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Fuzzy dot KM Ideals on K-Algebras

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ABSTRACT

In this paper, fuzzy dot KM ideals are introduced. Intersection of two Km ideals and compliment of Ideal are discussed and some properties are satisfied by our proposed dot KM ideals.

Keywords and Phrases: **K-algebras, KM Ideals, Fuzzy relations, Fuzzy dot KM Ideal.**

Subject to the classification: **06F35, 13A15**

1.Introduction:

Initially, the notion of the fuzzy sets and its functions are introduced by Zadeh[5]. Fuzzy ideals of K Algebras are introduced and their properties are verified [1] and [2] and the extension of Fuzzy KM an ideal on K-algebras is also introduced and checked [3].Fuzzy dot subalgebras of d-algebras are introduced [4] also fuzzy dot subalgebras and fuzzy dot Ideals of B- algebras are introduced[6] In this paper, fuzzy dot KM ideals on k algebras are introduced and investigate few properties .

2. Preliminaries:

Basic concepts of fuzzy K-algebra and its KM ideal are defined and discussed.

Definition 2.1:

Let $(G, \cdot, e)$ be a group with the identity e such that $x^2 \neq e$ for some $x(\neq e) \in G$.

A K-algebra built on G (briefly, K-algebra) is a structure $K = (G, ., \odot, e)$ where "$\odot$" is a binary operation on G which is induced from the operation "$\cdot$", that satisfies the following:

(k1) $(\forall a, x, y \in G)\ ((a \odot x) \odot (a \odot y) = (a \odot (y^{-1} \odot x^{-1})) \odot a)$,

(k2) $(\forall a, x \in G)\ (a \odot (a \odot x) = (a \odot x^{-1}) \odot a)$,

(k3) $(\forall a \in G)\ (a \odot a = e)$,

(k4) $(\forall a \in G)\ (a \odot e = a)$, (k5) $(\forall a \in G)$ $(e \odot a = a^{-1})$.

If G is abelian, then conditions (k1) and (k2) are replaced by:

(k1') $(\forall a, x, y \in G)$ $((a \odot x) \odot (a \odot y) = y \odot x)$,

(k2') $(\forall a, x \in G)$ $(a \odot (a \odot x) = x)$,

respectively. A nonempty subset H of a K-algebra K is called a subalgebra of K if it satisfies:

- $(\forall a, b \in H)$ $(a \odot b \in H)$.

Note that every subalgebra of a K-algebra K contains the identity e of the group $(G, \cdot)$.

Definition 2.2:

A fuzzy set η in a k-algebra k is called a fuzzy KM ideal of X if it satisfies:

(i) $(\forall x \in G)(\eta(e) \geq \eta(x))$
(ii) $(\forall x, y \in G)$ $(\eta(y) \geq \min\{\eta(y \odot x), \eta((x \odot (x \odot y))\})$

Definition 2.3:

Let X be a fuzzy set in a K-Algebra A. Then X is called a fuzzy dot KM ideal of A if it satisfies:

(i) $\aleph_X(0) \geq \aleph_X(a)$
(ii) $\aleph_X(b) \geq \aleph_X(b \odot a).\aleph_X(a \odot (a \odot b)) \forall a, b \epsilon A$.

Examples:

Let A = {0,1,2,3} be a set with the following cayley table

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then (A,*,0) is a K-Algebra.Let the fuzzy set X in A by $\aleph_X(0) = \aleph_X(1) = 0.9, \aleph_X(2) = 0.7$ and $\aleph_X(3) = 0.6$ for all $a \epsilon A$. Then X is a fuzzy dot KM ideal on X.

Definition 2.4:

A fuzzy set 'A' in X is called fuzzy KM ideal of X if it satisfies

(i) $\aleph_X(0) \geq \aleph_X(a)$
(ii) $\aleph_X(b) \geq M_i\{\aleph_X(ba).\aleph_X(a(ab)\} \forall a, b \epsilon A$.

Properties of fuzzy dot KM Ideal:

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Property: 1

Let X be a fuzzy dot KM ideal of A. If $a \leq b$ in A, then $\aleph_X(b) \geq \aleph_X(0)\aleph_X(a)$ for all $a, b \epsilon A$.

Proof:

Let $a, b \epsilon A$ such that $a \leq b$. Then $b \odot a = 0$ and thus

$$\aleph_X(b) \geq \aleph_X(b \odot a)\aleph_X\big(a \odot (b \odot a)\big)$$

$\geq \aleph_X(0)\aleph_X(a)$.

This Completes the proof.

Property: 2

Let X be fuzzy dot KM ideal of A. If the inequality $a \odot b \leq c$ holds in A, then $\aleph_X(b) \geq \aleph_X(0)\aleph_X(a)\aleph_X(c)$for all $a, b, c \epsilon A$.

Proof:

Let $a, b, c \epsilon A$, such that $a \odot b \leq c$.Then $((b \odot a) \odot c) = 0$, and thus

$$\aleph_X(b) \geq \aleph_X(b \odot a)\aleph_X\big(a \odot (b \odot a)\big)$$

$$\geq \aleph_X\big((b \odot a) \odot c\big).\aleph_X(a).\aleph_X(c))$$

$$= \aleph_X(0).\aleph_X(a).\aleph_X(c).$$

This Completes Proof.

Property: 3

If X_1 and X_2 be two fuzzy dot KM ideals of A, then $X_1 \cap X_2$ is also a fuzzy dot KM Ideal of A.

Proof:

Let X_1 and X_2 be two fuzzy dot KM Ideals of A. Then for any $a \epsilon A$,

$$\aleph_{X_1}(0) \geq \aleph_{X_1}(a)$$

and

$$\aleph_{X_2}(0) \geq \aleph_{X_2}(a)$$

Now

$$\aleph_{X_1 \cap X_2}(0) = Min\left\{\aleph_{X_1}(0), \aleph_{X_2}(0)\right\}$$

$$\geq Min\left\{\aleph_{X_1}(a), \aleph_{X_2}(a)\right\}$$

$$= \aleph_{X_1 \cap X_2}(a).$$

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Again for any $a, b \epsilon A$,

We have

$$\aleph_{X_1 \cap X_2}(b) = Min\left\{\aleph_{X_1}(b), \aleph_{X_2}(b)\right\}$$

$\geq Min\left\{\aleph_{X_1}(b \odot a).\aleph_{X_1}(a \odot (a \odot b)), \aleph_{X_2}(b \odot a).\aleph_{X_2}(a \odot (a \odot b))\right\}.$

$\geq Min\left\{\aleph_{X_1}(b \odot a), \aleph_{X_2}(b \odot a)).Min\left\{\aleph_{X_1}(a \odot (a \odot b), \aleph_{X_2}(a \odot (a \odot b)\right\}\right\}.$

$= \aleph_{X_1 \cap X_2}(b \odot a)\aleph_{X_1 \cap X_2}(a \odot (a \odot b)$

Hence $X_1 \cap X_2$ is a fuzzy dot KM Ideal of A.

Property : 4

If $\aleph$ and ב are fuzzy dot KM Ideals of a K-Algebra A one is confined in another then so is $\aleph \cup$ ב.

Proof:

Let $\aleph$, ב be fuzzy KM Ideals in A.

($\aleph \cup$ ב) 0 = Max { $\aleph$ (0), ב(0)}

$\geq$ Max { $\aleph$ (a), ב(a)} , $\forall$ $\boldsymbol{a\epsilon\ A}$

$\geq$ ($\aleph \cup$ ב) (a) , $\forall$ $\boldsymbol{a\epsilon\ A}$

($\aleph \cup$ ב) (b) = Max { $\aleph(b)$, ב(b)}

$\geq$ Max { $\aleph$ $(b \odot a)$, $\aleph(a \odot (a \odot b))$, ב $(b \odot a)$, ב$(a \odot (a \odot b))$ }

$\geq$ Max { $\aleph$ $(b \odot a)$, ב $(b \odot a)$ }. Max { $\aleph(a \odot (a \odot b))$, , ב$(a \odot (a \odot b))$ }

= ($\aleph \cup$ ב) $(b \odot a)$. ($\aleph \cup$ ב) $(a \odot (a \odot b))$

Hence ($\aleph \cup$ ב) is a Fuzzy dot KM Ideals of K-Algebra of A.

Property : 5

If $\aleph$ is a fuzzy dot KM Ieals of K-Algebra of A then prove that $\aleph^m$ is fuzzy dot KM Ideals of K-Algebra of A.

Proof:

$\aleph^m$ is a Fuzzy set on A defined by, where m is any positive Integer.

(i) $\aleph(0) \geq \aleph(a)$, $\forall$ $\boldsymbol{a\epsilon\ A}$

$$[\aleph(0)]^m \geq [\aleph(a)]^m$$
$$\aleph^m(0) \geq \aleph^m(a)$$

(ii) $\aleph_X(b) \geq \aleph_X(b \odot a).\aleph_X\big(a \odot (b \odot a)\big)$

$$[\aleph_X(b)]^m \geq \left[\aleph_X(b \odot a).\aleph_X\big(a \odot (b \odot a)\big)\right]^m$$

$$=[\aleph_X(b \odot a)]^m\left[\aleph_X\big(a \odot (b \odot a)\big)\right]^m$$
$$= {\aleph_X}^m(b \odot a).{\aleph_X}^m\big(a \odot (b \odot a)\big)$$

${\aleph_X}^m$ is a fuzzy dot KM-Ideals.

Property : 6

If $\aleph$ and its complement $\aleph^c$ are fuzzy dot KM-Ideals, Prove that $\aleph$ is a constant.

Proof:

$\aleph(0) \geq \aleph(a)$, $\forall$ $\boldsymbol{a \epsilon A}$ **………………….(1)**

$$\aleph^c(0) \geq \aleph^c(a)$$

$$1\text{-}\aleph(0) \geq 1 - \aleph(a)$$

$\aleph(0) \leq \aleph(a)$ …………………..(2)

From (1) & (2) $\aleph(0) = \aleph(a) \forall$ $\boldsymbol{a \epsilon A}$.

$\aleph$ is a constant fuzzy function.

Property: 7

Let $\varphi = \{A, \aleph_\varphi\}$ be a fuzzy KM Ideal of A, if the Inequality $ab \geq c$ holds in A then

$\aleph_\varphi(b) \geq T\{\aleph_\varphi(a(ab)), \aleph_\varphi(c)\}$.

Proof:

Let a,b,c$\in A$ be such that $ab \geq c$ then (ab)c=0

$$\aleph_\varphi(b) \geq T\{\aleph_\varphi(ba), \aleph_\varphi(a(ab))\}$$

$$\geq T\{T\{\aleph_\varphi(ba)c, \aleph_\varphi(c)\}, \aleph_\varphi(a(ab))\}$$

$$\geq T\{T\{\aleph_\varphi(0), \aleph_\varphi(c)\}, \aleph_\varphi(a(ab))\}$$

$$\geq T\{\aleph_\varphi(a(ab)), \aleph_\varphi(c)\}$$

Hence the Proof.

Property: 8

Let $f: X \to Y$ be an epimorphism of groups. If $\aleph^f$ is a fuzzy KM-Ideal of B, then $\aleph$ is fuzzy KM ideal of Y.

Proof:

Let $b \in B$, there exists $a \in A$ such that $f(a) = b$ then

$$\aleph(y) = \aleph\{f(a)\} = \aleph^f(a) \geq \aleph^f(e) = \aleph\big(f(e)\big) = \aleph(e)$$

Let $x, y \in Y$ then there exists $a, b \in X$ such that $f(a) = x \; and \; f(b) = y$.

$$\aleph^f(x) = \aleph\{f(a)\} = \aleph\{f(a)\}$$

$$\geq T\{\aleph^f(ba), \aleph^f(a(ab))\}$$

$$\geq T\{\aleph\big\{f(ba), \aleph\{f(a(ab))\}\big\}$$

$$\geq T\{\aleph\big\{f(ba), \aleph\{f(a).f(ab)\}\big\}$$

$\geq T\{\aleph\big\{f(yx), \aleph\{f(x(xy))\}\big\}$.

Hence ℵ is fuzzy KM-Ideal of Y.

Conclusion:

In this paper dot product of fuzzy KM ideal on K algebras properties are discussed.

References:

[1] Tapansenapathi, “ Translations of Intutionistic Fuzzy B- algebras”, Fuzzy information and Engineering,(2015)7:389-404.

[2] Muhammed Akram and Karamat H.Dar, “ Fuzzy Ideals of K- Algebras”, Annals of university of Craiova, Math.comp.sciser, Volume 34, 2007, page 11-20.

[3] S. Kailasavalli,D.Duraiaruldurgadevi, R.Jaya, M.Meenakshi, N. Nathiya, “ Fuzzy KM ideals on K-Algebras”, International journal of pure and Applied Mathematics, vol 119 No.15, 2018, 129-135.

[4] K.H.Kim, On Fuzzy dot subalgebras of D-algebra, International Mathematical forum,4(2009),645-651.

[5] L.A. Zadeh, “ Fuzzy set”, Information and Control, 8 (1965),338-353

[6] Tapansenapathi, Manoranjan Bhowmik,MadhumangalPal,” Fuzzy dot subalgebras abd Fuzzy dot Ideals of B-Algebras”,Journal of Uncertain Systems, Vol8,No.1,pp22-30(2014).