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Some New Separation Axioms Of (1,2)S_P-Open Sets In Bitopological Spaces

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ABSTRACT

In this paper, some new separation axioms in bitopological spaces using (1,2)S_p-open sets are defined and some of its characterizations are analysed.

Key Words:

(1,2)semi-open, (1,2)pre-open, (1,2)pre-closed, (1,2)S_p-open, (1,2)S_p-closed, (1,2)S_p-D space, (1,2)S_p-D_0 space, (1,2)S_p-D_1 space, (1,2)S_p-D_2 space.

AMS Classifications: 54A10.

1. INTRODUCTION

Semi-open sets were introduced and investigated by Levine [4] in 1963. In 1965, Njastad [7] studied certain classes of subsets in a topological space called α-sets and β-sets. In 1963, Kelly [3] initiated the study of the bitopological spaces, which is a triplet (X, τ_1, τ_2). Where X is a non-empty set and τ_1, τ_2 are topologies on X. In 1991, one type of open sets called a (1,2)α-open sets was defined and developed by Lellis Thivagar [6]. The purpose of this paper is to introduce new separation axioms and to discuss its various aspects by using (1,2)S_p-open sets.

. PRELIMINARIES

Definition 2.1 [5] A subset A of a bitopological space X is called a

(i) (1,2)semi-open if A ⊆ $\tau_1\tau_2$Cl($\tau_1 Int(A)$).

(ii) (1,2)pre-open if A ⊆ τ_1Int($\tau_1\tau_2 Cl(A)$).

(iii) (1,2)regular-open if A = τ_1Int($\tau_1\tau_2 Cl(A)$).

The collection of all (1,2)semi-open, (1,2)pre-open and (1,2)regular-open sets are denoted by (1,2)SO(X), (1,2)PO(X) and (1,2)RO(X) respectively.

Definition 2.2 [5] A subset A of a bitopological space X is called a

(i) (1,2)α-closed if τ_1Cl($\tau_1\tau_2$Int(τ_1Cl(A))) ⊆ A.
(ii) (1,2)semi-closed if $\tau_1\tau_2$Int(τ_1Cl(A)) ⊆ A.
(iii) (1,2)pre-closed if τ_1Cl($\tau_1\tau_2$Int(A)) ⊆ A.

(iv) (1,2)regular-closed if A = τ_1Cl($\tau_1\tau_2$Int(A)).

The set of all (1,2)α-closed, (1,2)semi-closed, (1,2)pre-closed and (1,2)regular-closed sets are denoted as (1,2)αCL(X), (1,2)SCL(X), (1,2)PCL(X) and (1,2)RCL(X) respectively. Also, for any subset A of X, the (1,2)α-closure, (1,2)semi-closure, (1,2)pre-closure and (1,2)regular-closure of A is denoted as (1,2)αCl(A), (1,2)SCl(A), (1,2)PCl(A) and (1,2)RCl(A) respectively.

Definition 2.3 [2] A (1,2)semi-open set A of a bitopological space X is called (1,2)S_p-open set if for each $x \in$ A, there exists a (1,2)pre-closed set F such that $x \in$ F ⊆ A.

Definition 2.4. [1**]** A bitopological space X is called (1,2)S_p-$\mathcal{T}_0$ space iff for every distinct points $x, y \in$ X there exists a (1,2)S_p-open set containing x but not y or a (1,2)S_p-open set containing y but not x.

Definition 2.5.[1**]** A bitopological space X is called (1,2)S_p-$\mathcal{T}_1$ space iff for every distinct points $x, y \in$ X there exists a (1,2)S_p-open set containing x but not y and a (1,2)S_p-open set containing y but not x.

Definition 2.6. [1**]** A bitopological space X is called (1,2)S_p-$\mathcal{T}_2$ space iff for every distinct points $x, y \in$ X there exists a (1,2)S_p-open sets U and V such that $x \in$ U and $y \in$ V.

2. (1,2)S_P-D_i(i = 0, 1, 2) IN BITOPOLOGICAL SPACES

Definition 3.1. A subset S of X is called a (1,2)S_p-difference set ((1,2)S_p-D set) if there exists two (1,2)S_p-open sets B_1 and B_2 such that S = ($B_1 \setminus B_2$) and $B_1 \neq$ X.

The family of all (1,2)S_p-D set is denoted by (1,2)S_p-D(X).

Remark 3.2. Every (1,2)S_p-open set is a (1,2)S_p-D set.

Proof. Let S be a (1,2)S_p-open set. Then there exist two (1,2)S_p-open sets B_1 and B_2 such that S = ($B_1 \setminus B_2$) with $B_2 = \Phi$. Hence S is a (1,2)S_p-D set.

Definition 3.3. A bitopological space X is called a (1,2)S_p-D_0 space if for $x, y \in$ X and $x \neq y$, there exists a (1,2)S_p-D set containing x but not y or containing y but not x.

Definition 3.4. A bitopological space X is called a (1,2)S_p-D_1 space if for $x, y \in$ X and $x \neq y$, there exist two (1,2)S_p-D sets G and H such that $x \in$ G, $x \notin$ H and $y \in$ H, $y \notin$ G.

Theorem 3.5. Every (1,2)S_p-$\mathcal{T}_0$ space is a (1,2)S_p-D_0 space and vice versa.

Proof. Every (1,2)S_p-$\mathcal{T}_0$ space is a (1,2)S_p-D_0 space by Remark 3.2.

Conversely, let X be (1,2)S_p-D_0 space. Then for every $x, y \in$ X and $x \neq y$, there exists a (1,2)S_p-D set S such that $x \in$ S and $y \notin$ S, where S = ($B_1 \setminus B_2$) and $B_1, B_2 \in$ (1,2)S_pO(X). Now, let $x \in$ S implies $x \in B_1$ and $x \notin B_2$. And, let $y \notin$ S implies $y \notin B_1$ or $y \in B_1$ and $\in B_2$. Hence the following two cases exist:

Case 1: $x \in B_1$ and $y \notin B_1$ implies X is (1,2)S_p-$\mathcal{T}_0$.

Case 2: $x \notin B_2$ and $y \in B_2$ implies X is (1,2)S_p-$\mathcal{T}_0$..

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Definition 3.6. A bitopological space X is called a (1,2)S_p-D$_2$ space if for x, y ∈ X and $x \neq y$, there exist two (1,2)S_p-D sets G and H such that x ∈ G and y ∈ H.

Remark 3.7. In a bitopological space X**,** the following holds good:

(i) If X is (1,2)S_p-D$_i$, then it is (1,2)S_p-D$_{i-1}$ (i = 1, 2).
(ii) If X is (1,2)S_p-$\mathcal{T}_i$, then it is (1,2)S_p-D$_i$ (i = 0, 1, 2).

Theorem 3.8. If a bitopological spaces X is (1,2)S_p-D$_1$ space, then it is a (1,2)S_p-$\mathcal{T}_0$ space.

Proof. Let X be (1,2)S_p-D$_1$space. Then X is (1,2)S_p-D$_0$ space. By Theorem 3.5, every (1,2)S_p-D$_0$ space is (1,2)S_p-$\mathcal{T}_0$ space. Hence X is (1,2)S_p-$\mathcal{T}_0$ space.

Remark 3.9. The converse of above Theorem 3.8 is not true as shown in the following example.

Example 3.10. Let X = {a, b, c, d}, τ_1 = {Φ, X, {a}, {c}, {a, c}, {a, b, d}}, τ_2 = {Φ, X, {d}}, (1,2)SO(X) = {Φ, X, {a}, {c}, {a, b}, {a, c}, {a, b, c}, {a, b, d}}, (1,2)PCL(X) = {X, Φ, {b}, {c}, {a, c}, {b, c}, {b, d}, {c, d}, {a, b, d}, {b, c, d}}, (1,2)S_pO(X) = {Φ, X, {c}, {a, c}, {a, b, c}, {a, b, d}}, (1,2)S_p-D(X) = {Φ, X, {a}, {b}, {c}, {a, b}, {a, c}, {a, b, c}, {a, b, d}}. Here, X is (1,2)S_p-$\mathcal{T}_0$ space but not (1,2)S_p-D$_1$ space.

Remark 3.11. The following example illustrates that (1,2)S_p-D_1 does not imply (1,2)S_p-$\mathcal{T}_1$ space.

Example 3.12. Let X = {a, b, c, d}, τ_1 = {Φ, X, {a}, {c}, {d}, {a, c}, {a, d}, {c, d}, {a, c, d}}, τ_2 = {Φ, X}, (1,2)SO(X) = {Φ, X, {a}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}, (1,2)PCL(X) = {X, Φ, {b}, {a, b}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}}, (1,2)S_pO(X) = {Φ, X, {a, b}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}}, (1,2)S_p-D(X) = {Φ, X, {a}, {c}, {d}, {a, b}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}}. Here, X is (1,2)S_p-D$_1$ space but not (1,2)S_p-$\mathcal{T}_1$ space.

Theorem 3.13. A bitopoloXgical space X is (1,2)S_p-D_1 space iff it is (1,2)S_p-D_2 space.

Proof. Let X is (1,2)S_p-D_2 space. Then X is (1,2)S_p-D_1 space (by Remark 3.7).

Conversely, suppose X is (1,2)S_p-D_1 space. Then for each x, y ∈ X and $x \neq y$, implies (1,2)S_p-D sets G_1 and G_2 such that $x \in G_1$, $x \notin G_2$ and $y \in G_2$, $y \notin G_1$, where $G_1 = (B_1 \setminus B_2)$ and $G_2 = (B_3 \setminus B_4)$, where $B_1, B_2, B_3, B_4 \in$ (1,2)S_pO(X). Now, $x \in G_1$implies $x \in B_1$ and $x \notin B_2$, $x \notin G_2$ implies $x \notin B_3$ or $x \in B_3$ and $x \in B_4$ and $y \in G_2$ implies $y \in B_3$ and $y \notin B_4$, $y \notin G_1$which implies $y \notin B_1$ or $y \in B_1$ and $y \in B_2$. Hence the following three cases exist.

Case (i): Let $x \notin G_2$ and $y \notin G_1$ which implies $x \notin B_3$ and $y \notin B_1$. If $x \in G_1$, then $x \in (B_1 \setminus B_2)$ implies $x \in [(B_1 \setminus B_3) \cup B_2]$ and if $y \in G_2$, then $y \in (B_3 \setminus B_4)$, which implies $y \in [(B_3 \setminus B_4) \cup B_1]$ and also $[(B_1 \setminus B_3) \cup B_2] \cap [(B_3 \setminus B_4) \cup B_1] = \Phi$.

Case (ii): Let $y \in B_1$ and $y \in B_2 \Longrightarrow x \in (B_1 \setminus B_2)$, $y \in B_2$ and $[(B_1 \setminus B_2) \cap B_2] = \Phi$.

Case (iii): Let $x \in B_3$ and $x \in B_4 \Longrightarrow y \in (B_3 \setminus B_4)$, $x \in B_4$ implies $[(B_3 \setminus B_4) \cap B_4] = \Phi$. Hence X is (1,2)$S_p$-$D_2$ space.

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