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Radio Harmonic Mean Labeling of Bistar Related Graphs

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ABSTRACT
A radio harmonic mean labeling of a connected graph G is a one to one map f from the vertex set V(G) to the set of natural numbers N such that for any two distinct vertices u and v of G satisfies the condition $d(u,v)+\left\lceil \frac{2\ f(u)f(v)}{f(u)+f(v)} \right\rceil \geq 1+diam(G)$ .The radio harmonic mean number of f, $rhmn(f)$ is the maximum number assigned to any vertex of G. The radio harmonic mean number of G, $rhmn(G)$ is the minimum value of $rhmn(f)$ taken over all radio harmonic mean labeling f of G. In this paper we have determined the radio harmonic mean number of special classes of bistar graphs.

Keywords: Radio harmonic mean labeling, Radio harmonic mean graph, Line graph of bistar, Square graph of bistar, Middle graph of bistar

1. Introduction

The channel assignment problems were introduced in 1980 by Hale [9]. The goal is to assign radio channels in a way so as to avoid interference between radio transmitters. Motivated by this Chartrand defined the concept of radio labeling of graphs in 2001[4]. Radio labeling, labels the vertices of a graph with non negative integers such that for any two vertices, the smaller the distance between the vertices, the greater the required difference in label. Radio labeling of graphs is applied in channel assignment problem, Sensor networks, TV and wireless networks etc... S.Ponraj et al. [12, 13] defined the concept of radio mean labeling of graphs. Motivated by the notion radio mean labeling we have introduced the radio harmonic mean labeling [2,3].In this paper, we have investigated the radio harmonic mean labeling of special classes of bistar graphs and determined the radio harmonic mean number of those graphs.
In this paper, Radio Harmonic Mean Labeling and Radio Harmonic Mean Number are referred as **RHML** and **RHMN** for briefness.

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2. *Preliminaries*

Definition 2.1

A radio harmonic mean labeling of a connected graph G is one to one map from the vertex set V(G) of G to N such that for two distinct vertices *u* and *v* of G satisfies the condition $d(u,v)+\left\lceil \frac{2\ f(u)f(v)}{f(u)+f(v)} \right\rceil \geq 1+diam(G)$.A graph which admits radio harmonic mean labeling is called radio harmonic mean graph.

Definition 2.2

Radio Harmonic mean number of graph G is denoted by $rhmn(G)$. It is defined as the lowest span taken over all radio labeling of graph G.

Definition 2.3

The span of a labeling f is the maximum integer that f maps to a vertex of graph G. The radio harmonic mean number of G, $rhmn(G)$ is the lowest span taken over all radio harmonic mean labeling of the graph G.

Definition 2.4

The **Line graph of a graph G,** denoted by *L(G),* is a graph whose vertices are the edges of G, and if $u,v \in E(G)$ then $u,v \in E(L(G))$ if *u* and *v* share a vertex in G.

Definition 2.5

For a simple connected graph G, **the square of graph G** is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in G.

3. Main Results

Theorem 3.1

The radio harmonic mean number of line graph of bistar $L[B_{n,n}]$ is $2n+1$ for $n \geq 3$.

Proof:

Let $V[L(B_{n,n})] = \left\{u, u_i, u_i^{'} : 1 \leq i \leq n\right\}$ and

$$E[L(B_{n,n})] = \left\{uu_i, uu_i^{'} : 1 \leq i \leq n\right\} \bigcup \left\{u_i u_j, u_i^{'} u_j^{'} : 1 \leq i \leq n-1,\ i+1 \leq j \leq n\right\}$$

The diameter of $L[B_{n,n}]$ is 2 for $n \geq 3$..

Define the vertex labels as follows

$$f(u) = 1$$

For $1 \leq i \leq n$,

$$f(u_i) = i+1$$

$$f(u_i^{'}) = n+1+i$$

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Now, check the radio harmonic mean condition.
In order to verify the definition of radio harmonic mean labeling

$$d(u,v)+\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \geq 3$$

for every pair of vertices of *L[Bn,n]*.

Case (i): Verify the pair (u,u_i) for $1 \leq i \leq n, d(u,u_i)=1$

$$d(u,u_i)+\left\lceil \frac{2f(u)f(u_i)}{f(u)+f(u_i)} \right\rceil = 1+\left\lceil \frac{2(1)(i+1)}{i+2} \right\rceil \geq 1+\left\lceil \frac{2(1)(2)}{3} \right\rceil \geq 3$$

Case (ii): Verify the pair (u_i,u_j) for $1 \leq i, j \leq n, j > i, d(u_i,u_j)=1$

$$d(u_i,u_j)+\left\lceil \frac{2f(u_i)f(u_j)}{f(u_i)+f(u_j)} \right\rceil = 1+\left\lceil \frac{2(i+1)(j+1)}{i+j+2} \right\rceil \geq 1+\left\lceil \frac{2(2)(3)}{5} \right\rceil \geq 4$$

Case (iii): Verify the pair $(u,u_i^{'})$ for $1 \leq i \leq n, d(u,u_i^{'})=1$

$$d(u,u_i^{'})+\left\lceil \frac{2f(u)f(u_i^{'})}{f(u)+f(u_i^{'})} \right\rceil = 1+\left\lceil \frac{2(1)(n+1+i)}{n+2+i} \right\rceil \geq 1+\left\lceil \frac{2(1)(5)}{6} \right\rceil \geq 3$$

Case (iv): Verify the pair $(u_i,u_j^{'})$ for $1 \leq i, j \leq n, d(u_i,u_j^{'})=2$

$$d(u_i,u_j^{'})+\left\lceil \frac{2f(u_i)f(u_j^{'})}{f(u_i)+f(u_j^{'})} \right\rceil = 2+\left\lceil \frac{2(i+1)(n+1+j)}{n+2+i+j} \right\rceil \geq 2+\left\lceil \frac{2(2)(5)}{7} \right\rceil \geq 5$$

Case (v): Verify the pair $(u_i^{'},u_j^{'})$ for $1 \leq i, j \leq n, j > i, d(u_i^{'},u_j^{'})=1$

$$d(u_i^{'},u_j^{'})+\left\lceil \frac{2f(u_i^{'})f(u_j^{'})}{f(u_i^{'})+f(u_j^{'})} \right\rceil = 1+\left\lceil \frac{2(n+1+i)(n+1+j)}{2n+2+i+j} \right\rceil \geq 1+\left\lceil \frac{2(5)(6)}{11} \right\rceil \geq 7$$

In all the above cases, *f* satisfies the radio harmonic mean condition (1).
Thus, *f* is a radio harmonic mean labeling and $L[B_{n,n}]$ is a radio harmonic mean graph.
Hence, the radio harmonic mean number of *L[Bn,n]* is 2*n*+2 for *n* ≥ 3.

Theorem 3.2

The radio harmonic mean number of Square graph of bistar $B_{n,n}^2$ is $2n+2$ for $n \geq 2$.

Proof:

Let $V(B_{n,n}^2)=\{u,u^{'},u_i,u_i^{'}: 1 \leq i \leq n\}$ and

$$E(B_{n,n}^2)=\{uu^{'},uu_i,uu_i^{'},u^{'}u_i,u^{'}u_i^{'}: 1 \leq i \leq n\}$$

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The diameter of $B_{n,n}^{2}$ is 2 for $n \geq 2$.

The vertex labels are defined by

$$f(u) = 1$$

$$f(u^{'}) = n+2$$

For $1 \leq i \leq n$,

$$f(u_i) = i+1$$

$$f(u_i^{'}) = n+2+i$$

Check the radio harmonic mean condition (1).

$$d(u,v) + \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \geq 3$$

for every pair of vertices of $B_{n,n}^{2}$.

Case (i): Verify the pair (u,u_i) for $1 \leq i \leq n, d(u,u_i) = 1$

$$d(u,u_i) + \left\lceil \frac{2f(u)f(u_i)}{f(u)+f(u_i)} \right\rceil = 1 + \left\lceil \frac{2(1)(i+1)}{i+2} \right\rceil \geq 1 + \left\lceil \frac{2(1)(2)}{3} \right\rceil \geq 3$$

Case (ii): Verify the pair (u_i, u_j) for $1 \leq i, j \leq n, j > i, d(u_i, u_j) = 2$

$$d(u_i,u_j) + \left\lceil \frac{2f(u_i)f(u_j)}{f(u_i)+f(u_j)} \right\rceil = 2 + \left\lceil \frac{2(i+1)(j+1)}{i+j+2} \right\rceil \geq 2 + \left\lceil \frac{2(2)(3)}{5} \right\rceil \geq 5$$

Case (iii): Verify the pair $(u,u^{'}),\ d(u,u^{'}) = 1$

$$d(u,u^{'}) + \left\lceil \frac{2f(u)f(u^{'})}{f(u)+f(u^{'})} \right\rceil = 1 + \left\lceil \frac{2(1)(n+2)}{n+3} \right\rceil \geq 1 + \left\lceil \frac{2(1)(4)}{5} \right\rceil \geq 3$$

Case (iv): Verify the pair $(u,u_i^{'})$ for $1 \leq i \leq n, d(u,u_i^{'}) = 1$

$$d(u,u_i^{'}) + \left\lceil \frac{2f(u)f(u_i^{'})}{f(u)+f(u_i^{'})} \right\rceil = 1 + \left\lceil \frac{2(1)(n+2+i)}{n+3+i} \right\rceil \geq 1 + \left\lceil \frac{2(1)(5)}{6} \right\rceil \geq 3$$

Case (v): Verify the pair $(u^{'},u_i^{'})$ for $1 \leq i \leq n, d(u^{'},u_i^{'}) = 1$

$$d(u^{'},u_i^{'}) + \left\lceil \frac{2f(u^{'})f(u_i^{'})}{f(u^{'})+f(u_i^{'})} \right\rceil = 1 + \left\lceil \frac{2(n+2)(n+2+i)}{2n+4+i} \right\rceil \geq 1 + \left\lceil \frac{2(4)(5)}{9} \right\rceil \geq 6$$

Case (vi): Verify the pair $(u^{'},u_i)$ for $1 \leq i \leq n, d(u^{'},u_i) = 1$

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$$d(u^{'},u_i)+\left\lceil \frac{2f(u^{'})f(u_i)}{f(u^{'})+f(u_i)} \right\rceil = 1+\left\lceil \frac{2(n+2)(i+1)}{n+3+i} \right\rceil \geq 1+\left\lceil \frac{2(4)(2)}{6} \right\rceil \geq 4$$

Case (vii): Verify the pair $(u_i, u_j^{'})$ for $1 \leq i,\ j \leq n, d(u_i, u_j^{'}) = 2$

$$d(u_i,u_j^{'})+\left\lceil \frac{2f(u_i)f(u_j^{'})}{f(u_i)+f(u_j^{'})} \right\rceil = 2+\left\lceil \frac{2(i+1)(n+2+j)}{n+3+i+j} \right\rceil \geq 2+\left\lceil \frac{2(2)(5)}{7} \right\rceil \geq 5$$

Case (viii): Verify the pair $(u_i^{'}, u_j^{'})$ for $1 \leq i,\ j \leq n,\ j > i, d(u_i^{'}, u_j^{'}) = 2$

$$d(u_i^{'},u_j^{'})+\left\lceil \frac{2f(u_i^{'})f(u_j^{'})}{f(u_i^{'})+f(u_j^{'})} \right\rceil = 2+\left\lceil \frac{2(n+2+i)(n+2+j)}{2n+4+i+j} \right\rceil \geq 2+\left\lceil \frac{2(5)(6)}{11} \right\rceil \geq 8$$

In all the above cases, *f* satisfies the radio harmonic mean condition (1).

Thus, *f* is a radio harmonic mean labeling and $B_{n,n}^2$ is radio harmonic mean graph.

Hence, the radio harmonic mean number of $B_{n,n}^2$ is $2n+2$ for $n \geq 2$.

Theorem 3.2

The radio harmonic mean number of Middle graph of bistar $M(B_{n,n})$ $4n+4$ for $n \geq 2$.

Proof:

Let $V[M(B_{n,n})] = \{u, v, w, u_i, u_i^{'}\ v_i, v_i^{'} : 1 \leq i \leq n\}$ and

$E[M(B_{n,n})] = \{uw, vw, uu_i, u_iu_i^{'}, u_iw, wv_i, vv_i, v_iv_i^{'} : 1 \leq i \leq n\}$

The diameter of $M(B_{n,n})$ is 4 for n≥2.

Define the vertex labels as follows

$f(u) = 4n+2$

$f(v) = 4n+4$

$f(w) = 4n+3$

For $1 \leq i \leq n$,

$f(u_i) = 3n+1+i$

$f(u_i^{'}) = i+1$

$f(v_i) = 2n+1+i$

$f(v_i^{'}) = n+1+i$

Check the radio harmonic mean condition (1).

In order to verify the definition of radio harmonic mean labeling

$$d(u,v)+\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \geq 5$$

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for every pair of vertices of $M(B_{n,n})$.

Case (1): Verify the pair (u,u_i) for $1\le i\le n, d(u,u_i)=1$

$$d(u,u_i)+\left\lceil \frac{2f(u)f(u_i)}{f(u)+f(u_i)}\right\rceil = 1+\left\lceil \frac{2(4n+2)(3n+1+i)}{7n+3+i}\right\rceil \ge 1+\left\lceil \frac{2(10)(8)}{18}\right\rceil \ge 10$$

Case (2): Verify the pair $(u,u_i^{'})$ for $1\le i\le n, d(u,u_i^{'})=2$

$$d(u,u_i^{'})+\left\lceil \frac{2f(u)f(u_i^{'})}{f(u)+f(u_i^{'})}\right\rceil = 2+\left\lceil \frac{2(4n+2)(i+1)}{4n+3+i}\right\rceil \ge 2+\left\lceil \frac{2(10)(2)}{12}\right\rceil \ge 6$$

Case (3): Verify the pair (u,w) , $d(u,w)=1$

$$d(u,w)+\left\lceil \frac{2f(u)f(w)}{f(u)+f(w)}\right\rceil = 1+\left\lceil \frac{2(4n+2)(4n+3)}{8n+5}\right\rceil \ge 1+\left\lceil \frac{2(10)(11)}{21}\right\rceil \ge 12$$

Case (4): Verify the pair (u,v) , $d(u,v)=2$

$$d(u,v)+\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)}\right\rceil = 2+\left\lceil \frac{2(4n+2)(4n+4)}{8n+6}\right\rceil \ge 2+\left\lceil \frac{2(10)(12)}{22}\right\rceil \ge 13$$

Case (5): Verify the pair (u,v_i) for $1\le i\le n, d(u,v_i)=2$

$$d(u,v_i)+\left\lceil \frac{2f(u)f(v_i)}{f(u)+f(v_i)}\right\rceil = 2+\left\lceil \frac{2(4n+2)(2n+1+i)}{6n+3+i}\right\rceil \ge 2+\left\lceil \frac{2(10)(6)}{16}\right\rceil \ge 10$$

Case (6): Check the pair $(u,v_i^{'})$ for $1\le i\le n, d(u,v_i^{'})=3$

$$d(u,v_i^{'})+\left\lceil \frac{2f(u)f(v_i^{'})}{f(u)+f(v_i^{'})}\right\rceil = 3+\left\lceil \frac{2(4n+2)(n+1+i)}{5n+3+i}\right\rceil \ge 3+\left\lceil \frac{2(10)(4)}{14}\right\rceil \ge 9$$

Case (7): Verify the pair (w,u_i) for $1\le i\le n, d(w,u_i)=1$

$$d(w,u_i)+\left\lceil \frac{2f(w)f(u_i)}{f(w)+f(u_i)}\right\rceil = 1+\left\lceil \frac{2(4n+3)(3n+1+i)}{7n+i+4}\right\rceil \ge 1+\left\lceil \frac{2(11)(8)}{19}\right\rceil \ge 11$$

Case (8): Verify the pair $(w,u_i^{'})$ for $1\le i\le n, d(w,u_i^{'})=2$

$$d(w,u_i^{'})+\left\lceil \frac{2f(w)f(u_i^{'})}{f(w)+f(u_i^{'})}\right\rceil = 2+\left\lceil \frac{2(4n+3)(i+1)}{4n+4+i}\right\rceil \ge 2+\left\lceil \frac{2(11)(2)}{13}\right\rceil \ge 6$$

Case (9): Verify the pair (v,w) for $1\le i\le n, d(v,w)=1$

$$d(v,w)+\left\lceil \frac{2f(v)f(w)}{f(v)+f(w)}\right\rceil = 1+\left\lceil \frac{2(4n+4)(4n+3)}{8n+7}\right\rceil \ge 1+\left\lceil \frac{2(12)(11)}{23}\right\rceil \ge 13$$

Case (10): Verify the pair (w,v_i) for $1\le i\le n, d(w,v_i)=1$

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$$d(w,v_i)+\left\lceil \frac{2f(w)f(v_i)}{f(w)+f(v_i)} \right\rceil = 1+\left\lceil \frac{2(4n+3)(2n+1+i)}{6n+4+i} \right\rceil \geq 1+\left\lceil \frac{2(11)(6)}{17} \right\rceil \geq 9$$

Case (11): Verify the pair $(w,v_i^{'})$ for $1 \leq i \leq n, d(w,v_i^{'})=2$

$$d(w,v_i^{'})+\left\lceil \frac{2f(w)f(v_i^{'})}{f(w)+f(v_i^{'})} \right\rceil = 2+\left\lceil \frac{2(4n+3)(n+1+i)}{5n+4+i} \right\rceil \geq 2+\left\lceil \frac{2(11)(4)}{15} \right\rceil \geq 8$$

Case (12): Verify the pair (v,v_i) for $1 \leq i \leq n, d(v,v_i)=1$

$$d(v,v_i)+\left\lceil \frac{2f(v)f(v_i)}{f(v)+f(v_i)} \right\rceil = 1+\left\lceil \frac{2(4n+4)(2n+1+i)}{6n+5+i} \right\rceil \geq 1+\left\lceil \frac{2(12)(6)}{18} \right\rceil \geq 9$$

Case (13): Verify the pair $(v,v_i^{'})$ for $1 \leq i \leq n, d(v,v_i^{'})=2$

$$d(v,v_i^{'})+\left\lceil \frac{2f(v)f(v_i^{'})}{f(v)+f(v_i^{'})} \right\rceil = 2+\left\lceil \frac{2(4n+4)(n+1+i)}{5n+5+i} \right\rceil \geq 2+\left\lceil \frac{2(12)(4)}{16} \right\rceil \geq 8$$

Case (14): Verify the pair (u_i,u_j) for $1 \leq i, j \leq n, j>i, d(u_i,u_j)=2$

$$d(u_i,u_j)+\left\lceil \frac{2f(u_i)f(u_j)}{f(u_i)+f(u_j)} \right\rceil = 2+\left\lceil \frac{2(3n+1+i)(3n+1+j)}{6n+2+i+j} \right\rceil \geq 2+\left\lceil \frac{2(8)(9)}{17} \right\rceil \geq 11$$

Case (15): Verify the pair $(u_i,u_j^{'})$ for $1 \leq i, j \leq n, d(u_i,u_j^{'})=1$

$$d(u_i,u_j^{'})+\left\lceil \frac{2f(u_i)f(u_j^{'})}{f(u_i)+f(u_j^{'})} \right\rceil = 1+\left\lceil \frac{2(3n+1+i)(j+1)}{3n+2+i+j} \right\rceil \geq 1+\left\lceil \frac{2(8)(2)}{10} \right\rceil \geq 5$$

Case (16): Verify the pair $(u_i^{'},u_j^{'})$ for $1 \leq i, j \leq n, j>i, d(u_i^{'},u_j^{'})=4$

$$d(u_i^{'},u_j^{'})+\left\lceil \frac{2f(u_i^{'})f(u_j^{'})}{f(u_i^{'})+f(u_j^{'})} \right\rceil = 4+\left\lceil \frac{2(i+1)(j+1)}{i+j+2} \right\rceil \geq 4+\left\lceil \frac{2(2)(3)}{15} \right\rceil \geq 7$$

Case (17): Verify the pair (v_i,v_j) for $1 \leq i, j \leq n, j>i, d(v_i,v_j)=2$

$$d(v_i,v_j)+\left\lceil \frac{2f(v_i)f(v_j)}{f(v_i)+f(v_j)} \right\rceil = 2+\left\lceil \frac{2(2n+1+i)(2n+1+j)}{4n+2+i+j} \right\rceil \geq 2+\left\lceil \frac{2(6)(7)}{13} \right\rceil \geq 9$$

Case (18): Verify the pair $(v_i^{'},v_j^{'})$ for $1 \leq i, j \leq n, j>i, d(v_i^{'},v_j^{'})=4$

$$d(v_i^{'},v_j^{'})+\left\lceil \frac{2f(v_i^{'})f(v_j^{'})}{f(v_i^{'})+f(v_j^{'})} \right\rceil = 4+\left\lceil \frac{2(n+1+i)(n+1+j)}{2n+2+i+j} \right\rceil \geq 4+\left\lceil \frac{2(4)(5)}{9} \right\rceil \geq 9$$

Case (19): Verify the pair $(u_i,v_j^{'})$ for $1 \leq i, j \leq n, d(u_i,v_j^{'})=3$

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$$d(u_i, v_j^{'}) + \left\lceil \frac{2f(u_i)f(v_j^{'})}{f(u_i)+f(v_j^{'})} \right\rceil = 3 + \left\lceil \frac{2(3n+1+i)(n+1+j)}{4n+2+i+j} \right\rceil \geq 3 + \left\lceil \frac{2(8)(4)}{12} \right\rceil \geq 9$$

Case (20): Verify the pair (u_i, v_j) for $1 \leq i, j \leq n, d(u_i, v_j) = 2$

$$d(u_i, v_j) + \left\lceil \frac{2f(u_i)f(v_j)}{f(u_i)+f(v_j)} \right\rceil = 2 + \left\lceil \frac{2(3n+1+i)(2n+1+j)}{5n+2+i+j} \right\rceil \geq 2 + \left\lceil \frac{2(8)(6)}{14} \right\rceil \geq 9$$

Case (21): Verify the pair (v, u_i) for $1 \leq i \leq n, d(v, u_i) = 2$

$$d(v, u_i) + \left\lceil \frac{2f(v)f(u_i)}{f(v)+f(u_i)} \right\rceil = 2 + \left\lceil \frac{2(4n+4)(3n+1+i)}{7n+5+i} \right\rceil \geq 2 + \left\lceil \frac{2(12)(8)}{20} \right\rceil \geq 12$$

Case (22): Verify the pair $(v, u_i^{'})$ for $1 \leq i \leq n, d(v, u_i^{'}) = 3$

$$d(v, u_i^{'}) + \left\lceil \frac{2f(v)f(u_i^{'})}{f(v)+f(u_i^{'})} \right\rceil = 3 + \left\lceil \frac{2(4n+4)(i+1)}{4n+5+i} \right\rceil \geq 3 + \left\lceil \frac{2(12)(2)}{14} \right\rceil \geq 7$$

Case (23): Verify the pair $(v_i, v_j^{'})$ for $1 \leq i, j \leq n, d(v_i, v_j^{'}) = 1$

$$d(v_i, v_j^{'}) + \left\lceil \frac{2f(v_i)f(v_j^{'})}{f(v_i)+f(v_j^{'})} \right\rceil = 1 + \left\lceil \frac{2(2n+1+i)(n+1+j)}{3n+2+i+j} \right\rceil \geq 1 + \left\lceil \frac{2(6)(4)}{10} \right\rceil \geq 6$$

Case (24): Verify the pair $(u_i^{'}, v_j)$ for $1 \leq i, j \leq n, d(u_i^{'}, v_j) = 3$

$$d(u_i^{'}, v_j) + \left\lceil \frac{2f(u_i^{'})f(v_j)}{f(u_i^{'})+f(v_j)} \right\rceil = 3 + \left\lceil \frac{2(i+1)(2n+1+j)}{2n+2+i+j} \right\rceil \geq 3 + \left\lceil \frac{2(2)(6)}{8} \right\rceil \geq 6$$

Case (25): Verify the pair $(u_i^{'}, v_j^{'})$ for $1 \leq i, j \leq n, j > i, d(u_i^{'}, v_j^{'}) = 4$

$$d(u_i^{'}, v_j^{'}) + \left\lceil \frac{2f(u_i^{'})f(v_j^{'})}{f(u_i^{'})+f(v_j^{'})} \right\rceil = 4 + \left\lceil \frac{2(i+1)(n+1+j)}{n+2+i+j} \right\rceil \geq 4 + \left\lceil \frac{2(2)(4)}{6} \right\rceil \geq 7$$

In all the above cases, *f* satisfies the radio harmonic mean condition (1).
Thus, *f* is a radio harmonic mean labeling and $M(B_{n,n})$ is a radio harmonic mean graph.
Hence, the radio harmonic mean number of $M(B_{n,n})$ is $4n+4$ for $n \geq 2$.

4.Conclusion

In this paper, the radio harmonic mean number of bistar related graphs was obtained. The radio labeling of graphs which would have played a very important role in the communication networks.

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References

[1] Amuthavalli.K and Dineshkumar.S, Radio odd mean number of complete graphs.International Journal of Pure and Applied mathematics, Volume 113, No 7,2017,8-15.
[2] Amuthavalii.K and Revathy.R, Radio harmonic mean labeling of some trees, Compliance engineering Journal, Vol 10,Issue 12,Dec 2019.
[3] Amuthavalli.K and Revathy.R,Radio harmonic mean labeling of splitting graph of Star, Adalya journal,Volume 9, Issue 1, Jan 2020.
[4] Chartrand.G, Erwin.D, Zhang.P and Harary.F, Radio labeling of graphs, Bull. Inst. Combin. Appl.33 (2001), 77 – 85.
[5] Chartrand.G, Erwin.D and Zhang.P, A graph labeling problem suggested by FM channel restrictions, Bull. Inst. Combin. Appl. 43 (2005) 43-57
[6] Dineshkumar.S and Amuthavalli.K , Radio odd mean number of Split Graph of Star and Bistar, International Journal of Research in Engineering application & Management, Volume-04, No 5, 2018,2652-2658.
[7] Dineshkumar.S and Amuthavalli.K, Radio odd mean number of Shadow Graph of Star and Bistar, International Journal of Research in Advent Technology, Vol-6, No10, 2018.
[8] Gallian J.A, A dynamic survey of graph labeling, Electron. J. Combin. 19 (2019), #DS6.
[9] Hale W.K, Frequency assignment: theory and applications, Proc. IEEE 68 (1980), 1497-1514
[10] Harary.F, Graph Theory, Addison Wesley, New Delhi (1969).
[11] HemalathaV, Mohanaselvi.V and Amuthavalli .K, Radio Geometric mean number of splitting graph of Star and Bistar, International Journal of Research in Advent Technology, Vol.6, No.7, pp.1696-1700 (2018).
[12] Ponraj.R, Sathish Narayanan.S and Kala.R, Radio mean labeling of a graph, AKCE International Journal of Graphs and Combinatorics 12 (2015), 224 – 228.
[13] Ponraj.R, Sathish Narayanan.S and Kala.R, On radio mean number of graphs, International J.Math. Combin. 3(2014), 41 – 48.