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An Algorithm of Edge Domination Number on Anti Fuzzy Graph

A. Sasireka, [1], R. Muthuraj [2], A. H. Nandhu Kishore[3]

[1] Assistant Professor, Department of Mathematics, PSNA College of Engineering and Technology, Dindigul-624 622, TamilNadu, India. sasireka.psna@gmail.com
[2]Assistant Professor, PG & Research Department of Mathematics, H.H. The Rajah's College, Pudukkottai – 622 001, TamilNadu, India. rmr1973@gmail.com
[3] Associate Professor, Department of Computer Science and Engineering, PSNA College of Engineering and Technology, Dindigul-624 622, TamilNadu, India. ahnkme.psna@gmail.com

ABSTRACT

In this paper we drive a algorithm for predicting the edge dominating set and edge domination number on anti fuzzy graph. This algorithm is implementing on Cartesian product on anti fuzzy graph and obtained the results on them.

Keywords: Anti fuzzy graph, Edge Dominating set, e-nodal anti fuzzy graph, uninodal anti fuzzy graph.

Mathematical Classification: 05C62, 05C69, 05C72, 05C76, 05E99.

I. Introduction

Mohamed Akram[5] introduced the concept of modified structure on fuzzy graph. The concept of edge domination was pioneered by Mitchell and Hedetniemi[4]. S.Arumugam and S.Velammal[1] conferred edge domination number of a graph. R.Seethalakshmi and R.B.Gnanajothi[11] introduced the definition of anti fuzzy graph. R.Muthuraj and A. Sasireka[5-8] defined some types of anti fuzzy graph also explained the concept of domination on anti fuzzy graph and anti cartesian product of anti fuzzy graph. In this paper, we derive the algorithm for predicting the edge dominating set on anti fuzzy graphs and the edge domination number on anti fuzzy graph. This algorithm is implemented on anti cartesian product of anti fuzzy graphs also.

II. Preliminaries

In this section, basic concepts of anti fuzzy graph are discussed. Notations and more formal definitions which are followed as in [3 - 8].

Definition 2.1 [7]

An anti fuzzy graph G = (σ, μ) is a pair of functions $\sigma : V\rightarrow [0,1]$ and $\mu : V\times V\rightarrow[0,1]$, where for all $u,v\in V$, we have $\mu(u,v) \geq \sigma(u) \vee \sigma(v)$ and it is denoted by $G_A(\sigma, \mu)$.

Note

μ is considered as symmetric. In all examples σ is chosen suitably. i.e., undirected anti fuzzy graphs are only considered.

Notation

Without loss of generality let us simply use the letter G_A to denote an anti fuzzy graph.

Definition 2.2 [7]

The order p and size q of an anti fuzzy graph $G_A = (V,\sigma,\mu)$ are defined to be $p=\sum_{x\in V}\sigma(x)$ and $q=\sum_{xy\in E}\mu(x, y)$ is denoted by O(G) and S(G).

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Definition 2.3 [7]

Two vertices u and v in G_A are called adjacent if $(½)[\sigma(u) \vee \sigma(v)] \leq \mu(u,v)$.

Definition 2.4 [7]

An anti fuzzy graph $G_A= (\sigma,\mu)$ is a strong anti fuzzy graph of $\mu(u,v) = \sigma(u) \vee \sigma(v)$ for all $(u,v)\in\mu^*$ and G_A is a complete anti fuzzy graph if $\mu(u,v) = \sigma(u) \vee \sigma(v)$ for all $(u,v) \in \mu^*$ and $u,v\in \sigma^*$. Two vertices u and v are said to be neighbors if $\mu(u,v) > 0$.

Definition 2.5 [7]

An edge e = {u, v} of an anti fuzzy graph G_A is called an effective edge if $\mu(u,v) = \sigma(u) \vee \sigma(v)$. An edge e = {u,v} of an anti fuzzy graph G_A is called an weak edge if $\mu(u,v) \neq \sigma(u) \vee \sigma(v)$.

Definition 2.6[8]

e_i is an edge in an anti fuzzy graph G_A then $N(e_i) = \{e_j$: e_i and e_j are incident to same vertex $u_i\}$ is called the neighborhood of e_i and $N[e_i] = N(e_i) \cup \{e_i\}$ is called closed neighborhood of e_i. $\sum_{e_1\in N(e_2)}\mu(e_1)$ is called the neighbourhood of edge degree of e_2 and is denoted by $d_N(e_2)$

Definition 2.7[8]

The strong neighbourhood of an edge e_i in an anti fuzzy graph G_A is $N_s(e_i) = \{e_j \in E(G)$ / e_j is an effective edge which have maximum fuzzy value in neighborhood of e_i in $G_A\}$.

III. Algorithms to Find Edge Dominating Set of Anti Fuzzy Graph

In this section, edge adjacency matrix and strong edge adjacency matrix is discussed on an anti fuzzy graph based on the definition of edge weighted edge adjacency matrix. Some properties of adjacency fuzzy matrix are iterated. Using strong edge adjacency matrix, an algorithm is derived for finding edge dominating set of anti fuzzy graph G_A.

Definition 3.1[3]

The edge-weighted edge-adjacency matrix, denoted by ${}^{ew}A$, has been introduced by Estrada. It is a square unsymmetric E × E matrix defined as:

$$[{}^{ew}A]_{ij} = \begin{cases} 1, & \text{if the edges i and j are adjacent} \\ k, & \text{if the edge i} - \text{j is weighted} \\ 0, & \text{otherwise} \end{cases}$$

Definition 3.2

An anti fuzzy graph $G_A = (\sigma,\mu)$ with the fuzzy relation μ to be reflexive and symmetric is completely determined by the adjacency fuzzy matrix and it is denoted by ${}^{e}A_{\mu}$.

Where, $$({}^{e}A_{\mu})_{i,j} = \begin{cases} \mu(e_j), & \text{for } i \neq j \text{ and } e_j \text{ is adjacent to } e_i \\ \mu(e_i), & \text{for } i = j \end{cases}$$

If σ^* contains n elements then ${}^{e}A_{\mu}$ is a square matrix of order n.

Definition 3.3

Let G_A be a simple connected anti fuzzy graph. Then the strong adjacency matrix $({}^{e}A_{\mu})'$ is defined as,

$$({}^{e}A_{\mu})' = \begin{cases} \mu(e_j), & \text{for } i \neq j \text{ and } e_j \text{ is a strong neighbourhood to } e_i \\ \mu(e_i), & \text{for } i = j \end{cases}$$

Properties 3.4

i. ${}^{e}A_{\mu}$ is a symmetric matrix whose all entries in main diagonal are $\mu(e_i)$.
ii. There is no row exists with 0 fuzzy value entries.
iii. In $({}^{e}A_{\mu})$, a i^{th} row contains r number of entries then the corresponding edge is a neighbour to atleast r vertices in G_A.
iv. Sum of the main diagonal elements gives q.
v. In $({}^{e}A_{\mu})'$, Sum of each row($R(e_i)$) is equal to sum of each column($C(e_i)$). That is, $\sum R(e_i) = \sum C(e_i)$
vi. $({}^{e}A_{\mu})'$ is asymmetric matrix whose all entries in main diagonal are $\mu(e_i)$.
vii. In $({}^{e}A_{\mu})'$, a i^{th} row contains r number of entries then the corresponding edge is a strong neighbour to r edges in G_A. but the reverse part it's may vary.

Algorithm 3.5

The algorithm is defined for finding an edge dominating set in an anti fuzzy graph G_A with m vertices.

Algorithm - Maxima Method

Step 1. For G_A, to construct the $({}^{e}A_{\mu})'$ and q = sum of main diagonal elements.
Step 2. Let D = ϕ.
Step 3. Find $\sum R(e_i)$ and $\sum C(e_i)$.
Step 4. In $({}^{e}A_{\mu})'$, choose the maximum value in $C(e_i)$. If tie occurs then choose an edge which have maximum $\mu(e_i)$ in main diagonal otherwise break it arbitrarily.
Step 5. Select the corresponding edge in the selected column as a dominated edge and write an edge in the set D.
Step 6. Cut the selected edge with corresponding to the selected row and column. In row, cut the edges which are incident with strong edge in D. The resulting reduced matrix is called as $({}^{e}A_{\mu})'_1$.
Step 7. If the reduced matrix $({}^{e}A_{\mu})'_1$ has any rows and columns goto step 3. Otherwise STOP.
Step 8. Finally get the edge dominating set D of G_A and it will be minimal edge dominating set of G_A.

Example 3.6

Consider the following Figure 1, the edge dominating set is found out using the maxima method algorithm.

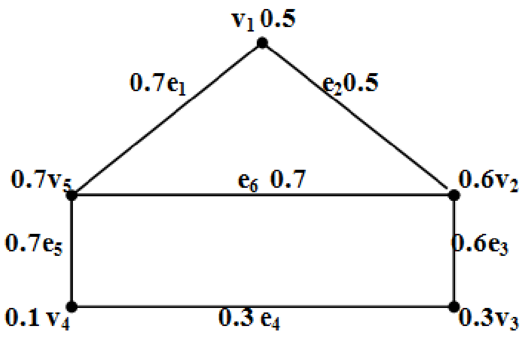

Figure 1. Anti fuzzy graph G_A

STEP 1: For the given anti fuzzy graph G_A, the strong edge adjacency matrix $({}^{e}A_{\mu})'$ is given as

$$({}^{e}A_{\mu})' = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 0.7 & 0 & 0 & 0 & 0.7 & 0.7 \\ 0.7 & 0.5 & 0 & 0 & 0 & 0.7 \\ 0 & 0 & 0.6 & 0 & 0 & 0.7 \\ 0 & 0 & 0 & 0.3 & 0.7 & 0 \\ 0.7 & 0 & 0 & 0 & 0.7 & 0.7 \\ 0.7 & 0 & 0 & 0 & 0.7 & 0.7 \end{bmatrix} \end{array}$$

STEP 2: Let the edge dominating set, D = ϕ.

STEP 3: e_1 e_2 e_3 e_4 e_5 e_6 $\Sigma R(e_i)$

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$$({}^{e}A_{\mu})' = \begin{array}{c} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{array}\begin{bmatrix} 0.7 & 0 & 0 & 0 & 0.7 & 0.7 \\ 0.7 & 0.5 & 0 & 0 & 0 & 0.7 \\ 0 & 0 & 0.6 & 0 & 0 & 0.7 \\ 0 & 0 & 0 & 0.3 & 0.7 & 0 \\ 0.7 & 0 & 0 & 0 & 0.7 & 0.7 \\ 0.7 & 0 & 0 & 0 & 0.7 & 0.7 \end{bmatrix}\begin{array}{c} 2.1 \\ 1.9 \\ 1.3 \\ 1.0 \\ 2.1 \\ 2.1 \end{array}$$

$$\Sigma\, C(e_i) \quad 2.8 \quad 0.5 \quad 0.6 \quad 0.3 \quad 2.8 \quad 3.5 \quad [10.5]$$

STEP 4: In $({}^{e}A_{\mu})'$, $C(e_6)$ has maximum value. Then select e_6 and D = {e_6}

STEP 5: Delete the selected column e_6 and the corresponding row e_1, e_2, e_3, e_5 and e_6 in $({}^{e}A_{\mu})'$. The resulting reduced matrix say $({}^{e}A_{\mu})'_1$.

STEP 6: $({}^{e}A_{\mu})'_1 = \begin{array}{c} \\ e_4 \end{array}\begin{array}{cc} e_4 & e_5 \\ [0.3 & 0.7] \end{array}$

$({}^{e}A_{\mu})'_1$ has two columns. Then goto step 3.

STEP 3A:

$$\begin{array}{cccc} & e_4 & e_5 & \Sigma\, R(e_i) \\ ({}^{e}A_{\mu})'_1 = e_4 & [0.3 & 0.7] & 1.0 \\ \Sigma\, C(e_i) & 0.3 & 0.7 & [1.0] \end{array}$$

STEP 4A: In $({}^{e}A_{\mu})'_1$, $C(e_5)$ has maximum value. Then select e_5 and D = {e_6, e_5}

STEP 5A: Delete the selected column e_5 and the corresponding row e_4 in $({}^{e}A_{\mu})'_1$.

Now $({}^{e}A_{\mu})'_1$ has no column. STOP.

The resulting set D = {e_6, e_5} is an edge dominating set of the given anti fuzzy graph G_A and which is also minimal. The edge domination number is $\gamma'_A = 1.4$.

Example 3.7

Consider a complete anti fuzzy graph $G_{A_1} = (\sigma_1, \mu_1)$ and $G_{A_2} = (\sigma_2, \mu_2)$ with $\sigma_1(u_i) = \{0.2, 0.3, 0.6\}$, $\sigma_2(v_j) = \{0.1, 0.4, 0.7\}$ for i, j = 1, 2, 3 and $\mu_1\{u_1u_2, u_2u_3, u_3u_1\} = \{0.3, 0.6, 0.6\}$, $\mu_2\{v_1v_2, v_2v_3, v_3v_1\} = \{0.4, 0.7, 0.7\}$. By the definition of anti cartesian product, get the Figure 2.

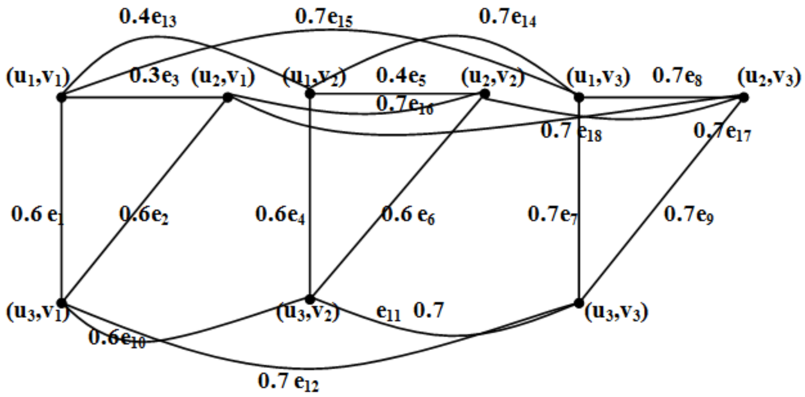

Figure 2. Anti fuzzy graph G_A

Algorithm 3.8

For the Figure 2, the minimal edge dominating set is obtained by using an algorithm. For applying the algorithm, it is necessary to construct a strong edge adjacency matrix for the resulting anti fuzzy graph.

STEP 1: For the given anti fuzzy graph $G_A = G_{A_1} \,\overline{\times}\, G_{A_2}$, the strong edge adjacency matrix $({}^{e}A_{\mu})'$ is given as

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$$({}^{e}A_{\mu})' = \begin{array}{ccc}
 & \begin{array}{cccccccccccccccc} e_1 & e_2 & \cdots & e_7 & e_8 & \cdots & e_{11} & \cdots & e_{15} & e_{16} & e_{17} & e_{18} \end{array} & \Sigma R(e_i) \\
\begin{array}{c} e_1 \\ \vdots \\ e_7 \\ \vdots \\ e_{13} \\ \vdots \\ e_{16} \\ e_{17} \\ e_{18} \end{array} &
\left[\begin{array}{cccccccccccccccc}
0.6 & 0 & & 0 & 0 & & 0 & & 0.7 & 0 & 0 & 0 \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & & 0.7 & 0.7 & & 0.7 & & 0.7 & 0 & 0 & 0 \\
\vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & & 0 & 0 & & 0 & & 0 & 0.7 & 0 & 0 \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & & 0 & 0 & & 0 & & 0 & 0.7 & 0.7 & 0.7 \\
0 & 0 & & 0 & 0 & & 0.7 & & 0.7 & 0.7 & 0.7 & 0.7 \\
0 & 0 & & 0.7 & 0.7 & & 0 & & 0 & 0.7 & 0.7 & 0.7
\end{array}\right] &
\begin{array}{c} 2 \\ \vdots \\ 4.9 \\ \vdots \\ 1.8 \\ \vdots \\ 2.1 \\ 3.5 \\ 3.5 \end{array}
\end{array}$$

$\Sigma C1(e_i)$	0.6	0.6	...	4.9	4.9	...	4.9	...	4.9	4.9	4.9	3.5	[51.7]
$\Sigma C2(e_i)$	0.6	0.6	...	↑	1.4	...	1.4	...	2.1	4.9	3.5	2.1	
$\Sigma C3(e_i)$	0.6	0	...	-	1.4	...	1.4	...	1.4	↑	0	0	
$\Sigma C4(e_i)$	0.6	0	...	-	↑	...	1.4	...	1.4	-	0	0	
$\Sigma C5(e_i)$	0.6	0	...	-	-	...	↑	...	1.4	-	0	0	
$\Sigma C6(e_i)$	0	0	..	-	-	...	-	...	↑	-	0	0	

$D = \{e_7, e_8, e_{11}, e_{15}, e_{16}\}$

The resulting set $D = \{e_7, e_8, e_{11}, e_{15}, e_{16}\}$ is an edge dominating set of the given anti fuzzy graph G_A and which is also minimal. The edge domination number is $\gamma' = 3.5$.

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