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Prime labeling of Franklin graph

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ABSTRACT
A graph G = (V, E) with n vertices is said to accept prime labelling if each pair of adjacent vertices can be labelled with different positive numbers not exceeding n, such that the label of each pair of adjacent vertices are relatively prime. Prime graph is a graph G that allows prime labelling. In this study, we look into prime labelling for a few different graph types. We focused on Franklin graph prime labelling in particular.

Keywords-Franklin graph, graph labelling, prime labelling, duplication, switching, and path union.

1.Introduction

All graphs considered here are finite, simple, undirected, connected and non – trivial graph. The graph G has vertex set V=$V(G)$ and the edge set E=$E(G)$. The number of elements of V, denoted as |V| called the order of the graph while the number of elements of E, denoted as |E| called the size of the graph. For notation and terminology we refer to J.A Bondy and U.S.R.Murthy [1]. The notion of the prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout.A(1982P365-368) [2]. Lee S(1998P59-67)[6] have proved that wheel W_n is a prime graph iff n is even. In [5] S.Meena and Vaithelingam have proved that the prime labeling for some fan related graphs .In [8] Dr V.Ganesan etal "Prime labeling of split graph of Star K1,n".In [9] Dr V.Ganesan proved "prime labeling of split graph of cycle C_n".

We will give brief summary of definitions and other information which are useful for the present task.

1. Preliminary definitions

Definition 1.1

Let G = (V(G), E(G)) be graph with P vertices. A bijection $f: V(G) \rightarrow \{1,2, \dots \dots \dots |V| \}$ is called a prime labeling if for each edge $e = uv, \gcd\big(f(u), f(v)\big) = 1$. A graph which admits prime labeling is called prime graph.

Definition 1.2

The Franklin graph is a 3-regular graph with 12 vertices and 18 edges.

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Definition 1.3

Duplication of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex v'_k with $N(v'_k) = N(v_k)$. In other words a vertex v'_k is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to v'_k.

Definition 1.4

A Vertex Switching G_v of a graph G is obtained by taking a vertex v of G, removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 1.5

Let $G_1, G_2, G_3, \dots \dots G_n$ be n copies of a fixed graph G. The graph obtained by adding an edge between G_i and G_{i+1} for i =1,2,…..n-1 is called the path union of G.

Illustration 1.6

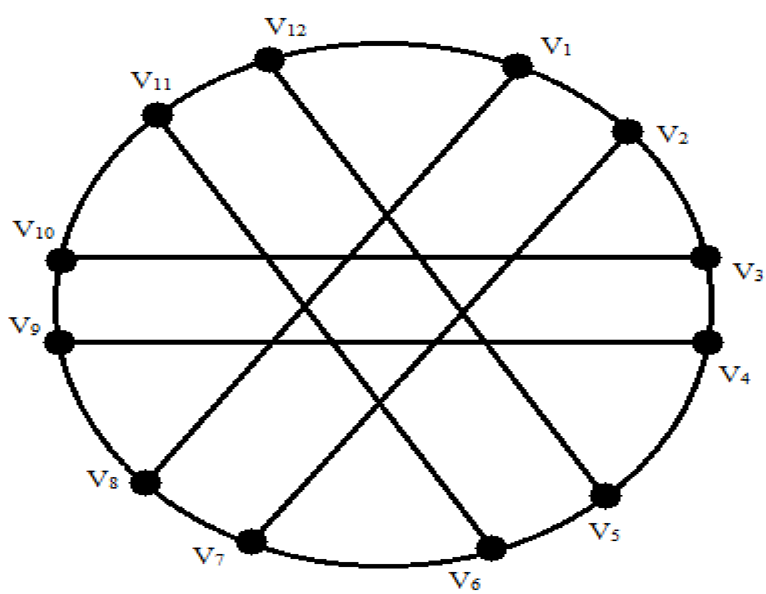

Figure 1.1 The Franklin graph

2.Main result

Theorem 2.1

The Franklin graph FG is a prime graph.

Proof

Let FG be the Franklin graph with 12 vertices and 18 edges. Let FG be the Franklin graph.

V(FG) = $\{v_1, v_2, v_3, v_4, \dots \dots \dots . v_{12}\}$

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E(FG) = $\{v_i v_{i+1}/1 \leq i \leq 11\} \cup \{v_{12}v_1\} \cup \{v_i v_{9-i}/1 \leq i \leq 2\} \cup \{v_i v_{13-i}/3 \leq i \leq 4\} \cup$

$\{v_i v_{17-i}/5 \leq i \leq 6\}$

|V(FG)|=12 and |E(FG)|=18

We define a function f: V(FG) →{1,2,3,……..12}

f(v_i) = i 1≤ $i \leq 12$

The relative prime of adjacent vertices have to be verify

We look at the following types of edges:

$\gcd\big(f(v_i), f(v_{i+1})\big) = 1$ for $1 \leq i \leq 11$

$\gcd\big(f(v_{12}), f(v_1)\big) = 1$

$\gcd\big(f(v_i), f(v_{9-i})\big) = 1$ for $1 \leq i \leq 2$

$\gcd\big(f(v_i), f(v_{13-i})\big) = 1$ for $3 \leq i \leq 4$

$\gcd\big(f(v_i), f(v_{17-i})\big) = 1$ for $5 \leq i \leq 6$

As a result, f meets the prime labelling condition
FG accept prime labelling.
As a result, FG is a prime graph.

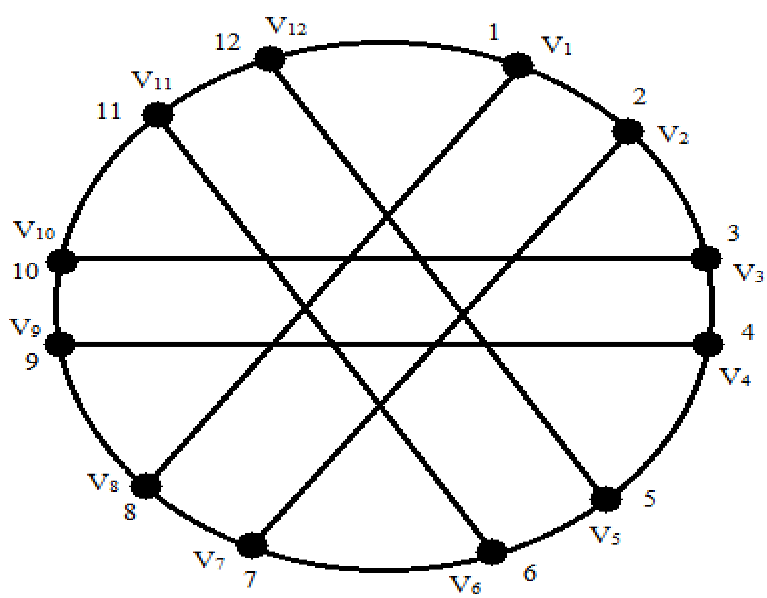

Fig 1.2 The Franklin graph admits prime labeling.

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Theorem 2.2

In a Franklin graph, the duplication of any vertex of degree 3 allows prime labelling.

Proof

Consider the Franklin graph, which has 12 vertices and 18 edges.

Let G be the graph generated by duplicating any vertex of degree 3 in the Franklin graph from FG.We can consider v_1 to be the duplicating vertex, and let v_1' be the duplication vertex $of\ \ v_1$.

V(G)=$\{v_1', v_1, v_2, v_3, v_4, \dots \dots \dots \dots .. v_{12}\}$

E(G) = $\{v_i v_{i+1}/1 \leq i \leq 11\} \cup \{v_{12} v_1\} \cup \{v_i v_{9-i}/1 \leq i \leq 2\} \cup \{v_i v_{13-i}/3 \leq i \leq 4\} \cup$

$\{v_i v_{17-i}/5 \leq i \leq 6\} \cup \{v_2 v_1'\} \cup \{v_{12} v_1'\} \cup \{v_8 v_1'\}$

Then |V(G)|=13 and |E(G)|=21

Define a function f:V(G)→{1,2,3,4,…….13}

Let $f(v_1') = 13$

$f(v_i) = i$ for 1≤ $i \leq 12$

We have to verify the relative prime of adjacent vertices

gcd$\big(f(v_i), f(v_{i+1})\big) = 1$ for $1 \leq i \leq 11$

gcd$\big(f(v_{12}), f(v_1)\big) = 1$

gcd$\big(f(v_i), f(v_{9-i})\big) = 1$ for $1 \leq i \leq 2$

gcd$\big(f(v_i), f(v_{13-i})\big) = 1$ for $3 \leq i \leq 4$

gcd$\big(f(v_i), f(v_{17-i})\big) = 1$ for $5 \leq i \leq 6$

gcd$\big(f(v_2), f(v_1')\big) = 1$

gcd$\big(f(v_{12}), f(v_1')\big) = 1$

gcd$\big(f(v_8), f(v_1')\big) = 1$

f fulfil the prime labelling condition
As a result, G admits prime labelling.
Hence G is a prime graph.

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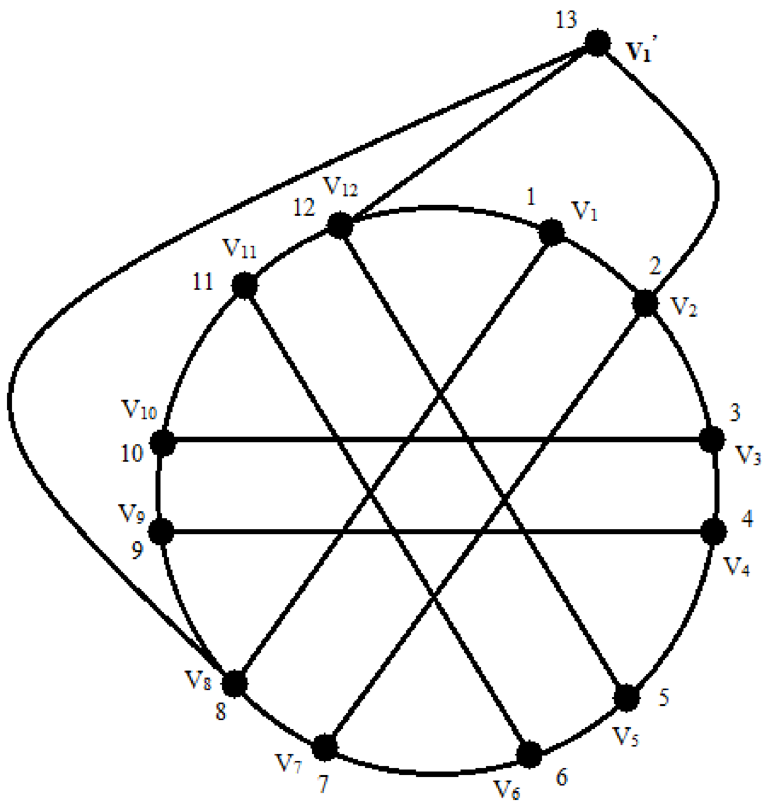

Figure 1.3 Duplication of the vertex v_1 in Franklin graph and its prime labeling

Theorem 2.3

The graph obtained by Switching of a vertex v_1 in a Franklin graph admits prime labeling.

Proof

Let FG be the Franklin graph with 12 vertices and 18 edges

G_u denotes the graph obtained by vertex switching of FG with respect to the vertex v_1

$\mathrm{V}(G_u) = \{v_1, v_2, v_3, v_4, v_5 \ldots \ldots \ldots \ldots \ldots v_{12}\}$

$\mathrm{E}(G_u) = \{v_i v_{i+1}/2 \leq i \leq 11\} \cup \{v_2 v_7\} \cup \{v_i v_{13-i}/3 \leq i \leq 4\} \cup \{v_i v_{17-i}/5 \leq i \leq 6\} \cup$

$\{v_1 v_{2+i}/1 \leq i \leq 5\} \cup \{v_1 v_{8+i}/1 \leq i \leq 4\}$

It is obvious that $|\mathrm{V}(G_u)| = 12$ and $|\mathrm{V}(G_u)| = 23$

Define a labeling f:V(G_u)→{1,2,3,……..12} as follows

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$f(v_i) = i$ for $1 \leq i \leq 12$

We have to verify the relative prime of adjacent vertices

$\gcd(f(v_i), f(v_{i+1})) = 1$ for $2 \leq i \leq 11$

$\gcd(f(v_2), f(v_7)) = 1$

$\gcd(f(v_i), f(v_{13-i})) = 1$ for $3 \leq i \leq 4$

$\gcd(f(v_i), f(v_{17-i})) = 1$ for $5 \leq i \leq 6$

$\gcd(f(v_1), f(v_{2+i})) = 1$ for $1 \leq i \leq 5$

$\gcd(f(v_1), f(v_{8+i})) = 1$ for $1 \leq i \leq 4$

Thus f is a prime labeling and consequently G_u is a prime graph

Therefore the switching of a vertex v_1 in a Franklin graph admits prime labeling.

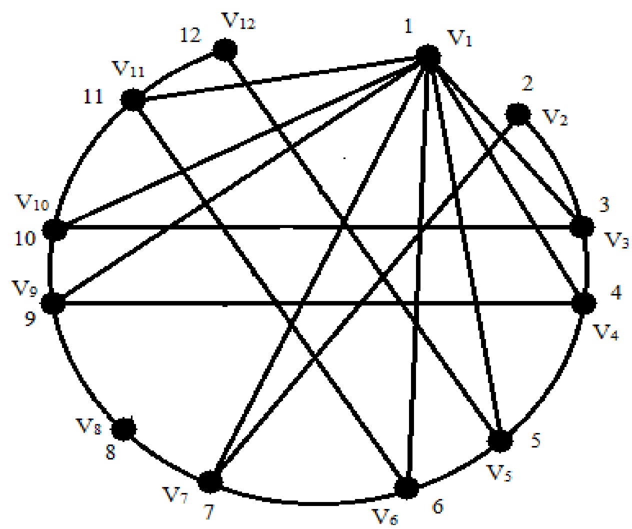

Fig 1.4 Switching of the vertex v_1 in Franklin graph admits prime labeling

Theorem 2.4

The graph obtained by path union of two pieces of Franklin graph admits prime labeling.

Proof

Consider two copies of Franklin graph FG and FG* respectively.

Let V(FG) = $\{v_1, v_2, v_3, \dots \dots \dots . v_{12}\}$

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E(FG) = $\{v_i v_{i+1}/1 \leq i \leq 11\} \cup \{v_{12} v_1\} \cup \{v_i v_{9-i}/1 \leq i \leq 2\} \cup \{v_i v_{13-i}/3 \leq i \leq 4\} \cup$

$\{v_i v_{17-i}/5 \leq i \leq 6\}$

V(FG*) = $\{u_1, u_2, u_3, \ldots \ldots \ldots . u_{12}\}$

E(FG*) = $\{u_i u_{i+1}/1 \leq i \leq 11\} \cup \{u_{12} u_1\} \cup \{u_i u_{9-i}/1 \leq i \leq 2\} \cup \{u_i u_{13-i}/3 \leq i \leq 4\} \cup$

$\{u_i u_{17-i}/5 \leq i \leq 6\}$

Let G_K be the graph obtained by the path union of two pieces of franklin graphs FG and FG*

V(G_K) = V(FG)∪V(FG*)

E(G_K) = E(FG)∪E(FG*) ∪ $\{v_1 u_1\}$

Define a labeling f:V(G_K)→ {1,2,3, … … .24} as follows

$f(v_i) = i \quad 1 \leq i \leq 12$

$f(u_i) = 12 + i \quad 1 \leq i \leq 12$

We have to verify the relative prime of adjacent vertices

gcd$\big(f(v_i), f(v_{i+1})\big) = 1$ for $1 \leq i \leq 11$

gcd$\big(f(v_{12}), f(v_1)\big) = 1$

gcd$\big(f(v_i), f(v_{9-i})\big) = 1$ for $1 \leq i \leq 2$

gcd$\big(f(v_i), f(v_{13-i})\big) = 1$ for $3 \leq i \leq 4$

gcd$\big(f(v_i), f(v_{17-i})\big) = 1$ for $5 \leq i \leq 6$

gcd$\big(f(u_i), f(u_{i+1})\big) = 1$ for $1 \leq i \leq 11$

gcd$\big(f(u_{12}), f(u_1)\big) = 1$

gcd$\big(f(u_i), f(u_{9-i})\big) = 1$ for $1 \leq i \leq 2$

gcd$\big(f(u_i), f(u_{13-i})\big) = 1$ for $3 \leq i \leq 4$

gcd$\big(f(u_i), f(u_{17-i})\big) = 1$ for $5 \leq i \leq 6$

gcd$\big(f(v_1), f(u_1)\big) = 1$

Thus f admits a prime labeling

Hence G_k is a prime graph

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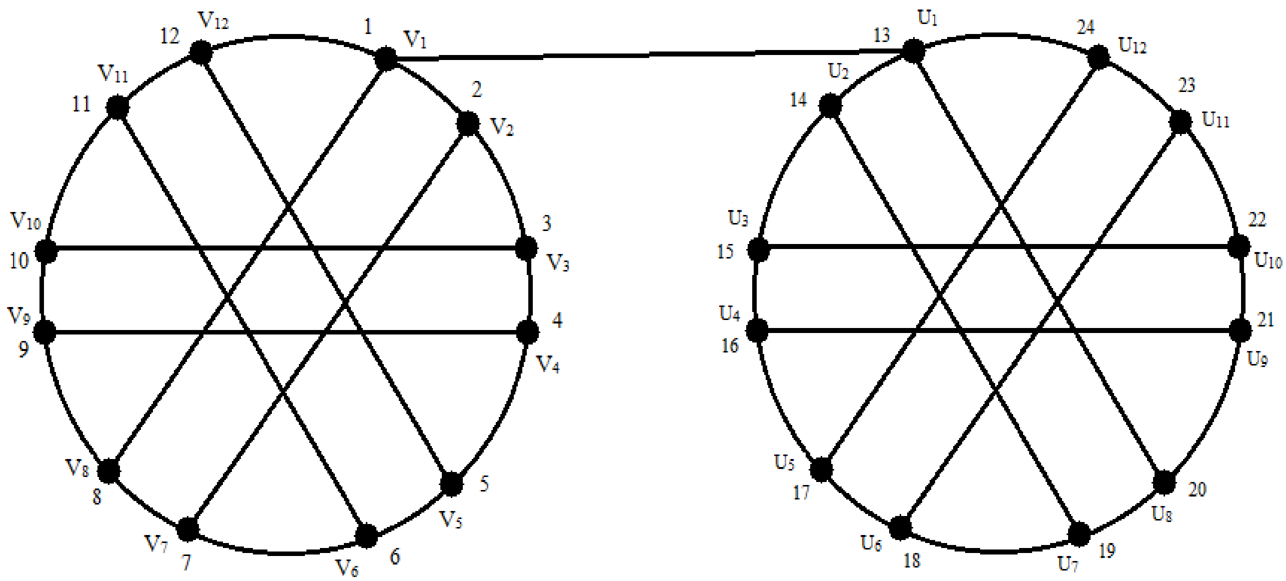

Figure 1.5 Path union of Franklin graph admits prime labeling

REFERENCES

[1] J.A.Bondy and U.S.R. Murthy, "Graph theory and Application",(North Holland), New York(1976)

[2] A Tout A.N.Dabboucy and K.Howalla "Prime labeling of graphs".Nat.Acad.Sci letter pp 365-368 1982

[3] J.A.Gallian, "A dynamic survey of Graph labeling ", the Electronic journal of Combinatories, Vol 18,2011.

[4] Dr V.Ganesan et al "prime labeling of Split graph of path graph P_n",International Journal of Applied and Advanced Scientific Research(IJAASR) Volume 3,issue 2,2018 .

[5] Meena .S and Vaithilingam. K "Prime labeling for some fan related graph",International journal of Engineering Research and Technology(IJERT) vol 1 issue 9,2012.

[6] S.M.Lee, Wui and J.Yen, on Amalagmation of prime graphs Bull. Mallisian Math .Soc.(second series)11,(1988) 59-67

[7] P.Haxel,O.Pikhurko and A.Taraz, "primality of tree",J.Comb.2(2011),481-500

[8] Dr.V.Ganesan " Prime labeling of split graph of Star K1,n. "IOSR Journal of Mathematics (IOSR-JM) 15.6(2019):04-07.

[9] Dr.V.Ganesan " Prime labeling of split graph of cycle C_n"Science,technology and Development Journal ISSN No:0950-0707 .