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Variational Iteration Method Approach for Fuzzy Initial Value Problem

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ABSTRACT

An application of variational iteration method(VIM) for linear and non-linear differential equation with fuzzy initial conditions is introduced. Using the VIM , it is very possible to find the exact or an approximate solution for the most of the proposed initial value problems without complications and with complete ease .Convergence analysis of the proposed method and also the maximum absolute truncation error are proved. Some illustrative numerical example are introduced to confirm the validity and powerfulness of the VIM through the results obtained.

Keywords: Variational iteration method, Fuzzy differential equation, Fuzzy number

1. Introduction

In the past years, both physicists and mathematicians have devoted considerable effort to fined stable and strong analytical and numerical methods for solving fuzzy differential eq- uations(FDEs) of physical interest[1]. Ji – Huan He proposed the VIM to find the solution of a differential equations by using an iterative scheme [2-4]. Several researches in series of scientific fields applied this method as well, Wazwaz [5] using the VIM for solving linear and nonlinear Volterra integral and integro-differential equations, Subashini [6] blended Sumudu transform and VIM to solve delay differential equations (DDEs) , Nabaa [7] presented VIM to approximate the solution of nonlinear fractional order ordinary differential equations, in [8] Harir, Melliani,
El Harfi and Chadli presented VIM and Transformation Method to find a solution for the model of nonlinear ordinary differential equations (ODEs) describing the so-called coronavirus (COVID-19), Mathankumar and Poornima[9] used VIM to solve the system of fuzzy Volterra integro-differential equations of first kind. Modified VIM applied in Faribrzi [10] and Tamer [11] for nonlinear ODEs. Some researchers have presented other methods for solving FDEs such as, Smita[12-13] improved Euler type method and orthogonal polynomials to obtain numerical solution of FDEs, Narayanamoorthy [14] proposed Adomian decomposition to solve fuzzy DDEs, Mine and Emine [15] presented Milne's predictor – corrector method to find numerical solution of FDEs , Mayada [16] suggested Runge-Kutta algorithms for solving FDEs to mention only a few.

The aim of this work is applied VIM to solve the linear and nonlinear FDEs, comparsion of VIM with another methods such as Adams , Milne and orthogonal polynomials, also convergence analysis is proved. This paper is organized as follows: In Sec.2, some basic definitions of fuzzy set theory are brought. In Sec.3, the VIM procdure for solving FDE is explained. In Sec.4, the convergence analysis and an estimation of the maximum absolute error is present. In Sec.5, several numerical examples are selected for solving by VIM to verify its efficiency and validity. Finally, in the last section a brief conclusion is drawn.

2. Preliminaries

Some fundamental definitions of fuzzy set theory are recalled in this section.

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Definition 2.1[19] If X is a collection of objects with generic element denoted by x,then a fuzzy set $\widetilde{A}$ in X with the membership function $\mu_{\widetilde{A}}(x)$ is a set of ordered pairs:

$\widetilde{A} = \{ (x, \mu_{\widetilde{A}}(x)) | x \in X \}$

Dfinition 2.2[19] The $\alpha -$ level set of fuzzy set $\widetilde{A}$ is the crisp set of elements that belong to the fuzzy set $\widetilde{A}$ at lest to the degree α i.e.

$A_\alpha = \{ x \in X \mid \mu_{\widetilde{A}}(x) \geq \alpha \}$

Definition 2.3:[18] A fuzzy number V is represented by an ordered pair of function $(\underline{V}(r), \overline{V}(r))$, $0 \leq r \leq 1$,which satisfies all the following requirements

(i) $\underline{V}(r)$ is a bounded, left continuous and non-decreasing function over $[0, 1]$

(ii) $\overline{V}(r)$ is a bounded, left continuous and non-increasing function over $[0, 1]$

(iii) $\underline{V}(r) \leq \overline{V}(r)$, $0 \leq r \leq 1$

3. Variational Iteration Method of FDE

To illustrate the basic idea of VIM technique we consider the general nonlinear equation:

$$L[u(t)] + N[u(t)] = g(t), \tag{1}$$

where L is represent a linear operator, N is represent a nonlinear operator, and g(t) is a given continuous function. The correction functional for Eq (1) is constructed as [1]

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda \{L[u_n(s)] + N[\tilde{u}_n(s)] - g(s)\} ds, \tag{2}$$

Where the λ symbol is represents a general Lagrange's multiplier which one can be recognised optimally via vartiational theory, u_n is the n^{th} approximate solution and $\tilde{u}_n$ is said to be a restricted variation, i.e., $\delta\tilde{u}_n = 0$. It is obligatory to determine the Lagrange multiplier λ which can be identified optimally by using a restricted variation and integration by parts. The general formula for Lagrange multiplier λ for the n^{th} order differential equation it was found in [5] is of the form:

$$\lambda = (-1)^n \frac{(s-t)^{(n-1)}}{(n-1)!} \tag{3}$$

Now after determined the Lagrange multiplier λ, the successive approximation u_{n+1} will be obtained through using any selective function u_0. Consequently, the solution is obtained by taking the limit:

$$u(t) = \lim_{n\to\infty} u_n(t) \tag{4}$$

More generally, so easy to see that the VIM can be simply extended to the n^{th} order FDE

$$u^{(n)}(t) + f\left(t, u(t), u'(t), u''(t), \dots u^{(n)}(t)\right) = 0, \quad t \in [0, 1]$$
$$\tilde{u}^{(k)}(0) = c_k \quad , \ 0 \leq k \leq n-1 . \tag{5}$$

where $c_k = (\underline{c}_k, \overline{c}_k)$, are given fuzzy number. To solve Eq. (5) , by using VIM we have,

$$\underline{\lambda}(s,t) = \overline{\lambda}(s,t) = (-1)^n \frac{(s-t)^{(n-1)}}{(n-1)!} \tag{6}$$

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and as a result, the iteration formula will be derived as following:

$$\underline{u}_{n+1}(t,r) = \underline{u}_n(t,r) + \int_0^t (-1)^n \frac{(s-t)^{(n-1)}}{(n-1)!} \left\{\frac{d^n}{ds^n} \underline{u}_n(s,r) + f\left(s, \underline{u}_n(s,r), \underline{u}_n{}'(s,r), \underline{u}_n{}''(s,r), \dots, \underline{u}_n{}^{(n)}(s,r)\right)\right\} ds \quad (7)$$

$$\overline{u}_{n+1}(t,r) = \overline{u}_n(t,r) + \int_0^t (-1)^n \frac{(s-t)^{(n-1)}}{(n-1)!} \left\{\frac{d^n}{ds^n} \overline{u}_n(s,r) + f\left(s, \overline{u}_n(s,r), \overline{u}_n{}'(s,r), \overline{u}_n{}''(s,r), \dots, \overline{u}_n{}^{(n)}(s,r)\right)\right\} ds$$

where $\underline{u}_0(t,r) = \sum_{k=0}^{n-1} \frac{\underline{c}_k}{k!} t^k$ and $\overline{u}_0(t,r) = \sum_{k=0}^{n-1} \frac{\overline{c}_k}{k!} t^k$.

4. Convergence Analysis

The VIM approach is derived using mathematical induction of Cauchy sequence in Banach space, according to that maximum absolute truncation error is estimated.

Theorem (4.1): the sequence u_{n+1} is converges to exact solution of the problem (1) if $\exists\, 0 \le q \le 1$ such that $\|B[u] - B[v]\| \le q\|u - v\|$, $u, v \in X$
where B is an operator from a Banach space X to X defined as

$$B[u(t)] = u(t) + \int_0^t \lambda \{L[u(s)] + N[u(s)] - g(s)\}\, ds.$$

Proof: Define the sequence u_{n+1} for arbitrary initial term c_0 as,

$$u_0 = c_0,\ \ u_1 = B[u_0],\ \ u_2 = B[u_1] \dots u_{n+1} = B[u_n]$$

To show $\|u_{n+1} - u_n\| \le q^n \|u_1 - u_0\|$

if $n = 0$ then $\|u_1 - u_0\| \le q^0 \|u_1 - u_0\|$, is clearly true

assume that $\|u_{n+1} - u_n\| \le q^n \|u_1 - u_0\|$, is true

$$\|u_{n+2} - u_{n+1}\| = \|B[u_{n+1}] - B[u_n]\| \le q\|u_{n+1} - u_n\| \le q\, q^n \|u_1 - u_0\| = q^{n+1}\|u_1 - u_0\|,$$

then its true for all $n \ge 0$.

suppose that $m > n$, $\forall\, m, n \in N$, then

$$\begin{aligned}\|u_m - u_n\| &\le \|u_m - u_{m-1}\| + \|u_{m-1} - u_{m-2}\| + \dots + \|u_{n+1} - u_n\| \\ &\le q^{m-1}\|u_1 - u_0\| + q^{m-2}\|u_1 - u_0\| + \dots + q^n\|u_1 - u_0\| \\ &\le q^n \|u_1 - u_0\| \left(1 + q + q^2 + \dots + q^{m-n-1}\right) \\ &= \frac{1-q^{m-n}}{1-q} q^n \|u_1 - u_0\|, \text{ since } 0 \le q \le 1, \text{ we get,}\end{aligned}$$

$\lim_{m,n\to\infty} \|u_m - u_n\| = 0$. thus, the sequence u_{n+1} is a Cauchy sequence in the Banach space X.

Theorem (4.2): If the sequence u_{n+1} is convergent as in theorem(4.1) then the maximum absolute truncation error of u_{n+1} is estimated by the following inquality

$$E_n(t) = \|u(t) - u_n(t)\| \leq \frac{q^n}{1-q}\|u_1 - u_0\|$$

Proof: From theorem (4.1), we have,

$$\|u_m - u_n\| \leq \frac{1-q^{m-n}}{1-q} q^n \|u_1 - u_0\|$$

Now, as $m \to \infty$ we have, $u_m(t) \to u(t)$. So,

$$\|u(t) - u_n(t)\| \leq \frac{1-q^{m-n}}{1-q} q^n \|u_1 - u_0\|.$$

Also, since $0 \leq q \leq 1$ then $(1 - q^{m-n}) \leq 1$. Therefore we have,

$$\|u(t) - u_n(t)\| \leq \frac{q^n}{1-q} \|u_1 - u_0\|$$

5. Numerical examples

Three test examples are illustrated below to solve fuzzy differential equations by VIM where, $y(x,r)$ the approximate solution by the proposed method and the error $= |Y(x,r) - y(x,r)|$, where $Y(x,r)$ the exact solution.

Example(5.1): Consider the first order FDE:

$$y'(t) = -y(t), \quad t \in [0,1]$$
$$\tilde{y}(0) = (0.96 + 0.04r, 1.01 - 0.01r) \tag{8}$$

the exact fuzzy solution [15]:

$$\underline{Y}(t,r) = (0.96 + 0.04r)e^{-t}$$
$$\overline{Y}(t,r) = (1.01 - 0.01r)e^{-t} \tag{9}$$

Now to apply the VIM, we will be rewrite Eqs. (8) in the form

$$L[\,\underline{y}\,(t)\,] + N[\,\underline{y}\,(t)\,] = 0,$$
$$L[\,\overline{y}\,(t)\,] + N[\,\overline{y}\,(t)\,] = 0 \tag{10}$$

where $L[\,\underline{y}\,(t)\,] = \frac{d}{dt}\,\underline{y}\,(t) + \underline{y}\,(t)$, $L[\,\overline{y}\,(t)\,] = \frac{d}{dt}\overline{y}\,(t) + \overline{y}\,(t)$ symbolize the linear terms and $N[\,\underline{y}\,(t)\,] = 0, N[\,\overline{y}\,(t)\,] = 0$ for the nonlinear terms. Thus, by taking the variation with respect to $\underline{y}_n$ and $\overline{y}_n$, and noticing that $\delta\underline{y}_n(0,r) = \delta\overline{y}_n(0,r) = 0$,

$$\delta\underline{y}_{n+1}(t,r) = \delta\underline{y}_n(t,r) + \delta\int_0^t \underline{\lambda}(s,t)\left\{\frac{d}{ds}\,\underline{y}_n\,(s,r) + \underline{y}_n\,(s,r)\right\} ds$$
$$= \delta\underline{y}_n(t,r) + \underline{\lambda}(s,t)\delta\underline{y}_n\,(s,r)|_0^t + \int_0^t\left\{-\frac{\partial}{\partial s}\underline{\lambda}(s,t) + \underline{\lambda}(s,t)\right\}\delta\underline{y}_n\,(s,r)\,ds = 0,$$

$$\delta\overline{y}_{n+1}(t,r) = \delta\overline{y}_n(t,r) + \delta\int_0^t \overline{\lambda}(s,t)\left\{\frac{d}{ds}\,\overline{y}_n(s,r) + \overline{y}_n(s,r)\right\} ds$$
$$= \delta\overline{y}_n(t,r) + \overline{\lambda}(s,t)\delta\overline{y}_n\,(s,r)|_0^t + \int_0^t\left\{-\frac{\partial}{\partial s}\overline{\lambda}(s,t) + \overline{\lambda}(s,t)\right\}\delta\overline{y}_n\,(s,r)\,ds = 0.$$

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Thus Euler-Lagrange equations [3] given by

$$-\frac{\partial \underline{\lambda}(s,t)}{\partial s}+\underline{\lambda}(s,t)=0 \quad , \quad -\frac{\partial \overline{\lambda}(s,t)}{\partial s}+\overline{\lambda}(s,t)=0 \tag{11}$$

with natural boundary conditions:

$$1+\underline{\lambda}(t,t)=0 \quad , \quad 1+\overline{\lambda}(t,t)=0 \tag{12}$$

Now, by solve (11) and substituting the natural boundary conditions (12), we get:

$$\underline{\lambda}(s,t)=\overline{\lambda}(s,t)=-e^{(s-t)}. \tag{13}$$

As a results, the iteration formulation is:

$$\underline{y}_{n+1}(t,r)=\underline{y}_n(t,r)-\int_0^t e^{(s-t)}\left\{\frac{d}{ds}\underline{y}_n(s,r)+\underline{y}_n(s,r)\right\}ds,$$
$$\overline{y}_{n+1}(t,r)=\overline{y}_n(t,r)-\int_0^t e^{(s-t)}\left\{\frac{d}{ds}\overline{y}_n(s,r)+\overline{y}_n(s,r)\right\}ds, \quad n\geq 0. \tag{14}$$

If we begin with $\underline{y}_0(t,r)=0.96+0.04r$ and $\overline{y}_0(t,r)=1.01-0.01r$, then

$$\underline{y}_1(t,r)=(0.96+0.04r)e^{-t}$$
$$\overline{y}_1(t,r)=(1.01-0.01r)e^{-t}$$

which is represent the exact explicit solution itself. Results at $t=0.1$ are shown, in table (1)-(2), where lower approximation $\underline{y}(t,r)$, lower exact $\underline{Y}(t,r)$, upper approximation $\overline{y}(t,r)$, upper exact $\overline{Y}(t,r)$. Error of VIM compare with error of Adams, Milne methods given in [15]

Table 1: Lower results of example 5.1 at $t=0.1$

r	$\underline{Y}(t,r)$	$\underline{y}(t,r)$	$\underline{y}$(VIM Error)	$\underline{y}$(Adams Error)	$\underline{y}$(Milne Error)
0	0.8686439	0.8686439	0.0000000	0.0002857	0.0000082
0.2	0.8758826	0.8758826	0.0000000	0.0002286	0.0000065
0.4	0.8831213	0.8831213	0.0000000	0.0001714	0.0000049
0.6	0.8903600	0.8903600	0.0000000	0.0001143	0.0000032
0.8	0.8975987	0.8975987	0.0000000	0.0000571	0.0000016
1	0.9048374	0.9048374	0.0000000	0.0000000	0.0000000

Table 2: Upper results of example 5.1 at $t=0.1$

r	$\overline{Y}(t,r)$	$\overline{y}(t,r)$	$\overline{y}$(VIM Error)	$\overline{y}$(Adams Error)	$\overline{y}$(Milne Error)
0	0.9138857	0.9138857	0.0000000	0.0002857	0.0000082
0.2	0.9120761	0.9120761	0.0000000	0.0002286	0.0000065
0.4	0.9102664	0.9102664	0.0000000	0.0001714	0.0000049
0.6	0.9084567	0.9084567	0.0000000	0.0001143	0.0000032

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0.8	0.9066470	0.9066470	0.0000000	0.0000571	0.0000016
1	0.9048374	0.9048374	0.0000000	0.0000000	0.0000000

It is clear from table 1 and 2 that the one-iteration of VIM approximate solution is more accurate than results from the Adams and Milne numerical solution when compared of with exact solution of Eq.(8) when $t = 0.1$ for all $r \in [0,1]$.

Example(5.2): Consider the second order FDE [13]:

$$y''(t) - 4y'(t) + 4y(t) = 0, \qquad t \in [0,1]$$
$$\tilde{y}(0) = (2 + r, 4 - r) \qquad (15)$$
$$\tilde{y}'(0) = (r + 5\,, 7 - r)$$

The exact fuzzy solution of Eq.(15) is as follows:

$$\underline{Y}(t,r) = (2 + r)e^{2t} + (1 - r)te^{2t},$$
$$\overline{Y}(t,r) = (4 - r)e^{2t} + (r - 1)te^{2t}. \qquad (16)$$

To solve Eq.(15) by means of VIM, we can obtain the Lagrange multiplier $\underline{\lambda}(s,t) = \overline{\lambda}(s,t) = (s - t)$, and the following variational iteration can be obtained

$$\underline{y}_{n+1}(t,r) = \underline{y}_n(t,r) + \int_0^t (s - t)\left\{\frac{d^2}{ds^2}\underline{y}_n(s,r) - 4\frac{d}{dt}\underline{y}_n(s,r) + 4\underline{y}_n(s,r)\right\} ds,$$
$$\overline{y}_{n+1}(t,r) = \overline{y}_n(t,r) + \int_0^t (s - t)\left\{\frac{d^2}{ds^2}\overline{y}_n(s,r) - 4\frac{d}{dt}\overline{y}_n(s,r) + 4\overline{y}_n(s,r)\right\} ds \qquad n \geq 0,$$

If we start with the initial approximation guesses of Eq.(15) $\underline{y}_0(t,r) = (2 + r) + (5 + r)t$ and $\overline{y}_0(t,r) = (4 - r) + (7 - r)t$. Then the results of example (5.2) at $t = 0.01$, and error of VIM compare with error of methods given in [13] are shown in Table 3 and 4.

Table 3: Lower results of Example 5.2 at $t = 0.01$.

r	$\underline{Y}$(Exact)	$\underline{y}$(VIM)	$\underline{y}$(VIM) Error	$\underline{y}$ (Legendra) Error	$\underline{y}$ (Chebyshev) Error
0	2.050604693	2.050604692	0.000000001	0.000461836	0.000461835
0.2	2.252604558	2.252604558	0.000000000	0.000490272	0.000490272
0.6	2.656604289	2.656604288	0.000000001	0.000547146	0.000547146
0.8	2.858604154	2.858604154	0.000000000	0.000575582	0.000575582
1	3.060604020	3.060604019	0.000000001	0.000604020	0.000604020

Table 4. Upper results of Example 5.2 at $t = 0.01$.

r	$\overline{Y}$(Exact)	$\overline{y}$(VIM)	$\overline{y}$(VIM) Error	$\overline{y}$ (Legendra) Error	$\overline{y}$ (Chebyshev) Error
0	4.070603346	4.070603346	0.000000000	0.000746203	0.000746204
0.2	3868603481	3868603480	0.000000001	0.000717767	0.000717767
0.6	3.464603750	3.464603750	0.000000000	0.000660893	0.000660893

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0.8	3.262603885	3.262603884	0.000000001	0.000632457	0.000632457
1	3.060604020	3.060604019	0.000000001	0.000604020	0.000604020

We can see that From table 3 and 4 that the 3- iterations of VIM approximate solution is more accurate than results from the numerical solution of methods given in [13] when compared of with exact solution of Eq.(15) when $t = 0.01$ for all $r \in [0,1]$.

Example(5.3): Consider the nonlinear FDE [17] :

$$y'(t) = y^2(t), \quad t \in [0,1]$$
$$\tilde{y}(0) = (0.4 + 0.2r, 0.9 - 0.3r) \tag{17}$$

with the exact fuzzy solution

$$\underline{Y}(t,r) = \frac{0.4+0.2r}{1-(0.4+0.2r)t}$$
$$\overline{Y}(t,r) = \frac{0.9-0.3r}{1-(0.9-0.3r)t} \tag{18}$$

Now, to solve Eq.(17) by means of VIM, we can obtain the Lagrange multiplier $\underline{\lambda}(s,t) = \overline{\lambda}(s,t) = -1$ .As a results, we obtain the following iteration formulation:

$$\underline{y}_{n+1}(t,r) = \underline{y}_n(t,r) - \int_0^t \left\{\frac{d}{ds}\underline{y}_n(s,r) - \underline{y}_n{}^2(s,r)\right\} ds,$$
$$\overline{y}_{n+1}(t,r) = \overline{y}_n(t,r) - \int_0^t \left\{\frac{d}{ds}\overline{y}_n(s,r) - \overline{y}_n{}^2(s,r)\right\} ds \quad n \geq 0,$$

If we start with the initial approximation $\underline{y}_0(t,r) = 0.4 + 0.2r$ and $\overline{y}_0(t,r) = 0.9 - 0.3r$, then the VIM approximation solution of example (5.3) at $t = 0.01$ are shown in Table 5.

Table 5. Upper and Lower VIM Approximate and Exact Solution of Example 5.3 at $t = 0.01$.

r	$\underline{Y}$ (Exact)	$\underline{y}$ (VIM)	$\overline{Y}$ (Exact)	$\overline{y}$ (VIM)	$\underline{y}$ (VIM Error)	$\overline{y}$ (VIM Error)
0	0.40160642570	0.40160642570	0.90817356205	0.90817356205	0.00000000000	0.00000000000
0.2	0.44194455604	0.44194455604	0.84711577248	0.84711577248	0.00000000000	0.00000000000
0.4	0.48231511254	0.48231511254	0.78613182826	0.78613182825	0.00000000000	0.00000000000
0.6	0.52271813429	0.52271813429	0.72522159548	0.72522159548	0.00000000000	0.00000000000
0.8	0.56315366049	0.56315366049	0.66438494060	0.66438494060	0.00000000000	0.00000000000
1	0.60362173038	0.60362173038	0.60362173038	0.60362173038	0.00000000000	0.00000000000

The results of table 5 are obtained by using 4-iterations of approximate solution which explaining the accurate of the method when compared of with exact solution of Eq.(15) at $t = 0.01$.

Comparison of VIM approximate and exact solutions for the given examples are showing in the following figures.

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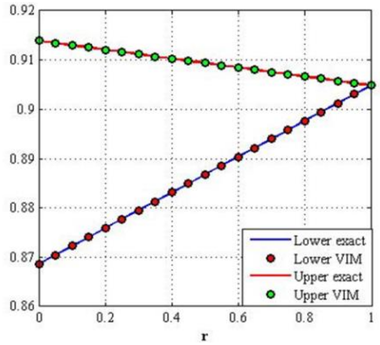

Fig.1

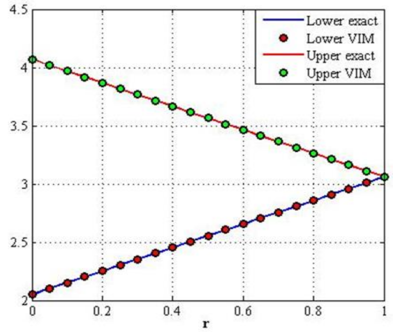

Fig.2

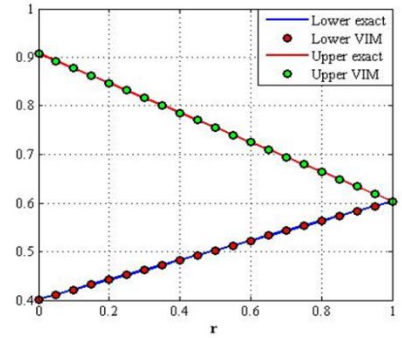

Fig.3

Where, figure 1, figure 2 and figure 3 represent the results of example (5.1) at $t = 0.1$, example (5.2) at $t = 0.01$ and example (5.3) at $t = 0.01$ respectively .Which shows the accuracy, efficiency and validity of the VIM through the comparison between the exact solution and approximate solution .

We use $r \in [0,1]$ where we calculate the error of the obtained fuzzy solution and exact solution with VIM. Tables show the convergence conduct of the method.the exact and obtained solution of fuzzy differential equations at $t = 0.1$ or $t = 0.01$ and $r \in [0,1]$ are shown in figures, also the results compare with results in [13] and [15] to show the convergence of the method.

6. Conclusion

It is easy to see that the VIM gives fast convergent successive approximations without any transformation, through determining the general Lagrange multipliers. The theorems of onvergence and maximum absolute truncation error estimation have been discussed. The numerical results , of the proposed method have been presented, and compared the obtained results with the existing results to show the efficiency and powerfulness of the method. As a future work we suggest fuzzy delay differantioal equations.

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