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Total Mean Cordial Labeling in Different Graphs

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ABSTRACT

In a graph G(V,E) is a mapping $f:V\rightarrow\{0,1,2\}$ such that $f(uv)=\left\lceil\frac{f(u)+f(v)}{2}\right\rceil, u,v\in V$ and the mapping satisfies the condition $|e*(i)-e*(j)|\leq 1,\ for\ i,j=\{0,1,2\}$ where $e*(i)$ donates the total number of edges labeled with $i\in\{0,1,2\}$. The graph consist total mean cordial labeling it named as a total mean cordial graph. In this paper, we will investigate the total mean cordial labeling in various graphs like brush graph, ladder graph, triangular ladder graph. Further explain the theorem with examples.

1. Introduction

Cordial labeling is one of the well-known research topic in graph theory. There are different cordial labeling in graphs like SD prime cordial , sum divisor cordial, intersection cordial labeling etc. In this paper we will explain the concept of total mean cordial labeling. In a graph G(V,E) is a mapping $f:V\rightarrow\{0,1,2\}$ such that $f(uv)=\left\lceil\frac{f(u)+f(v)}{2}\right\rceil, u,v\in V$ and the mapping satisfies the condition $|e*(i)-e*(j)|\leq 1,\ for\ i,j=\{0,1,2\}$ where $e*(i)$ donates the total number of edges labeled with $i\in\{0,1,2\}$. The graph consist total mean cordial labeling it named as a total mean cordial graph. Further we investigated the total mean cordial labeling in various graphs like brush graph, ladder graph, triangular ladder graph. Further explain the theorem with examples.

2. Total mean cordial labeling in various graphs

In this section the idea of total mean cordial labeling in different graphs like brush graph, ladder graph, triangular ladder graph.

Definition 2.1: In a graph G(V,E) is a mapping $f:V\rightarrow\{0,1,2\}$ such that $f(uv)=\left\lceil\frac{f(u)+f(v)}{2}\right\rceil, u,v\in V$ and the mapping satisfies the condition $|e*(i)-e*(j)|\leq 1,\ for\ i,j=\{0,1,2\}$ where $e*(i)$ donates the total number of edges

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labeled with $i \in \{0,1,2\}$. The graph consist total mean cordial labeling it named as a total mean cordial graph.

Theorem 2.1: The Brush graph B_n is a total mean cordial graph.

Proof: The Brush graph B_n constructed by the path P_n and P_1^n . Therefore Brush graph B_n having the set of vertices $V = \{u_i, v_i, 1 \le i \le n\}$. Note that there is a $'n'$ different $u_i v_i$, P_1 paths in B_n it is denoted by $P_1^n, 1 \le i \le n$ and also it contain P_n paths in B_n . Therefore edges set $E(B_n) = \{u_i v_{i+1} | 1 \le i \le n-1\} \cup \{u_i v_i | 1 \le i \le n\}$. This implies order and size of B_n are $2n$ and $2n-1$.Construct the mapping $f : V(B_n)) \to \{0,1,2\}$ as follows ,

Case(i): If n=0 (mod3)

Let $t = \dfrac{n}{3}$ now we construct the mapping $f : V(B_n)) \to \{0,1,2\}$ as follows

$$f(U_i) = \begin{cases} 2, \ for\, 1 \le i \le t \\ 1, \ for\left(t+1\right) \le i \le 2t \\ 0, Otherwise \end{cases} \qquad f(V_i) = \begin{cases} 2, \ for\, 1 \le i \le t \\ 1, \ for\left(t+1\right) \le i \le 2t \\ 0, Otherwise \end{cases}$$

We note that from the above mapping $e_{f^*}(0) = \left\lceil \dfrac{(2n-1)}{3} \right\rceil - 1$, $e_{f^*}(1) = \left\lceil \dfrac{(2n-1)}{3} \right\rceil$ and $e_{f^*}(2) = \left\lceil \dfrac{(2n-1)}{3} \right\rceil$

Case(ii): If n=1 (mod3)

Let $t = \left\lfloor \dfrac{n}{3} \right\rfloor$ now we construct the mapping $f : V(B_n)) \to \{0,1,2\}$ as follows

$$f(U_i) = \begin{cases} 2, \ for\, 1 \le i \le t \\ 1, \ for\left(t+1\right) \le i \le 2t \\ 0, Otherwise \end{cases} \qquad f(V_i) = \begin{cases} 2, \ for\, 1 \le i \le t \\ 1, \ for\left(t+1\right) \le i \le 2t \\ 0, Otherwise \end{cases}$$

We note that from the above mapping $e_{f^*}(0) = \left\lfloor \dfrac{(2n-1)}{3} \right\rfloor + 1$, $e_{f^*}(1) = \left\lfloor \dfrac{(2n-1)}{3} \right\rfloor$ and $e_{f^*}(2) = \left\lfloor \dfrac{(2n-1)}{3} \right\rfloor$

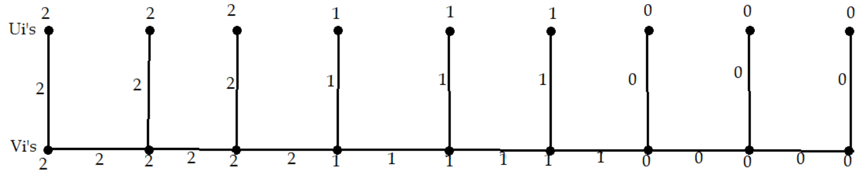
Case(iii): If n=2 (mod3)

Let $t = \left\lceil \dfrac{n}{3} \right\rceil$ now we construct the mapping $f : V(B_n)) \to \{0,1,2\}$ as follows

$$f(U_i) = \begin{cases} 2, \ for\ 1 \le i \le t \\ 1, \ for\ \left(t+1\right) \le i \le 2t \\ 0, Otherwise \end{cases} \qquad f(V_i) = \begin{cases} 2, \ for\ 1 \le i \le \left(t-1\right) \\ 1, \ for\ t \le i \le 2(t-1) \\ 0, Otherwise \end{cases}$$

We note that from the above mapping $e_{f^*}(0) = \dfrac{(2n-1)}{3}$, $e_{f^*}(1) = \dfrac{(2n-1)}{3}$ and $e_{f^*}(2) = \dfrac{(2n-1)}{3}$.Therefore from the above cases the Brush graph B_n under consideration satisfies the conditions $|e*(i) - e*(j)| \le 1, \ for\, i, j = \{0,1,2\}$. Hence Brush graph B_n is a total mean cordial graph.

Example 2.1: The total mean cordial labeling of the graph B_9, B_7 & B_8



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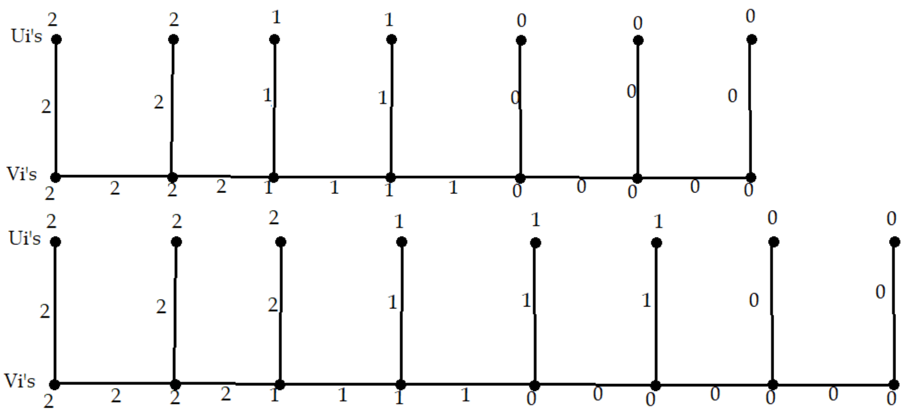

Figure 2.1: Total mean cordial labeling of the graph $B_9, B_7 \,\&\, B_8$

In the above graph B_9, the number of edges labeled with 0,1 or 2 is $e_{f*}(0)=5$, $e_{f*}(1)=6$ and $e_{f*}(2)=6$. The graph B_7 , the number of edges labeled with 0,1 or 2 is $e_{f*}(0)=5$, $e_{f*}(1)=4$ and $e_{f*}(2)=4$. Finally the graph B_8, the number of edges labeled with 0,1 or 2 is $e_{f*}(0)=5$, $e_{f*}(1)=5$ and $e_{f*}(2)=5$. Therefore $B_9, B_7 \,\&\, B_8$ satisfies the condition $|e*(i)-e*(j)|\le 1,\ for\ i,j=\{0,1,2\}$. Hence the graph $B_9, B_7 \,\&\, B_8$ are Total mean cordial graph.

Theorem 2.2: The ladder graph L_n is a total mean cordial graph.

Proof: The ladder graph L_n, constructed by the graphs $P_1 \,\&\, P_n$. Therefore the graph L_n having the set of vertices $V=\{u_i|1\le i\le n\}\cup\{v_i|1\le i\le n\}$. Note that there is $(2n)$ vertices in L_n. The contains the graphs P_2, P_n and P_2^n . Therefore the edges set of ladder graph L_n is $E(B_n)=\{u_iv_{i+1}|1\le i\le n-1\}\cup\{v_iv_{i+1}|1\le i\le n-1\}\cup\{u_iv_i|1\le i\le n\}$. This implies size of L_n are $3n-2$. Construct the mapping $f:V(B_n))\to\{0,1,2\}$ as follows ,

Case(i): If n=0 (mod3)

Let $t=\dfrac{n}{3}$ now we construct the mapping $f:V(B_n))\to\{0,1,2\}$ as follows

$$f(U_i)=\begin{cases}2, & for\ 1\le i\le t\\ 1, & for\ (t+1)\le i\le 2t\\ 0, & Otherwise\end{cases}\qquad f(V_i)=\begin{cases}2, & for\ 1\le i\le t\\ 1, & for\ (t+1)\le i\le (2t-1)\\ 0, & Otherwise\end{cases}$$

We note that from the above mapping we get $e_{f^*}(0)=n-1$, $e_{f^*}(1)=n-1$ and $e_{f^*}(2)=n$.

Case(ii): If n=1 (mod3)

Let $t=\left\lfloor\dfrac{n}{3}\right\rfloor$ now we construct the mapping $f:V(B_n))\to\{0,1,2\}$ as follows

$$f(U_i)=\begin{cases}2, & for\ 1\le i\le t\\ 1, & for\ (t+1)\le i\le 2t\\ 0, & Otherwise\end{cases}\qquad f(V_i)=\begin{cases}2, & for\ 1\le i\le t\\ 1, & for\ (t+1)\le i\le 2t\\ 0, & Otherwise\end{cases}$$

We note that from the above mapping $e_{f^*}(0)=n$, $e_{f^*}(1)=n-1$ and $e_{f^*}(2)=n-1$

Case(iii): If n=2 (mod3)

Let $t=\left\lceil\dfrac{n}{3}\right\rceil$ now we construct the mapping $f:V(B_n))\to\{0,1,2\}$ as follows

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$$f(U_i)=\begin{cases}2, \ for\ 1\le i\le t\\ 1, \ for\left(t+1\right)\le i\le\left(2t-1\right)\\ 0, Otherwise\end{cases} \qquad f(V_i)=\begin{cases}2, \ for\ 1\le i\le\left(t-1\right)\\ 1, \ for\ t\le i\le\left(2t-1\right)\\ 0, Otherwise\end{cases}$$

We note that from the above mapping $e_{f^*}(0)=n-1$, $e_{f^*}(1)=n-1$ and $e_{f^*}(2)=n$.Therefore from the above cases the graph L_n under consideration satisfies the conditions $\left|e^*(i)-e^*(j)\right|\le 1, \ for\ i, j=\{0,1,2\}$. Hence ladder graph L_n is a total mean cordial graph.

Example 2.2: The total mean cordial labeling of ladder graph $L_6, L_7 \,\&\, L_8$

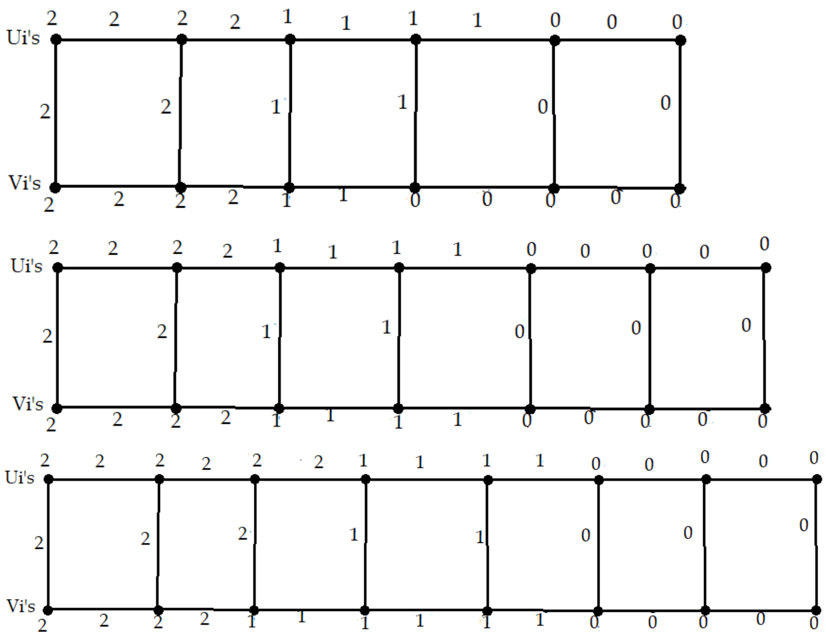

Figure 2.2: The total mean cordial labeling of ladder graph $L_6, L_7 \,\&\, L_8$

In the above graph L_6, the number of edges labeled with 0,1 or 2 is $e_{f^*}(0)=5$, $e_{f^*}(1)=6$ and $e_{f^*}(2)=6$. The graph L_7, the number of edges labeled with 0,1 or 2 is $e_{f^*}(0)=7$, $e_{f^*}(1)=6$ and $e_{f^*}(2)=6$. Finally the graph L_8, the number of edges labeled with 0,1 or 2 is $e_{f^*}(0)=7$, $e_{f^*}(1)=7$ and $e_{f^*}(2)=8$. Therefore $B_9, B_7 \,\&\, B_8$ satisfies the condition $\left|e^*(i)-e^*(j)\right|\le 1, \ for\ i, j=\{0,1,2\}$. Hence the graph $L_9, L_7 \,\&\, L_8$ are total mean cordial graph.

Theorem 2.3: The triangular ladder graph $(\mathrm{T}L_n), n\ is\ divisible\ by\ 3$ is a total mean cordial graph.

Proof: The triangular ladder graph $(\mathrm{T}L_n)$ constructed by the path P_n and P_{2n} . The triangular ladder graph $(\mathrm{T}L_n)$ contains two P_n paths and a P_{2n} path. Therefore triangular ladder graph $(\mathrm{T}L_n)$ having the set of vertices $V=\{u_i, v_i, 1\le i\le n\}$. The edges set $E(TL_n)=\{u_1u_2, u_2u_3, u_{n-1}u_n\}\cup\{v_1v_2, v_2v_3, v_{n-1}v_n\}\cup\{v_1u_1, u_1v_2, .. u_{n-1}v_n, v_nu_n\}$. This implies order and size of $(\mathrm{T}L_n)$ are $2n$ and $4n-3$.Let $t=\frac{n}{3}$ now we construct the mapping $f: V(TL_n)) \to \{0,1,2\}$ as follows

$$f(U_i)=\begin{cases}2, \ for\ 1\le i\le t\\ 1, \ for\left(t+1\right)\le i\le 2t\\ 0, Otherwise\end{cases} \qquad f(V_i)=\begin{cases}2, \ for\ 1\le i\le (t-1)\\ 1, \ for\ t\le i\le 2\left(t-1\right)\\ 0, Otherwise\end{cases}$$

We note that from the above mapping

$e_{f^*}(0)=\frac{(4n-3)}{3}$, $e_{f^*}(1)=\frac{(4n-3)}{3}$ and $e_{f^*}(2)=\frac{(4n-3)}{3}$. Therefore from the above cases the graph (TL_n) under consideration satisfies the conditions $|e^*(i)-e^*(j)|\le 1,\ for\ i,j=\{0,1,2\}$. Hence triangular ladder graph $(TL_n), n\ is\ divisible\ by\ 3$ is a total mean cordial graph.

Example 2.3: The total mean cordial labeling of the graph (TL_9)

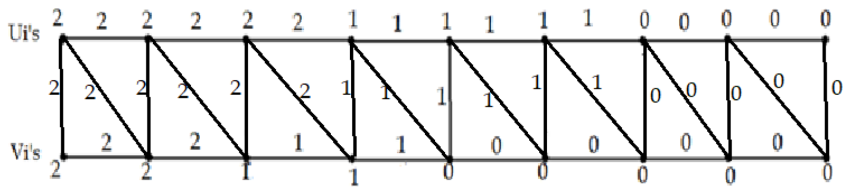

Figure 2.3: The total mean cordial labeling of triangular ladder graph (TL_9)

In the above graph TL_9, the number of edges labeled 0,1 and 2 is $e_{f^*}(2)=11$, $e_{f^*}(1)=11$ and $e_{f^*}(0)=11$. Therefore the triangular ladder graph TL_9 satisfies the condition $|e^*(i)-e^*(j)|\le 1,\ for\ i,j=\{0,1,2\}$. Hence the graph TL_9 is a total mean cordial graph.

Theorem 2.4: The sunlet graph S_n is a total mean cordial graph.

Proof: The sunlet graph S_n constructed by the cycle C_n and P_1^n . Therefore sunlet graph S_n having the set of vertices $V=\{u_i,v_i,1\le i\le n\}$. Note that there is a '$n$' different u_iv_i , P_1 paths in S_n and also it contain C_n paths in S_n. Therefore edge set $E(S_n)=\{(v_1v_2),(v_2v_3),\ldots(v_{(n-1)}v_n),(v_nv_1)\}\cup\{(u_1v_1),(u_2v_2),\ldots(u_{(n-1)}v_{(n-1)}),(u_nv_n)\}$.This implies order and size of S_n is $2n$ .Construct the mapping $f:V(S_n))\to\{0,1,2\}$ as follows ,

Case(i): If n=0 (mod3)

Let $t=\frac{n}{3}$ now we construct the mapping $f:V(B_n))\to\{0,1,2\}$ as follows

$$f(U_i)=\begin{cases}2, & for\ 1\le i\le t\\ 1, & for\ (t+1)\le i\le 2t\\ 0, & Otherwise\end{cases}\qquad f(V_i)=\begin{cases}2, & for\ 1\le i\le t\\ 1, & for\ (t+1)\le i\le (2t-1)\\ 0, & Otherwise\end{cases}$$

We note that from the above mapping, we get $e_{f^*}(0)=\frac{2n}{3}$, $e_{f^*}(1)=\frac{2n}{3}$ and $e_{f^*}(2)=\frac{2n}{3}$

Case(ii): If n=1 (mod3)

Let $t=\left\lfloor\frac{n}{3}\right\rfloor$ now we construct the mapping $f:V(B_n))\to\{0,1,2\}$ as follows

$$f(U_i)=\begin{cases}2, & for\ 1\le i\le t\\ 1, & for\ (t+1)\le i\le 2t\\ 0, & Otherwise\end{cases}\qquad f(V_i)=\begin{cases}2, & for\ 1\le i\le t\\ 1, & for\ (t+1)\le i\le 2t\\ 0, & Otherwise\end{cases}$$

We note that from the above mapping $e_{f^*}(0)=\left\lceil\frac{2n}{3}\right\rceil$, $e_{f^*}(1)=\left\lceil\frac{2n}{3}\right\rceil$ and $e_{f^*}(2)=\left\lceil\frac{2n}{3}\right\rceil-1$.

Case(iii): If n=2 (mod3)

Let $t=\left\lceil\frac{n}{3}\right\rceil$ now we construct the mapping $f:V(B_n))\to\{0,1,2\}$ as follows

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$$f(U_i)=\begin{cases}2, \ for \ 1\le i\le t\\ 1, \ for \ (t+1)\le i\le (2t-1)\\ 0, Otherwise\end{cases} \qquad f(V_i)=\begin{cases}2, \ for \ 1\le i\le (t-1)\\ 1, \ for \ t\le i\le 2(t-1)\\ 0, Otherwise\end{cases}$$

We note that from the above mapping $e_{f^*}(0)=\left\lfloor \frac{2n}{3}\right\rfloor$, $e_{f^*}(1)=\left\lfloor \frac{2n}{3}\right\rfloor+1$ and $e_{f^*}(2)=\left\lfloor \frac{2n}{3}\right\rfloor$

Therefore from the above cases the sunlet graph S_n under consideration satisfies the conditions $|e^*(i)-e^*(j)|\le 1, \ for \ i, \ j=\{0,1,2\}$. Hence sunlet graph S_n is a total mean cordial graph.

Example 2.4: The total mean cordial labeling of the sunlet graph S_6

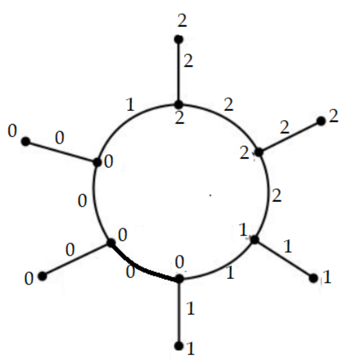

Figure 2.4: Total mean cordial labeling of the sunlet graph S_6

In the above graph S_6, the number of edges labeled with 0,1 or 2 is $e_{f^*}(0)=4$, $e_{f^*}(1)=4$ and $e_{f^*}(2)=4$. Therefore S_6 satisfies the condition $|e^*(i)-e^*(j)|\le 1, \ for \ i, \ j=\{0,1,2\}$. Hence the graph S_6 are Total mean cordial graph.

3. Conclusion

In this paper we will explain the concept of total mean cordial labeling. The graph consist total mean cordial labeling it named as a total mean cordial graph. Further we investigated the total mean cordial labeling in various graphs like brush graph, ladder graph, triangular ladder graph. Further explain the theorem with examples. In future

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