 is an effective edge in $G_2(V_2, E_2)$. This implies the edge $(a_1a_2)(b_1b_2) \in G_1 \times G_2$ such that

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$$
\begin{aligned}
(t_{12}\times t_{22})\big((a_1a_2)(b_1b_2)\big) &\le t_{12}(a_1v_1)\wedge t_{22}(a_2b_2)\\
&= t_{11}(a_1)\wedge t_{21}(a_2)\wedge t_{21}(b_2)\\
&= t_{11}(a_1)\wedge t_{21}(a_2)\wedge t_{11}(a_1)\wedge t_{21}(b_2)\\
(t_{12}\times t_{22})\big((a_1a_2)(b_1b_2)\big) &= (t_{12}\times t_{22})(a_1a_2)\wedge(t_{12}\times t_{22})(b_1b_2)\\
(i_{12}\times i_{22})\big((a_1a_2)(b_1b_2)\big) &\ge i_{12}(a_1v_1)\vee i_{22}(a_2b_2)\\
&= i_{11}(a_1)\vee i_{21}(a_2)\vee i_{21}(b_2)\\
&= i_{11}(a_1)\vee i_{21}(a_2)\vee i_{11}(a_1)\vee i_{21}(b_2)\\
(i_{12}\times i_{22})\big((a_1a_2)(b_1b_2)\big) &= (i_{12}\times i_{22})(a_1a_2)\vee(i_{12}\times i_{22})(b_1b_2)\\
(f_{12}\times f_{22})\big((a_1a_2)(b_1b_2)\big) &\ge f_{12}(a_1b_1)\vee f_{22}(a_2b_2)\\
&= f_{11}(a_1)\vee f_{21}(a_2)\vee f_{21}(b_2)\\
&= f_{11}(a_1)\vee f_{21}(a_2)\vee f_{11}(a_1)\vee f_{21}(b_2)\\
(f_{12}\times f_{22})\big((a_1a_2)(b_1b_2)\big) &= (f_{12}\times f_{22})(a_1a_2)\vee(f_{12}\times f_{22})(b_1b_2)
\end{aligned}
$$

Therefore the edges $(a_1a_2)(b_1b_2)\in G_1\times G_2$ is an effective edge . This implies the vertices $(a_1a_2)\,\&\,(b_1b_2)$ are diffrent color in $G_1\bullet G_2$.

Case(ii): If the vertices a_1,b_1 are same color in $G_1(V_1,E_1)$ and the vertices $a_2=b_2$ in $G_2(V_2,E_2)$. Therefore the edges a_2b_2 is an effective edges in $G_2(V_2,E_2)$. This implies the edge $(a_1a_2)(b_1b_2)\in G_1\times G_2$ such that

$$
\begin{aligned}
(t_{12}\times t_{22})\big((a_1a_2)(b_1b_2)\big) &\le t_{12}(a_1b_1)\wedge t_{22}(a_2b_2)\\
&= t_{11}(a_1)\wedge t_{11}(b_1)\wedge t_{21}(a_2)\\
&= t_{11}(a_1)\wedge t_{21}(a_2)\wedge t_{11}(b_1)\wedge t_{21}(a_2)\\
(t_{12}\times t_{22})\big((a_1a_2)(b_1b_2)\big) &= (t_{12}\times t_{22})(a_1a_2)\wedge(t_{12}\times t_{22})(b_1b_2)\\
(i_{12}\times i_{22})\big((a_1a_2)(b_1b_2)\big) &\ge i_{12}(a_1b_1)\vee i_{22}(a_2b_2)\\
&= i_{11}(a_1)\vee i_{11}(b_1)\vee i_{21}(a_2)\\
&= i_{11}(a_1)\vee i_{21}(a_2)\vee i_{11}(b_1)\vee i_{21}(a_2)\\
(i_{12}\times i_{22})\big((a_1a_2)(b_1b_2)\big) &= (i_{12}\times i_{22})(a_1a_2)\vee(i_{12}\times i_{22})(b_1b_2)\\
(f_{12}\times f_{22})\big((a_1a_2)(b_1b_2)\big) &\ge f_{12}(a_1b_1)\vee f_{22}(a_2b_2)\\
&= f_{11}(a_1)\vee f_{11}(b_1)\vee f_{21}(a_2)\\
&= f_{11}(a_1)\vee f_{21}(a_2)\vee f_{11}(b_1)\vee f_{21}(a_2)\\
(f_{12}\times f_{22})\big((a_1a_2)(b_1b_2)\big) &= (f_{12}\times f_{22})(a_1a_2)\vee(f_{12}\times f_{22})(b_1b_2)
\end{aligned}
$$

Therefore the edges $(a_1a_2)(b_1b_2)\in G_1\times G_2$ is an effective edge . This implies the vertices $(a_1a_2)\,\&\,(b_1b_2)$ are diffrent color in $G_1\times G_2$.

From case (i) and (ii) $G_1\times G_2$ is $k_1\ or\ k_2$ color graph. Hence $\chi_v(G_1\times G_2)=\min\{k_1,k_2\}$.

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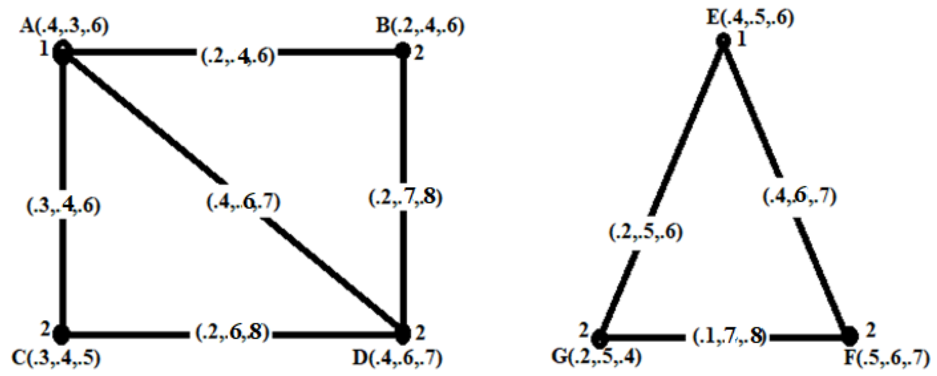
Example 2.4.



$G_1(V_1, E_1)$ $G_2(V_2, E_2)$

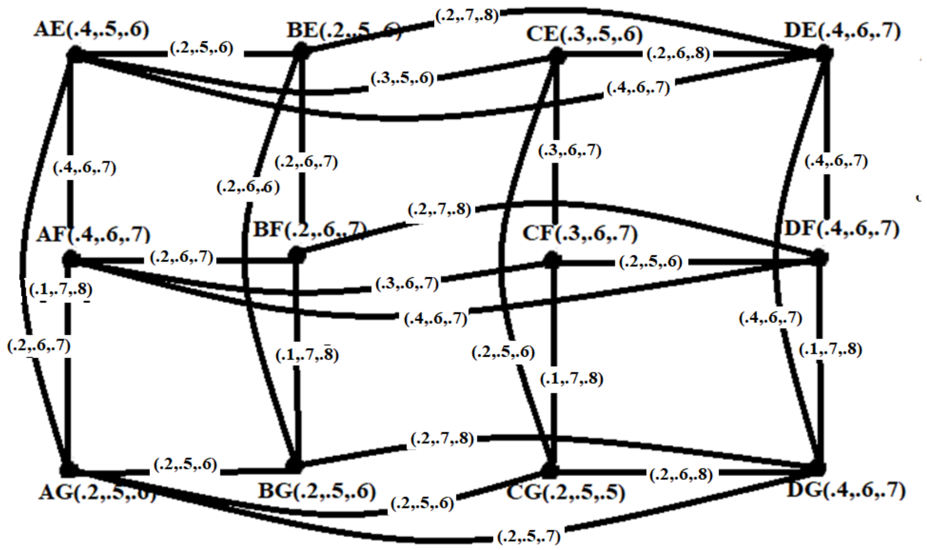


Figure 2.4: The Cartesian product graph $G_1 \times G_2$

In the above example, the effective coloring number of $G_1(A_1, B_1)$ and $G_2(A_2, B_2)$ are $\chi_v(G_1) = 2$ and $\chi_v(G_2) = 2$ recpectively .The effective coloring number of $G_1 \times G_2$ is $\chi_v(G_1 \times G_2) = 2$.

Definition 2.6. Let $G_1(A_1, B_1)$ and $G_2(A_2, B_2)$ be a SVNG of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ respectively. The composition $G_1 \circ G_2$ is defined as SVNG $G(A, B)$ of $G^*(A, B)$ such that,

$(t_{11} \circ t_{21})(a_1a_2) = t_{11}(a_1) \wedge t_{21}(a_2)$, $\forall\, a_1a_2 \in V_1 \times V_2$

$(i_{11} \circ i_{21})(a_1a_2) = i_{11}(a_1) \wedge i_{21}(a_2)$, $\forall\, a_1a_2 \in V_1 \times V_2$

$(f_{11} \circ f_{21})(a_1a_2) = f_{11}(a_1) \vee f_{21}(a_2)$, $\forall\, a_1a_2 \in V_1 \times V_2$

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and

$(t_{12} \circ t_{21})((a_1a_2)(b_1b_2)) = t_{11}(a_1) \wedge t_{22}(a_2b_2), \forall a_2b_2 \in E_2 \,\&\, a_1 = b_1$
$(i_{12} \circ i_{21})((a_1a_2)(b_1b_2)) = i_{11}(a_1) \wedge i_{22}(a_2b_2), \forall a_2b_2 \in E_2 \,\&\, a_1 = b_1$
$(f_{12} \circ f_{21})((a_1a_2)(b_1b_2)) = f_{11}(a_1) \vee f_{22}(a_2b_2), \forall a_2b_2 \in E_2 \,\&\, a_1 = b_1$

$(t_{12} \circ t_{21})((a_1a_2)(b_1b_2)) = t_{12}(a_1b_1) \wedge t_{21}(b_2), \forall u_1v_1 \in E_1 \,\&\, a_2 = b_2$
$(i_{12} \circ i_{21})((a_1a_2)(b_1b_2)) = i_{12}(a_1b_1) \wedge i_{21}(b_2), \forall u_1v_1 \in E_1 \,\&\, a_2 = b_2$
$(f_{12} \circ f_{21})((a_1a_2)(b_1b_2)) = f_{12}(a_1b_1) \vee f_{21}(b_2), \forall u_1v_1 \in E_1 \,\&\, a_2 = b_2$

$(t_{12} \circ t_{21})((a_1a_2)(b_1b_2)) = t_{12}(a_1b_1) \wedge t_{21}(a_2) \wedge t_{21}(b_2), \forall a_1b_1 \in E_1 \,\&\, a_2 \neq b_2$
$(i_{12} \circ i_{21})((a_1a_2)(b_1b_2)) = i_{12}(a_1b_1) \vee i_{21}(a_2) \wedge i_{21}(b_2) \forall a_1b_1 \in E_1 \,\&\, a_2 \neq b_2$
$(f_{12} \circ f_{21})((a_1a_2)(b_1b_2)) = f_{12}(a_1b_1) \vee f_{21}(a_2) \vee f_{21}(b_2) \forall a_1b_1 \in E_1 \,\&\, a_2 \neq b_2$

Theorem 2.5. Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are k_1 *and* k_2 coloring single valued neutrosophic graphs respectively. Then $\chi_v(G_1 \circ G_2) = k_1 + k_2$.

Proof: Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are k_1 *and* k_2 coloring single valued neutrosophic graphs respectively.

Case(i): If the vertices u_2, v_2 are same color in $G_2(V_2, E_2)$ and the vertices $a_1 = b_1$ in $G_1(V_1, E_1)$. Therefore the edges a_2b_2 is an effective edges in $G_2(V_2, E_2)$. This implies the edge $(a_1a_2)(b_1b_2) \in G_1 \times G_2$ such that

$$\begin{aligned}
(t_{12} \circ t_{22})\big((a_1a_2)(b_1b_2)\big) &\leq t_{12}(a_1b_1) \wedge t_{22}(a_2b_2) \\
&= t_{11}(a_1) \wedge t_{21}(a_2) \wedge t_{21}(b_2) \\
&= t_{11}(a_1) \wedge t_{21}(a_2) \wedge t_{11}(a_1) \wedge t_{21}(b_2) \\
(t_{12} \circ t_{22})\big((a_1a_2)(b_1b_2)\big) &= (t_{12} \circ t_{22})(a_1a_2) \wedge (t_{12} \circ t_{22})(b_1b_2) \\
(i_{12} \circ i_{22})\big((a_1a_2)(b_1b_2)\big) &\geq i_{12}(a_1b_1) \vee i_{22}(a_2b_2) \\
&= i_{11}(a_1) \vee i_{21}(a_2) \vee i_{21}(b_2) \\
&= i_{11}(a_1) \vee i_{21}(a_2) \vee i_{11}(a_1) \vee i_{21}(b_2) \\
(i_{12} \circ i_{22})\big((a_1a_2)(b_1b_2)\big) &= (i_{12} \circ i_{22})(a_1a_2) \vee (i_{12} \circ i_{22})(b_1b_2) \\
(f_{12} \circ f_{22})\big((a_1a_2)(b_1b_2)\big) &\geq f_{12}(a_1b_1) \vee f_{22}(a_2b_2) \\
&= f_{11}(a_1) \vee f_{21}(a_2) \vee f_{21}(b_2) \\
&= f_{11}(a_1) \vee f_{21}(a_2) \vee f_{11}(a_1) \vee f_{21}(b_2) \\
(f_{12} \circ f_{22})\big((a_1a_2)(b_1b_2)\big) &= (f_{12} \circ f_{22})(a_1a_2) \vee (f_{12} \circ f_{22})(b_1b_2)
\end{aligned}$$

Therefore the edges $(a_1a_2)(b_1b_2) \in G_1 \circ G_2$ is an effective edge . This implies the vertices $(a_1a_2) \,\&\, (b_1b_2)$ are diffrent color in $G_1 \circ G_2$.

Case(ii): If the vertices u_1, v_1 are same color in $G_1(V_1, E_1)$ and the vertices $a_2 = b_2$ in $G_2(V_2, E_2)$. Therefore the edges a_2b_2 is an effective edges in $G_2(V_2, E_2)$. This implies the edge $(a_1a_2)(b_1b_2) \in G_1 \circ G_2$ such that

$$(t_{12} \circ t_{22})((a_1a_2)(b_1b_2)) \le t_{12}(a_1b_1) \wedge t_{22}(a_2b_2)$$
$$= t_{11}(a_1) \wedge t_{11}(b_1) \wedge t_{21}(a_2)$$
$$= t_{11}(a_1) \wedge t_{21}(a_2) \wedge t_{11}(b_1) \wedge t_{21}(a_2)$$
$$(t_{12} \circ_{22})((a_1a_2)(b_1b_2)) = (t_{12} \circ t_{22})(a_1a_2) \wedge (t_{12} \circ t_{22})(b_1b_2)$$
$$(i_{12} \circ i_{22})((a_1a_2)(b_1b_2)) \ge i_{12}(a_1b_1) \vee i_{22}(a_2b_2)$$
$$= i_{11}(a_1) \vee i_{11}(b_1) \wedge i_{21}(a_2)$$
$$= i_{11}(a_1) \vee i_{21}(a_2) \vee i_{11}(b_1) \vee i_{21}(a_2)$$
$$(i_{12} \circ i_{22})((a_1a_2)(b_1b_2)) = (i_{12} \circ i_{22})(a_1a_2) \vee (i_{12} \circ i_{22})(b_1b_2)$$
$$(f_{12} \circ f_{22})((a_1a_2)(b_1b_2)) \ge f_{12}(a_1b_1) \vee f_{22}(a_2b_2)$$
$$= f_{11}(a_1) \vee f_{21}(b_1) \vee f_{21}(a_2)$$
$$= f_{11}(a_1) \vee f_{21}(a_2) \vee f_{11}(b_1) \vee f_{21}(a_2)$$
$$(f_{12} \circ f_{22})((a_1a_2)(b_1b_2)) = (f_{12} \circ f_{22})(a_1a_2) \vee (f_{12} \circ f_{22})(b_1b_2)$$

Therefore the edges $(a_1a_2)(b_1b_2) \in G_1 \circ G_2$ is an effective edge . This implies the vertices $(a_1a_2)\,\&\,(b_1b_2)$ are diffrent color in $G_1 \circ G_2$.

Case(iii): If the vertices a_1, b_1 are different color in $G_1(V_1, E_1)$ and the vertices a_2, b_2 are different color in $G_2(V_2, E_2)$. Therefore the edges a_1b_1 and a_2b_2 are effective edges in $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively . This implies the edge $(a_1a_2)(b_1b_2) \in G_1 * G_2$ such that

$$(t_{12} * t_{22})((a_1a_2)(b_1b_2)) \le t_{12}(a_1b_1) \wedge t_{22}(a_2b_2)$$
$$= t_{11}(a_1) \wedge t_{11}(b_1) \wedge t_{21}(a_2) \wedge t_{21}(b_2)$$
$$= t_{11}(a_1) \wedge t_{21}(a_2) \wedge t_{11}(b_1) \wedge t_{21}(b_2)$$
$$(t_{12} * t_{22})((a_1a_2)(b_1b_2)) = (t_{12} * t_{22})(a_1a_2) \wedge (t_{12} * t_{22})(b_1b_2)$$
$$(i_{12} * i_{22})((a_1a_2)(b_1b_2)) \ge i_{12}(a_1b_1) \vee i_{22}(a_2b_2)$$
$$= i_{11}(a_1) \vee i_{11}(b_1) \vee i_{21}(a_2) \vee i_{21}(b_2)$$
$$= i_{11}(a_1) \vee i_{21}(a_2) \vee i_{11}(b_1) \vee i_{21}(b_2)$$
$$(i_{12} * i_{22})((a_1a_2)(b_1b_2)) = (i_{12} * i_{22})(a_1a_2) \vee (i_{12} * i_{22})(b_1b_2)$$
$$(f_{12} * f_{22})((a_1a_2)(b_1b_2)) \ge f_{12}(a_1b_1) \vee f_{22}(a_2b_2)$$
$$= f_{11}(a_1) \vee f_{11}(b_1) \vee f_{21}(a_2) \vee f_{21}(b_2)$$
$$= f_{11}(a_1) \vee f_{21}(a_2) \vee f_{11}(b_1) \vee f_{21}(b_2)$$
$$(f_{12} * f_{22})((a_1a_2)(b_1b_2)) = (f_{12} * f_{22})(a_1a_2) \vee (f_{12} * f_{22})(b_1b_2)$$

Therefore the edges $(a_1a_2)(b_1b_2) \in G_1 \circ G_2$ is an effective edge . This implies the vertices $(a_1a_2)\,\&\,(b_1b_2)$ are different color in $G_1 \circ G_2$.

From case (i) , (ii) and (ii) $G_1 \circ G_2$ is k_1 *and* k_2 color graph. Hence $\chi_v(G_1 \circ G_2) = k_1 + k_2$.

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Example 2.5

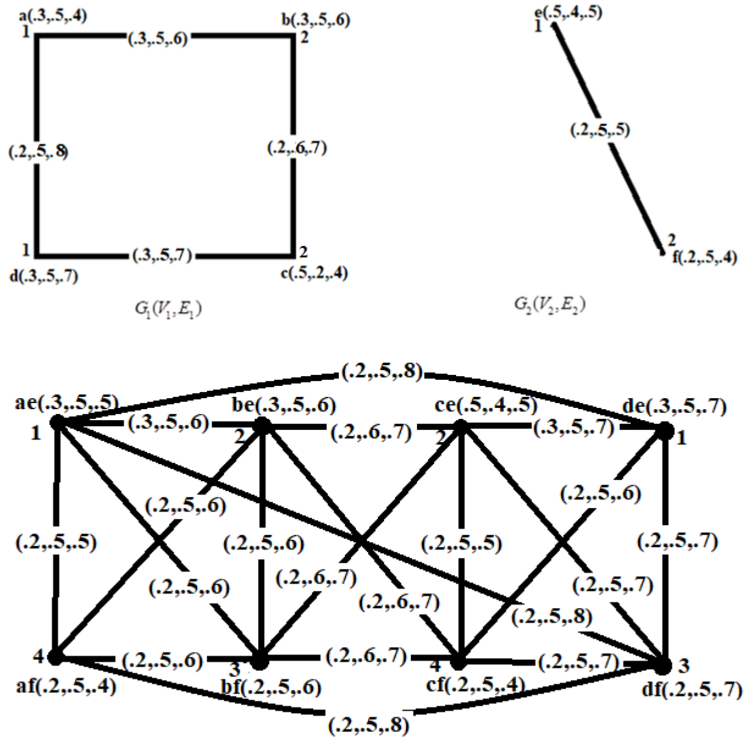

Figure 2.5: The Composition product graph $G_1 \circ G_2$

In the above example, the effective coloring number of $G_1(A_1, B_1)$ and $G_2(A_2, B_2)$ are $\chi_v(G_1) = 2$ and $\chi_v(G_2) = 2$ recpectively .The effective coloring number of $G_1 \circ G_2$ is $\chi_v(G_1 \circ G_2) = 4$.

CONCLUSION:

In this paper, the concept of effective colouring is introduced. Further investigate the effective colouring in operation of SVNG like, join, strong product, semi product, composition etc. In future we will define various colouring in SVNG and investigate an effective colouring in operation of SVNG like, join, strong product, semi product, composition etc.

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