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Some Generalization of Micro S_P-Locally Closed Sets in Micro Topological Spaces

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ABSTRACT

In this paper, a new class of sets called Mic S_P-locally closed set in a Micro topological space is introduced. Also introduce their generalizations and study some of its properties.

Keywords: Micro S_P-open, Micro S_P-closed, Micro S_Pg-open, Micro S_Pg-closed, Micro S_P-locally closed, Micro S_Pg-locally closed.

1. Introduction

In 1966, the study of locally closed sets was introduced by Bourbaki [3]. In 1989, Ganster and Raily [6] studied locally closed sets extensively in topological spaces. In [1], Balachandran et. al introduced the concept of Generalization of locally closed sets in Topology. In [2], K. Bhuvaneswari et. al introduced Nano Generalized Locally closed sets in Nano topological spaces. In [1], Subramanian et.al introduced the classes of m-lc-set, mg-lc- set, mg-lc*- set, mg-lc**-set. In this paper, Micro S_P-locally closed set in Micro topological spaces is introduced and some of its generalizations are defined and analyzed.

2. Preliminaries

Definition 2.1. [9] Let (U, $\tau_R(X)$) be a Nano topological space. Then $\mu_R(X) = \{N \cup (N' \cap \mu): N, N' \in \tau_R(X)$ and $\mu \notin \tau_R(X)\}$ is called the Micro topology on U with respect to X. The triplet (U, $\tau_R(X)$, $\mu_R(X)$) is called Micro topological space and the elements of $\mu_R(X)$ are Micro open sets and the complement of Micro open set is called a Micro closed set.

Definition 2.3. [1] A subset A of a space (U, $\tau_R(X)$, $\mu_R(X)$) is called Micro generalized closed (briefly Mg-closed} set if MCl (A) $\subseteq$ T whenever A $\subseteq$ T and T is Micro open in (U, $\tau_R(X)$, $\mu_R(X)$). The complement of Mg-closed set is Mg-open set.

Definition 2.3. [9] For any A $\subseteq$ U, Mg-Int (A) is defined as the union of all Mg-open sets contained in A. i.e., Mg-Int (A) = $\cup$ {G: G $\subseteq$ A and G is Mg-open}.

Definition 2.4. **[9]** For any A $\subseteq$ U, Mg-Cl (A) is defined as the intersection of all Mg-closed sets containing A. i.e., Mg-Cl (A) = $\cap$ {F: F is Mg-closed and A $\subseteq$ F}.

Proposition 2.5. [1] For any A $\subseteq$ U, the following holds.

1) Mg-Int(A) is the largest Mg-open set contained in A.
2) A is Mg-open if and only if Mg-Int (A) = A
3) Mg-Int (A $\cap$ B) = Mg-Int (A) $\cap$ Mg-Int (B).
4) Mg-Int (A $\cup$ B) $\supseteq$ Mg-Int (A) $\cup$ Mg-Int (B).
5) If A $\subseteq$ B, then Mg-Int (A) $\subseteq$ Mg-Int (B).
6) Mg-Int (U) = U and Mg-Int (ϕ) = ϕ.
7) Mg-Cl (A) is the smallest Mg-closed set containing A.
8) A is Mg-closed if and only if Mg-Cl (A) = A.
9) If A $\subseteq$ B, then Mg-Cl (A) $\subseteq$ Mg-Cl (B).
10) Mg-Cl (A $\cap$ B) $\subseteq$ Mg-Cl (A) $\cap$ Mg- Cl (B).

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11) (Mg-Int (A))^c = Mg-Cl (A^c).
12) Mg-Int (A) = (Mg-Cl (A^c))^c.
13) Mg-Cl (A) = (Mg-Cl (A^c))^c.

Definition 2.6. [8] Let (U, τ_R(X), μ_R(X)) be a Micro topological space and B $\subseteq$ U is called Micro S_P-closed (briefly Mic S_P-closed) if and only if its complement is Micro S_P-open.

The set of all Micro S_P-closed sets is denoted by Mic S_P-CL (U, X).

Definition 2.7. [8] A subset A of a Micro topological space (U, τ_R(X), μ_R(X)) is said to be a Micro S_P-generalized closed (briefly Mic S_Pg-closed)set if Mic S_P-Cl (A) $\subseteq$ V whenever A $\subseteq$ V and V $\in$ Mic S_P-O (U, X).

The set of all Mic S_Pg-closed sets is denoted by Mic S_PG-CL (U, X).

Definition 2.8. [9] A subset A of a Micro topological space(U, τ_R(X), μ_R(X)) is called a Micro locally closed (briefly M-lc) set if A = V $\cap$ W where V is Micro open and W is Micro closed.

The class of all M-locally closed sets in a Micro topological space (U, τ_R(X), μ_R(X)) is denoted by MLC (U).

Definition 2.9. [9] A subset A of a Micro topological space (U, τ_R(X), μ_R(X)) is called a Micro generalized locally closed (briefly Mg-lc) set if A = E $\cap$ F where E is Mg-open and F is Mg-closed.

The class of all Mg-locally closed sets in Micro topological spaces (U, τ_R(X), μ_R(X)) is denoted by MGLC (U).

3. Micro S_P-Locally closed sets

Definition 3.1. A subset A of a Micro topological space (U, τ_R(X), μ_R(X)) is called a Micro S_P-locally closed (briefly Mic S_P-lc) set if A = C $\cap$ D where C is Mic S_P-open and D is Mic S_P-closed.

The family of all Micro S_P-locally closed sets in a Micro topological space is denoted by Mic S_P-LC (U, X).

Remark 3.2. Every Micro S_P-closed (resp. Micro S_P-open) set is Micro S_P-locally closed set but the converse need not always be true and is shown in the following example.

Example 3.3. Consider U = {p , q , r , s} with U|R = {{p}, {q , r , s}} and X = {p , q}. Then
τ_R(X) = {U, ϕ, {p}, {q}, {p , q}}. If μ = {s} then μ_R(X) = {U, ϕ, {p}, {q}, {s}, {p , q}, {p, s}, {q , s}, {p , q , s}}, Mic S_P-O (U, X) = {U, ϕ, {p, r}, {q, r}, {r, s}, {p, q, r}, {p, r, s}, {q , r , s}}, Mic S_P-CL (U, X) = {ϕ, U, {q, s}, {p, s}, {p, q}, {s}, {q}, {p}} and Mic S_P-LC (U, X) = {ϕ, U, {p}, {q}, {s}, {p, q}, {p, r}, {p, s}, {q, r}, {q, s}, {r, s}, {p, q, r}, {p, r, s}, {q, r, s}}.

Here {p, r} and {q, r} are in Mic S_P-LC (U, X) but not in Mic S_P-CL (U, X).

Definition 3.4. A subset A of a Micro topological space (U, τ_R(X), μ_R(X)) is called a Micro S_P-generally locally closed (briefly Mic S_Pg-lc) set if A = E $\cap$ F where E is Micro S_Pg-open and F is Micro S_Pg-closed.

The family of all Micro S_P-generalized locally closed sets in Micro topological space is denoted by Mic S_PG-LC (U, X).

Remark 3.5. Every Micro S_Pg-closed (Micro S_Pg-open) set is Micro S_Pg-locally closed set but the converse need not be true as shown in the following example.

Example 3.6. In example 3.3, Mic S_PG-CL (U, X) = { U , ϕ , {p}, {q}, {s} , {p , q} , {p , s}, {q , s}, {p , q , s}} and Mic S_PG-LC (U, X) = {U, ϕ, {p}, {q}, {r}, {s} , {p, q}, {p , r}, {p ,s},
{q ,r} , {q , s} , {r , s} , {p , q , r} , {p , r , s}, {q , r , s}, {p , q , s}}.

Here {p , q , r} $\in$ Mic S_PG-LC (U, X) but {p ,q , r} $\notin$ Mic S_Pg-CL (U, X).

Remark 3.7. Every Micro S_P-lc set is Micro S_Pg-lc set but the converse is not true which is shown by the following example.

Example 3.8. In example 3.3, Mic S_PG-CL (U, X) = {U, ϕ, {p} , {q} , {s} , {p , q} , {p , s} , {q , s}, {p , q , s}}, Mic S_PG-O (U, X) = {ϕ, U, {q , r , s}, {p , r , s}, {p , q , r}, {r, s}, {q , r}, {p, r} , {r}} and Mic S_PG-LC (U, X) = {U, ϕ, {p} , {q} , {s} , {p , q} , {p , r} , {p , s} , {q , r}, {q , s}, {r , s}, {p, q, r}, {p , r, s}, {q , r, s}}, Mic S_P-LC (U, X) = {U, ϕ, {p} , {q} , {s} , {p , q} , {p , r}, {p , s} , {q , r} , {q , s} , {r , s} , {p , q , r} , {p , r, s} , {q , r , s}}.

Here {r} $\in$ Mic S_PG-LC (U, X) but {r} $\notin$ Mic S_P-LC (U, X).

Definition 3.9. A subset A of a Micro topological space (U, τ_R(X), μ_R(X)) is called

1) Mic S_Pg-lc* set if A = B $\cap$ E, where B is Mic S_Pg-open in (U, τ_R(X), μ_R(X)) and E is Mic S_P-closed in (U, τ_R(X), μ_R(X)).

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2) Mic S_Pg-lc** set if A = C ∩ F, where C is Mic S_P-open in (U, $\tau_R(X)$, $\mu_R(X)$) and F is Mic S_Pg-closed in (U, $\tau_R(X)$, $\mu_R(X)$).

The collection of all Mic S_Pg-lc* (resp. Mic S_Pg-lc**) sets in a Micro topological space (U, $\tau_R(X)$, $\mu_R(X)$) is denoted by Mic S_PG-LC*(U, X) (resp. Mic S_PG-LC**(U, X)

Remarks 3.10.

(i) Every Mic S_P-lc set is Mic S_Pg-lc* set,
(ii) Every Mic S_P-lc set is Mic S_Pg-lc** set,
(iii) Every Mic S_P-lc* set is Mic S_Pg-lc set,
(iv) Every Mic S_Pg-lc** set is Mic S_Pg-lc set

The reverse of the above implications are may not be true as shown in the following example.

Example 3.11. In example 3.3, Mic S_P-LC (U, X) = { U, ϕ, {p} , {q} , {s} , {p , q} , {p , r}, {p , s} , {q , r}, {q , s}, {r , s}, {p , q , r} , {p , r , s} , {q , r, s}}, Mic S_PG-LC (U, X) = { U, ϕ, {p}, {q}, {r}, {s}, {p, q}, {p, r} , {p, s}, {q, r} {q, s}, {r, s},{p, q, r}, {p, r, s}, {q, r, s}, {p, q, s}}. Mic S_PG-LC* (U, X) = {U, ϕ, {p} , {q} , {r} , {s} , {p , q} , {p , r}, {p , s}, {q , r}, {q , s}, {r , s}, {p , q , r}, {p , q , s}, {p ,r , s} , {q , r , s}} and Mic S_PG-LC**(U, X) = {U, ϕ, {p}, {q}, {s}, {p, q}, {p, r}, {p, s}, {q, r},{q, s}, {r, s}, {p, q, r}, {p, q, s}, {p, r, s}, {q, r, s}.

(i) Here {r} ∈ Mic S_PG-LC* but {r} ∉ Mic S_P-LC.
(ii) Here {p,q,s} ∈ Mic S_PG- LC** but {p,q,s} ∉ Mic S_P-LC set.
(iii) Here {p,q,s} ∈ Mic S_PG-LC set but {p,q,s} ∉ Mic S_Pg-LC* set.
(iv) Here {r} ∈ Mic S_PG-LC set but {r} ∉ Mic S_Pg-LC** set.

4. Characterizations of Mic S_Pg-lc sets and Mic S_Pg-lc* sets and Mic S_Pg-lc** sets.

Theorem 4.1.

For a subset A of (U, $\tau_R(X)$, $\mu_R(X)$), the following statements are equivalent:

(i) A ∈ Mic S_PG-LC (U, X).
(ii) A = V ∩ Mic S_Pg-Cl (A) for some Mic S_Pg-open set V.
(iii) Mic S_Pg-Cl (A) – A is Mic S_Pg-closed.
(iv) A ∪ [(Mic S_Pg-Cl (A)]^c is Mic S_Pg-open.
(v) A ⊆ Mic S_Pg-Int [A ∪ (Mic S_Pg-Cl (A))^c]^c

Proof: (i) ⇒ (ii) Let A ∈ Mic S_PG-LC (U, X). Then A= V ∩ W where V is Mic S_Pg-open and W is Mic S_Pg-closed. Since A ⊆ W, Mic S_Pg-Cl (A) ⊆ W and so V ∩ Mic S_Pg-Cl (A) ⊆ V ∩ W = A. Also, since A ⊆ V and A ⊆ Mic S_Pg-Cl (A) which implies A ⊆ V ∩ Mic S_Pg-Cl (A). Therefore A = V ∩ Mic S_Pg-Cl (A).

(ii) ⇒ (iii) A = V ∩ Mic S_Pg-Cl (A) which implies Mic S_Pg-Cl (A) – A = Mic S_Pg-Cl (A) ∩ V^c which is Mic S_Pg-closed. Since V^c is Mic S_Pg-closed and Mic S_Pg- Cl (A) is Mic S_Pg-closed. Hence Mic S_Pg-Cl (A) – A is also Mic S_Pg-closed.

(iii) ⇒ (iv) A ∪ [Mic S_Pg-Cl (A)]^c = [Mic S_Pg-Cl (A) – A]^c and by assumption, [Mic S_Pg-Cl (A) – A]^c is Mic S_Pg-open and so is A ∪ [Mic S_Pg-Cl (A)]^c.

(iv) ⇒ (v) By assumption, A ∪ [Mic S_Pg-Cl (A)]^c = Mic S_Pg-Int [A ∪ (Mic S_Pg-Cl (A))^c] and hence A ⊆ Mic S_Pg-Int [A ∪ (Mic S_Pg-Cl (A))^c].

(v) ⇒ (i) By assumption and since A ⊆ [Mic S_Pg-Cl (A)], A = Mic S_Pg-Int[A ∪ (Mic S_Pg-Cl (A))^c] ∩ Mic S_Pg-Cl (A). Therefore A ∈ Mic S_PG-LC (U, X).

Theorem 4.2.

For a subset A of (U, $\tau_R(X)$, $\mu_R(X)$), the following statements are equivalent:

(i) A ∈ Mic S_PG-LC*(U, X).
(ii) A = V∩ Mic S_P-Cl (A) for some Mic S_Pg-open set V.
(iii) Mic S_P-Cl (A) – A is Mic S_Pg-closed.
(iv) A ∪ [Mic S_P-Cl (A)]^c is Mic S_Pg-open.

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Proof: (i) $\Rightarrow$ (ii) Let A $\in$ Mic S_PG-LC*(U, X). Then there exists a Mic S_Pg-open set V and Mic S_P-closed set W such that A = V $\cap$ W Since A $\subseteq$ V and A $\subseteq$ Mic S_P-Cl (A), A $\subseteq$ V $\cap$ Mic S_P-Cl (A). Also, since Mic S_Pg-Cl (A) $\subseteq$ W, V $\cap$ Mic S_P-Cl (A) $\subseteq$ V $\cap$ W = A. Therefore A= V $\cap$ Mic S_P-Cl (A).

(ii) $\Rightarrow$ (i) Since V is Mic S_Pg-open and Mic S_P-Cl (A) is Mic S_P-closed, A = V $\cap$ Mic S_P-Cl (A) $\in$ Mic S_PG-LC* (U, X).

(ii) $\Rightarrow$ (iii) Since Mic S_P-Cl (A) – A = Mic S_P-Cl (A) $\cap$ $V^{c,}$ Mic S_P-Cl (A) – A is Mic S_Pg-closed, since V^c is Mic S_Pg-closed.

(iii) $\Rightarrow$ (ii) Let V = [Mic S_P-Cl (A) – A]^c. Then by assumption V is Mic S_Pg-open in (U, $\tau_R(X)$, $\mu_R(X)$) and A = V $\cap$ Mic S_P-Cl (A).

(iii) $\Rightarrow$ (iv) Let W = Mic S_P-Cl (A) – A. Then W^c = A $\cup$ [Mic S_P-Cl (A)]^c and hence A $\cup$ [Mic S_P-Cl (A)]^c is Mic S_Pg-open.

(iv) $\Rightarrow$ (iii) Let V = A $\cup$ (Mic S_P-Cl (A)]^c . Then V^c is Mic S_Pg-closed and V^c = Mic S_P-Cl (A) – A and so Mic S_P-Cl (A) – A is Mic S_Pg-closed.

Theorem 4.3. Let A be a subset of (U, $\tau_R(X)$, $\mu_R(X)$). Then A $\in$ Mic S_PG-Lc** (U, X) if and only if A = V $\cap$ Mic S_Pg-Cl(A) for some Micro S_P-open set V.

Proof: Let A $\in$ Mic S_PG-LC** (U, X). Then A = V $\cap$ W. where V is Micro S_P-open and G is Micro S_Pg-closed. Since A $\subseteq$ W, Mic S_Pg-Cl (A) $\subseteq$ G. Thus A = A $\cap$ Mic S_Pg-Cl (A) = V $\cap$ W $\cap$ Mic S_Pg-Cl (A) = V $\cap$ Mic S_Pg-Cl (A).

Conversely, since V is Micro S_P-open and Mic S_Pg-Cl (A) is a Mic S_Pg-closed set, A = V $\cap$ Mic S_Pg-Cl (A) $\in$ Mic S_PG-LC** (U, X).

Corollary 4.4. Let A be a subset of (U, $\tau_R(X)$, $\mu_R(X)$). If A $\in$ Mic S_PG-LC** (U, X), then Mic S_Pg-Cl (A) – A is Mic S_Pg-closed and A $\cup$ [Mic S_Pg-Cl (A)]^c is Mic S_Pg-open.

Proof: Let A $\in$ Mic S_PG-LC** (U, X). Then by Theorem 4.3, A = V $\cap$ Mic S_Pg-Cl (A) for some Micro S_P-open set V and Mic S_Pg-Cl (A) – A = Mic S_Pg-Cl (A) $\cap$ V^c is Mic S_Pg-closed in (U, $\tau_R(X)$, $\mu_R(X)$). If W = Mic S_Pg-Cl (A) – A , then W^c = A $\cup$ [Mic S_Pg-Cl (A)]^c and W^c is Mic S_Pg-open and so is A $\cup$ [Mic S_Pg-Cl (A)]^c.

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