JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p.3789-3795
https://publishoa.com
ISSN: 1309-3452

Critical Study of Flow Shop Scheduling

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Received 2022 March 15; **Revised** 2022 April 20; **Accepted** 2022 May 10.

Abstract:
Flow-shop scheduling is an optimization problem. It is a form of optimal job scheduling. In a basic job-scheduling problem, n jobs J1, J2,, Jn are given, each with a different processing time. These jobs must be scheduled on m machines, each with a different processing power, while attempting to reduce the makespan, or the overall length of the schedule (that is, when all the jobs have finished processing). Each task in the particular variation known as flow-shop scheduling has exactly m operations. The i-th machine must carry out the i-th operation of the job. No machine can carry out more than one task at once. Execution time is given for each job's operation. This paper reflects critical study of Flow Shop Scheduling .
Keywords : optimizing , scheduling , managing , characterized , materials

1. Scheduling

In the context of production and industry, scheduling may be characterized as a method for organizing, managing, and optimizing work and workloads. Human resources, production techniques, production planning, and the acquisition of production materials are all part of the process. Two common processes are involved in a scheduling problem: planning or deciding on a job's order, and determining when each activity will begin and finish. The scheduling process requires comprehensive knowledge of the types and characteristics of all production resources in order to successfully accomplish all activities. As long as the resources are known, the scheduling boundary may be established.

In addition, each operation is categorized in terms of the resources necessary, the length of achievement, the earliest start and finish periods, and so on, in the scheduling process. Scheduling is important in both manufacturing and non-manufacturing, and it is described using the same language. Resources and tasks are often referred to as machines and jobs, respectively, in every scheduling context. The collection of activities (operations) to be executed by the restricted resources available in industries and manufacturing firms is the subject of production scheduling (machines). Producing schedule planning is all about finding out how many resources you have on hand and how long it will take them to complete specific tasks. A key aspect of an industry's planning role is making choices about how to allocate resources and perform activities (operations). Information on the design and quality of the product to be produced, accessible manufacturing technology, and necessary production parts are all part of the industrial planning function. As a result, the industrial planning function is a representation of the available production resources and the order in which these resources will be used. Product quality, delivery time, and product pricing are all becoming more competitive in today's industrial environment. Incredible advances in information and communication technology have brought manufacturers and consumers closer together than ever before. There are a number of factors that are used to evaluate a company's success in today's competitive marketplace:

(a)Technological dimension: The quality and quantity of industrial products are constantly promoted by rapid technological advancement, which allows the manufacturing unit to satisfy the market's expectations promptly and expertly.

(b) Organizational dimension: To increase industry and manufacturing enterprise performance, the organizational dimension focuses on inventory management, delivery time, production time and so on. In order to keep a tight rein on output, businesses need sophisticated tools and processes at their disposal. Due to the highly competitive market and the high expectations of customers, these dimensions have grown significantly in significance. That is why effective procedures and strategies are required in any production management in order to meet client needs on time and properly while maintaining a strong market reputation.

There are a number of critical responsibilities, including scheduling, that play a significant part in the overall administration of an organization in order to attain these goals. When it comes to managing human and technology resources, scheduling is all about making sure that all customers' needs are met as well as the needs that arise as a result of a company's production plan. Organization and problem-solving abilities are essential for this planning function, which is responsible for managing the restricted production resources needed to complete the tasks and occupations (machines). As a result, it is up to this role to make judgments on product production. Furthermore, the relationship between the firm and its clients is critical to its success or failure. Structured manufacturing has been a part of every sector for many years now. As a result, in today's competitive market, a huge number of tasks must be handled on a limited number of resources in the quickest time feasible.

1.1 PROMINENCE OF SCHEDULING

Organizational effectiveness and customer satisfaction are directly related to the effectiveness and efficiency of the scheduling process in both manufacturing and service sectors. Scheduling is concerned with maximizing one or more goals by allocating resources to activities in the most efficient manner possible over a certain period of time. An organization's duties, resources, and goals may take a variety of shapes. Industrial/manufacturing equipment, processing units, input/output devices linked to computers, mechanics at a service station, runways at airports and many more are examples of the resources. The same is true for a wide range of other duties, such as those that take place in manufacturing, on computers, at service stations, and in airfields, amongst others. Scheduling's goals might be performance-, cost-, or deadline-related. scheduling's performance goals include minimizing the time it takes to complete the final job, as well as the time it takes to complete all tasks on the last machine. As a result, cost-related goals include cutting setup costs, cutting down on rental/leasing fees and so on. Due date-related goals include minimizing the number of works completed beyond the due date, minimizing lateness, and reducing tardiness, among other things. Scheduling is a critical component of many production and service systems, as well as many information processing settings. The following points illustrate the necessity of scheduling in all of these businesses:

(i) Inadequate scheduling may result in nescient utilization of available production resources. A notable indicator of ineffective scheduling is the idleness of machines, equipment's, and human resources waiting for processing the orders. This may lead to an increase in production and manufacturing cost.

(ii) Inefficient scheduling may lead to delay in the delivery of orders through the production channels, resulting in calls for some advance steps that again results in high production time and production cost.

1.2 CLASSIFICATION OF SCHEDULING MODELS

There are a number of different types of scheduling models, but they all aim at defining the sequence in which tasks (e.g., Jobs) flow through a given system of resources (e.g., Machines). Grave's categorization of scheduling models is based on the performance of scheduling in different industrial and manufacturing organizations.

a) The process of creating requirements

Requirement generation determines whether a scheduling model is an open shop or a closed shop model. There is no stock of merchandise in open shop scheduling models, but there is in closed shop scheduling models. Closing off the list of closed shop scheduling models, we have the Flow Shop and the Job Shop scheduling models. Figure 1 shows a comprehensive classification of scheduling models based on the creation of requirements.

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ISSN: 1309-3452

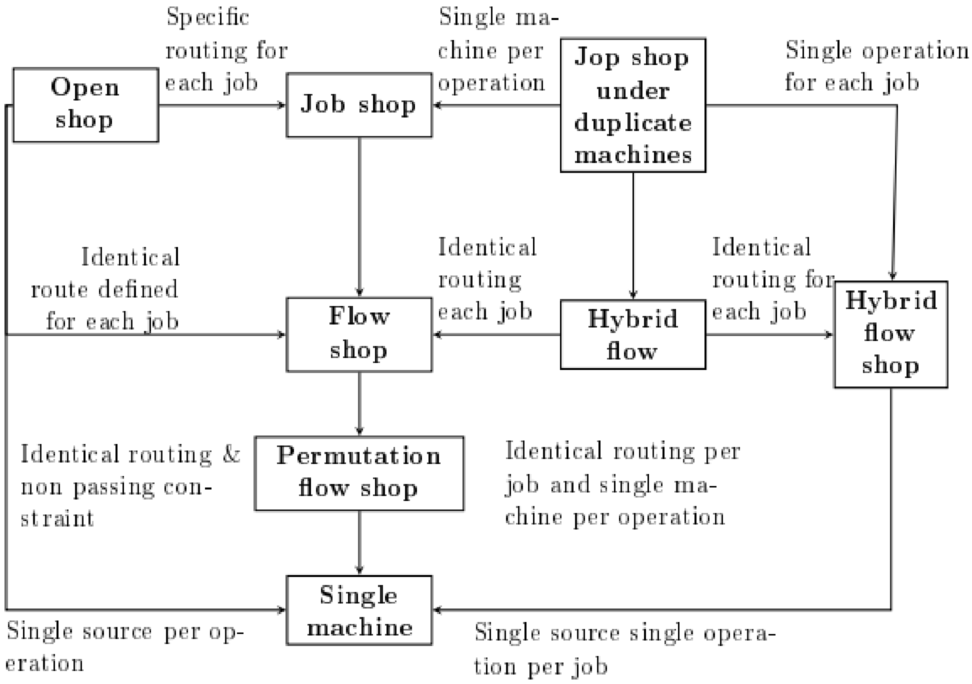

Figure 1. Classification of scheduling models based on requirement generation

(b) Processing intricacy

The number of manufacturing stages and workstations in a given production facility correlates with the processing complexity. Scheduling models may be categorized according to this dimension:

(i) Single stage, single machine
(ii) Single stage, multiple machines
(iii) Multistage multiple machine ow shop
(iv) Multistage multiple machine job shop

Scheduling models for multistage, multiple-machine flow shops use a particular scheduling environment in which each job comprises of many tasks to be executed by various resources (machines), but the route for all jobs is same. Scheduling routes for works in a multistage job shop might be varied in order to accommodate diverse production methods.

(c) Scheduling criteria

This means that the goal is to be maximized via the use of scheduling criteria. We have a wide range of goals, some of which are incompatible. There are a variety of scheduling criteria that are often used: minimize overall tardiness, minimize the number of late tasks, optimize the usage of systems/resources, reduce the time spent on system utilization, minimize the amount of in-process stock, etc.

(d) Parameter unpredictability

The degree of uncertainty or ambiguity in various scheduling parameters is shown by parameter unpredictability. Stochastic or probabilistic scheduling may be described as stochastic or deterministic if there is a little amount of uncertainty.

(e) Scheduling environment

The scheduling model's dynamic and static nature protects the scheduling environment. Static scheduling challenges are those in which the quantity of tasks and the time at which they will begin are fully known in advance. Dynamic scheduling, on the other hand, refers to scheduling issues in which the number of tasks and their processing parameters fluctuate over a certain period of time.

1.3 DIFFERENT CATEGORIES OF SCHEDULING

A distinct aspect of optimization theory, scheduling, has a wide range of practical applications in many industries. Every producer and service provider, in general, has experienced a certain form of scheduling

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challenge. Airports, airlines, institutions, manufacturers, and so on all confront a variety of scheduling and routing issues, to name a few. According to Graham et al. [GLLK79], the properties of scheduling were expressed by the three eld notation /. Because it describes the scheduling environment, we may expect this to often have only one element. is a representation of the processing features and restrictions that are in effect during computation. Scheduling optimization is shown in. Scheduling issues may be broken down into the following categories:

(a) Project scheduling

Project scheduling generally aims at sequencing of different activities subject to eminence constraints and allocating the available resources to these activities in the production project. Similar to parallel machines the project scheduling consists of an infinite number of machines. The objective is to minimize the make span. The techniques used for solving the problem of project scheduling are critical path method (CPM) and program evaluation and review technique (PERT).

(b) Single machine scheduling

A single machine scheduling challenge arises when a single machine or facility is required to handle a series of work orders. Each task in the single machine scheduling issue has just one action that must be completed by a single machine.

(c) Flow shop scheduling

It is not permissible to schedule a flow shop scheduling issue, which is a scheduling problem in which each machine in the system performs a specific function and the tasks travel through these machines in a predetermined order. Pre-emission of tasks is often not permitted in this scheduling system.

(d) Job shop scheduling

The scheduling of n tasks across a set of m machines is the focus of job shop scheduling difficulties. The machines work together to do the many tasks. Neither a single machine nor two machines can process the same work in the same period of time. There is no flow shop scheduling challenge when it comes to scheduling jobs in a job shop since each work has its own unique path through the set of equipment.

(e) Flexible flow shop scheduling

It is possible to extend the flow shop issues by addressing the flexible flow-shop scheduling challenge. As an alternative to traditional flow shop scheduling difficulties, FFSP has m stages with each stage having more than one machine conducting the same activity, but the speed of these machines may change.

(f) Flexible job shop scheduling

When it comes to work shop scheduling, flexible scheduling follows the same technology constraints as FFSP.

1.4 FLOW SHOP SCHEDULING MODEL

If you've got m machines (M1, M2, M3, Mm) deployed along a predetermined path, the flow shop scheduling model can handle your production environment, which includes these machines, as well as n tasks (J1, J2, J3, Jn) to be processed on them. The scheduler's major goal is to establish the best sequence in which these tasks should be processed on the available system of computers. Job J1 must go via machine Mi before machine Mi+1, and vice versa, due to the technical limitations of this environment. This holds true for all other tasks in the system as well. In addition, each finished task must be transferred to the next machine in the sequence. This technical limitation may also be referred to by the following acronym:

Jobs Processing route
J1 M1 M2 M3.......Mm
J2 M1 M2 M3.......Mm
....
Jn M1 M2 M3.......Mm

According to the above-described model, there are, m.(n!). The number of separate schedules in the flow shop scheduling environment is limited to n! In fact, even in medium-sized problems, the reduced number is rather big and recognized to be NP hard by Garey and Johnson [GJS76], Gonzalez and Shani [GS78b], Pinedo [Pin10], and many others. The flow shop scheduling issue for a system of m machines may be written as follows using the scheduling triplet notation: Fm/perm/. Typical performance metrics for the field include reducing total completion time (makes pan), limiting average lateness (or tardiness), minimizing maximum lateness, and maximizing the number of tardy tasks, minimizing average flow time, etc.

1.4.1 PERFORMANCE INDICATORS IN FLOW SHOP SCHEDULING

Models for scheduling flow shops include a plethora of goals, many of which are contradictory and difficult to achieve. Regularity is defined as the rate at which a performance indicator increases as a function of the completion time of all tasks. Schedules are designed to reduce the performance indicator. The literature on flow shop scheduling contains the most frequent indications of performance, which are also known as scheduling criteria. Regular performance metrics that get the greatest attention include:

(I) Make span (Cmax):

Make span, also known as elapsed time, refers to the greatest amount of time required to complete any given task schedule using the current machine system. A schedule's "makespan" is the time it takes to complete all of the jobs in the schedule from start to finish. Make span is a crucial scheduling tool in all manufacturing and production systems in order to boost productivity and optimism the exploitation of available resources (machines).

The permutation flow shop scheduling issue with minimal makes pan, Fm/perm/Cmax, may be modelled using mixed integer linear programming (MILP).

Min Z= $\sum_{i=1}^{m-1} \quad \sum_{j=1}^{n}(Yj1 * Pij) + \sum_{j=1}^{n-1} Imj + \sum_{j=1}^{n} Pmj$

Subject to

$I_{ik}+$

$\sum_{j=1}^{n}\big(Yj(k+1)\big) * P\,ij + Wi(k+1) - Wik - \sum_{j=1}^{n}(Yjk + P(i+1)j) - Ii + 1k =$

$0\ \forall k = 1,2..,n-1;\ i = 1,2..,m-1$

$\sum_{j=1}^{n} Yjk = 1\ \forall k = 1,2..,n$

$\sum_{k=1}^{n} Yjk = 1\ \forall j = 1,2..,n$

$W_{ik} =0\ \forall i = 1,2,..,m-1$

$I_{1k} =0\ \forall k - 1,2,..,n$

$Y_{jk} \in \{0,1\}\ \forall j = 1,2,\ldots,n;\ k = 1,2,.,n$

Waiting time for the work planned between machines I and I + 1) is Wik in this example. When a machine I is idle for a certain amount of time, it is known as iik I = 1) Yijk is an integer variable that has a range of zero to one.

1.4.2 BASIC RESULTS AND OBSERVATIONS:

If the goal is to plan n tasks in an available machine environment in the shortest amount of time feasible, the goal of flow shop scheduling issues is to minimize the overall completion time of these jobs. Flow shop scheduling has been researched extensively by scholars and practitioners, and the following findings have been established:

Result 1. If the sequence of n-jobs {J1, J2, ...Jn} is to be processed on the machinesM1 and M2 then optimal schedule minimizing the total elapsed time, Cmax, is given by following decision rule:

Job α precedes job β, if

min {p1α, p2β,} <min {p1β, p2α}

Result 2. F3/perm/Cmax, is strongly NP-hard.

Scheduling more than two machines in a flow shop setting with the goal of minimizing make-span is NP-hard, according to the aforementioned findings. This means that the producers and planners will have to settle for an approximation rather than a precise answer in this situation.

(II) Mean flow time (F¯): Any production or manufacturing system may be described as having a mean flow time. Making progress on the more difficult task of minimizing mean flow time is more difficult than making

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ISSN: 1309-3452

progress on the simpler issue of making pan smaller. For Fm/perm/F, the following findings have been generated during the last many decades.

Result 3. As part of the issue solving process, it is necessary to take into account the task schedules for the first two machines and for the final two machines.

Result 4. Even if the input length is calculated as the average of the job lengths, the scheduling issue for a two-machine flow shop is NP-complete.

(III) Total tardiness (ƐTj): Tardiness is measured by the amount of time it takes a project to complete beyond its due date. Customers now demand on-time delivery of their many purchases in the current production age. On-time delivery is thus the most important goal for all manufacturers and industrialists, since missing a deadline (or due date) may lead to a loss of customer interest and market competitiveness. Many producers and manufacturers have been paying attention to the scheduling issues with due date-related goals because of the rise in industrial rivalry.

A mathematical model for Fm/perm/$\sum_{j=1}^{n} Tj$ can be formulated as follows:

Min Z= $\sum_{j=1}^{n} Tj$

Subject to

$C_{(i+j)j}$ - $C_{ij} \geq \sum_{k=1}^{n}[Ykj * (Pi+1j+ \delta max)]$ $\forall i$ = 1,2,.., m-1;j=1,2,3,..n

$C_{(i+j)j}$ - $C_{ij} \leq \sum_{k=1}^{n}[(Ykj * (P(i+1)j+ \delta max)]$ $\forall i$ = 1,2,.., m-1;j=1,2,3,..n

C_{ij} - $C_{i(j-1)} \geq \sum_{k=1}^{n}(Ykj * Pij)$ $\forall i$ = 1, 2..., m; j=1,2, 3...n

T_j -C_{mj} + $\sum_{k=1}^{n}(dj * Yjk) \geq 0$ $\forall j = 1,2,3..,n$

$\sum_{j=1}^{n} Yjk = 1$ $\forall k = 1,2,..,n$

$\sum_{k=1}^{n} Yjk = 1$ $\forall j = 1,2,..,n$

$T_j \geq 0$ *and* $Y_{jk} \in \{0,1\}$ $\forall j = 1,2,..,n$; k=1, 2...n

Where δ_{ij}^{min} is minimum time lag and δ_{ij}^{max} maximal time lag between machines I and (i+j) while processing the job j ($\forall i$ = 1, 2..., m-1; j=1,2, 3...n)

Some Results and Observations:

Result 5. Let S is an optimal schedule of n jobs $\{J1, J2, ... Jn\}$ on one machine. For each job 1 $\leq k \geq$ n-1, if there is a job $J_i \in G_k(s)$ (set of job scheduled in S after k^{th} scheduled job) such that $p_{Ji} \leq p_{Jj}$ and $d_{Ji} <$ max (Ck (S) + p_{Jj}, d_{Jj})($\forall Jj \in Gk(S)$) then there is an optimal schedule S' such that F_l (S')=F_l(S) for each 1<l<k and F_{k+1}(S') is the job J_i

Result 6. Let σ be an optimal schedule of n jobs $\{J1, J2, ... Jn\}$ in single machine environment. For each job 1 $\leq$ k < n, if there is a job $J_j \in G_k$ (σ)(set of job scheduled in σ after k_{th} scheduled job) such that $p_{Ji} \geq p_{Jj}$ and d_{Jj} > $C_k(\sigma)$ + p_{Jj} ($\forall J_i \in G_k(\sigma)$) then there exist an optimal scheduled σ'such that $F_l(\sigma')$ = $F_l(\sigma)$ for each 1 < l< k.

Result 7. There is always an optimal scheduled $S\ for\ n\ job\ which\ is\ canonical\ and\ \sum_{j=1}^{n} Tj(S) \geq \omega_0$. Here, equality holds if and only if for two partitions of job $\{V11, V21, ..., VN2\}$ we have, $\sum_{j=1}^{n} V1j$=$\sum_{j=1}^{n} V2j$

Minimum manufacturing costs and maximum productivity are required concurrently in today's competitive climate, and this may be done via efficient resource utilization and due date concerns. The flow shop scheduling environment takes into account other performance indicators such as makes pan, flow time, and tardiness in addition to the usual hiring/leasing cost (the cost of renting/leasing the equipment) (Sharma, Gupta and Sharma [SGS12]).

Due to the rapid growth in competitive marketplaces, there is a danger of company failure in contemporary production and operations management. Industrialists also have to deal with the challenge of starting up innovative manufacturing businesses on the cheap. Renting manufacturing and service equipment is always preferable than buying it in these scenarios. As a real-world example, the health care sector necessitates the purchase of costly medical equipment, such as x-ray, ultrasound, and MRI machines, as well as patient monitoring devices and pulse oximeters, during the first phases of setting up health facilities. However, renting or leasing equipment is preferable than purchasing it outright. Hospitals may save money by renting medical equipment when they don't have enough money to purchase it. The major independent medical equipment leasing and rental organizations in the healthcare market are the Made One Group in the United States, Scottsdale's worldwide banking and leasing services industry, and many more. These businesses supply dealers with simple and customized leasing options that have a positive impact on their ability to sell. Renting/leasing also allows for the reduction of working capital and the use of cutting-edge technologies. The practical existence of multiple rental/leasing strategies in different industrial and manufacturing firms shows that minimizing rental/leasing costs is a significant goal of production scheduling. There are other factors that contribute to production costs when resources are taken on rent/lease, such as machinery, equipment, and tools. Today, the primary goal of contemporary industries and manufacturing organizations is to meet consumer demand in a timely manner with high utilization of production equipment and a cheap cost of production.

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