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On Total Edge Irregularity Strength of Certain Classes of Extended Duplicate Graphs

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ABSTRACT

Measuring irregular strength of a network mathematically is a problem of concern in communication networks. This can be achieved by establishing edge irregular k- labeling of graph of the corresponding network. A function from collection of vertices and edges to first k- nonnegative integers set is an edge irregular k-labeling if the labels of edges induced as sum of labels of the edges together with the labels at end vertices are all different. The least k-value of all such edge irregular k-labeling is known as total edge irregular strength of the graph and denoted by tes(G). Baca M has found lower bound for all graphs. In this paper, we find total edge irregularity strength of extended duplicate graphs of few path generated graphs.

Keywords: Total edge irregular k-labeling, duplicate and extended duplicate graphs, Path, Comb, Twig, Ladder graphs.
AMS Subject Classification: 05C78

1. PRELIMINARIES

Definition 1.1: Consider a graph G with vertex set V. The duplicate graph DG of G is obtained as follows: It is graph with vertex set $V \cup V'$ where V and V' are disjoint and for each vertex v in V there is a vertex v' in V' such that uv is an edge in the graph G if and only if uv' and $u'v$ are edges in its duplicate graph DG.

Definition 1.2 : A path $\boldsymbol{P_n}$ is a sequence of arrangement of vertices $v_1, v_2, \dots, v_n$ where the edges are in the form of $v_i v_{i+1}, i = 1,2, \dots, n-1$. A path $\boldsymbol{P_n}$ has *n* vertices and *n-1* edges and its duplicate graph has *2n* vertices and *2n-2* edges. As this duplicate graph is disconnected, connecting any two vertices one from $v_{i,}$ and one from v_i' (preferably other than terminal vertices) makes it connected and called extended duplicate graph of $\boldsymbol{P_n}$ and denoted by $\boldsymbol{ED(P_n)}$

Definition 1.3: A comb $\boldsymbol{CB_n}$ is obtained by connecting a new pendant edge with each vertex of path P_n. Comb $\boldsymbol{CB_n}$ has $2n$ vertices and $2n-1$ edges and its duplicate graphs has $4n$ vertices and $4n-2$ edges. As this duplicate graph is disconnected , connecting any two vertices one from v_i and one from v_i' makes it connected and called extended duplicate graph of $\boldsymbol{CB_n}$. Here, the vertices v_{2n-1}, v'_{2n-1} are connected by an edge to get extended duplicate graph $\boldsymbol{ED(CB_n)}$.

Definition 1.4: A twig $\boldsymbol{T_n}$ is a graph obtained by connecting two new pendant edges with each of the internal vertices in $\boldsymbol{P_{n+2}}$. Twig $\boldsymbol{T_n}$ has $3n+2$ vertices and $3n+1$ edges and its duplicate graph has $6n+4$ vertices and $6n+2$ edges. The vertices v_{3n-1}, v'_{3n-1} are connected by an edge to get extended duplicate graph $\boldsymbol{ED(T_n)}$.

Definition 1.5: The ladder graph L_n is a planar undirected graph with $2n$ vertices and $2n-2$ edges. It is obtained as the Cartesian product of two path graphs, one of which has only one edge: $L_n = P_n X P_1$. Its duplicate graph has $4n$ vertices and $4n-4$ edges. The vertices v_{2n-1}, v'_{2n-1} are connected to get extended duplicate graph $EDG(L_n)$

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2. MAIN RESULTS

Definition 2.1: A graph G with vertex set V and edge set E, has an edge irregular total k-labeling (k is a positive integer) if there is a function $f: V \cup E \rightarrow \{1,2,\dots,k\}$ such that

(i)the edge induced sums f^* defined for an edge xy in the graph by $f^*(xy) = f(x) + f(y) + f(xy)$

(ii) for any two edges xy and uv in the graph $f^*(xy) \neq f^*(uv)$

Definition 2.2 :The minimum k for which G has an edge irregular total k-labeling is called as total edge irregularity strength of G and denoted by tes(G).

We state the following theorem without proof due to 'Baca' which is a major significant result giving a lower bound for *tes(G)*.

Theorem 2.3: For any graph G which maximum vertex degree $\Delta(G)$, it follows that

$$tes(G) \geq \max\left\{\frac{|E(G)|+2}{3}, \frac{|\Delta(G)|+1}{2}\right\}.$$

Algorithm 2.4 : Mapping labels to the vertices and edges of extended duplicate graph of path graph.

The vertices of $\boldsymbol{EDG(P_n)}$ are labeled as below:

$$\chi(v_i) = \begin{cases} \dfrac{i+1}{2} + \left\lfloor \dfrac{i+1}{6} \right\rfloor, & i = 5,7,\dots,2\left\lceil \dfrac{n}{2} \right\rceil - 1 \\ \dfrac{i}{2} + 1 + \left\lfloor \dfrac{i}{6} \right\rfloor, & i = 2,4,6 \dots,2\left\lfloor \dfrac{n}{2} \right\rfloor \end{cases}$$

$$\chi(v'_i) = \begin{cases} \dfrac{i+1}{2} + \left\lceil \dfrac{i-1}{6} \right\rceil, & i = 3,5,\dots,2\left\lceil \dfrac{n}{2} \right\rceil - 1 \\ \dfrac{i}{2} + 1 + \left\lfloor \dfrac{i-1}{6} \right\rfloor, & i = 4,6 \dots,2\left\lfloor \dfrac{n}{2} \right\rfloor \end{cases}$$

$$\chi(v_1) = 1 \ ; \quad \chi(v_3) = 3 \ ; \quad \chi(v'_1) = 2 \ ; \quad \chi(v'_2) = 1 \ ;$$

The edges of $\boldsymbol{EDG(P_n)}$ are labeled as below:

$$\chi(v_1 v'_2) = 1 \ ; \ \chi(v'_1 v_2) = 1 \ ; \ \chi(v_2 v'_2) = 1 \ ; \ \chi(v_3 v'_4) = 2$$

$$\chi(v_{2i-1} v'_{2i}) = i + \left\lceil \frac{i-1}{3} \right\rceil, \quad i = 3,4,5,6,\dots,\left\lfloor \frac{n}{2} \right\rfloor$$

$$\chi(v_{2i} v'_{2i+1}) = i + 1 + \left\lceil \frac{i-1}{3} \right\rceil, \quad \begin{cases} i = 1,2,3,\dots,\left\lfloor \dfrac{n}{2} \right\rfloor & if \ \ n \ is \ odd \\ i = 1,2,3,\dots,\left\lfloor \dfrac{n}{2} \right\rfloor - 1, & if \ \ n \ is \ \ even \end{cases}$$

$$\chi(v'_{2i-1} v_{2i}) = i + \left\lceil \frac{i}{3} \right\rceil, \quad i = 2,3,\dots,\left\lfloor \frac{n}{2} \right\rfloor$$

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$$\chi(v'_{2i}v_{2i+1}) = i + \left\lceil\frac{i+1}{3}\right\rceil, \qquad \begin{cases} i = 1,2,3,\dots, \left\lfloor\frac{n}{2}\right\rfloor & if \quad n\ is\ odd \\ i = 1,2,3,\dots, \left\lfloor\frac{n}{2}\right\rfloor - 1, & if\ \ n\ is\ \ even \end{cases}$$

Theorem 2.5 : Total edge irregularity strength $tes(EDG(P_n))$ of the extended duplicate graph of path graph is $\left\lceil\frac{2n+1}{3}\right\rceil$

Proof: By algorithm 2.5 , it is clear that the vertex v'_n has maximum label $\frac{n}{2} + \left\lceil\frac{n}{6}\right\rceil = \left\lceil\frac{2n+1}{3}\right\rceil$ when n is even and $\frac{n+1}{2} + \left\lceil\frac{n-1}{6}\right\rceil = \left\lceil\frac{2n+1}{3}\right\rceil$ when n is odd and all other vertices and edges are labeled with lesser numbers.

The induced edge sums (weights) of the edges are computed using

$\chi^*(uv) = \chi(u) + \chi(uv) + \chi(v)$ as

$\chi^*(v_1v'_2) = 3$; $\chi^*(v'_1v_2) = 5$; $\chi^*(v_2v'_2) = 4$; $\chi^*(v_3v'_4) = 8$

$$\chi^*(v_{2i-1}v'_{2i}) = \frac{2i-1+1}{2} + \left\lfloor\frac{2i-1+1}{6}\right\rfloor + i + \left\lceil\frac{i-1}{3}\right\rceil + \frac{2i}{2} + 1 + \left\lfloor\frac{2i-1}{6}\right\rfloor = 4i, \qquad i = 3,4,5,6,\dots, \left\lfloor\frac{n}{2}\right\rfloor$$

$$\chi^*(v_{2i}v'_{2i+1}) = \frac{2i}{2} + 1 + \left\lfloor\frac{2i}{6}\right\rfloor + i + 1 + \left\lceil\frac{i-1}{3}\right\rceil + \frac{2i+2}{2} + \left\lceil\frac{2i+1-1}{6}\right\rceil$$

$$= 4i + 3, \qquad \begin{cases} i = 1,2,3,\dots, \left\lfloor\frac{n}{2}\right\rfloor & if \quad n\ is\ odd \\ i = 1,2,3,\dots, \left\lfloor\frac{n}{2}\right\rfloor - 1, & if\ \ n\ is\ \ even \end{cases}$$

$$\chi^*(v'_{2i-1}v_{2i}) = \frac{2i-1+1}{2} + \left\lceil\frac{2i-1-1}{6}\right\rceil + i + \left\lfloor\frac{i}{3}\right\rfloor + \frac{2i}{2} + 1 + \left\lfloor\frac{2i}{6}\right\rfloor = 4i + 1, \qquad i = 2,3,\dots, \left\lfloor\frac{n}{2}\right\rfloor$$

$$\chi^*(v'_{2i}v_{2i+1}) = \frac{2i}{2} + 1 + \left\lfloor\frac{2i-1}{6}\right\rfloor + i + \left\lceil\frac{i+1}{3}\right\rceil + \frac{2i+1+1}{2} + \left\lfloor\frac{2i+1+1}{6}\right\rfloor$$

$$= 4i + 2, \qquad \begin{cases} i = 1,2,3,\dots, \left\lfloor\frac{n}{2}\right\rfloor & if \quad n\ is\ odd \\ i = 1,2,3,\dots, \left\lfloor\frac{n}{2}\right\rfloor - 1, & if\ \ n\ is\ \ even \end{cases}$$

All the induced edge sums (weights) are distinct, and so it is a $\left\lceil\frac{2n+1}{3}\right\rceil -$ labeling and hence the extended duplicate graph of path graph has total edge irregularity strength $tes(EDG(P_n)) = \left\lceil\frac{2n+1}{3}\right\rceil$. Illustration of irregular 4 labeling is given in Figure 1. □

Algorithm 2.6: Assignment of labels to the vertices and edges of extended duplicate of Ladder graph .

The vertices of $EDG(L_n)$ are labeled as follows:

For $i = 1\ to\ 2n$

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$$\chi(v_i)=\begin{cases} n-\left(\frac{i-2}{2}\right), & i\equiv 0 \ mod\ 4\\ n-\left(\frac{i-1}{2}\right), & i\equiv 1 \ mod\ 4\\ n+\left(\frac{i}{2}\right), & i\equiv 2 \ mod\ 4\\ n+\left(\frac{i-1}{2}\right), & i\equiv 3 \ mod\ 4\end{cases} \qquad \chi(v'_i)=\begin{cases} n+\left(\frac{i}{2}\right), & i\equiv 0 \ mod\ 4\\ n+\left(\frac{i+1}{2}\right), & i\equiv 1 \ mod\ 4\\ n-\left(\frac{i-2}{2}\right), & i\equiv 2 \ mod\ 4\\ n-\left(\frac{i-1}{2}\right), & i\equiv 3 \ mod\ 4\end{cases}$$

The edges of $EDG(L_n)$ are labeled as below:

$$\chi(v_{2i-1}v'_{2i+1})=\begin{cases} n-i, & i=1,3,5,\dots,2\left\lceil\frac{n-1}{2}\right\rceil-1\\ n+i, & i=2,4,6,\dots,2\left\lfloor\frac{n-1}{2}\right\rfloor\end{cases}$$

$$\chi(v_{2i-1}v'_{2i})=\begin{cases} n-i+1, & i=1,3,5,\dots,2\left\lfloor\frac{n+1}{2}\right\rfloor-1\\ n+i, & i=2,4,6,\dots,2\left\lceil\frac{n-1}{2}\right\rceil\end{cases}$$

$$\chi(v'_{2i-1}v_{2i+1})=\begin{cases} n+i, & i=1,3,5,\dots,2\left\lceil\frac{n-1}{2}\right\rceil-1\\ n-i, & i=2,4,6,\dots,2\left\lfloor\frac{n-1}{2}\right\rfloor\end{cases}$$

$$\chi(v'_{2i-1}v_{2i})=\begin{cases} n+i-1, & i=1,3,5,\dots,2\left\lfloor\frac{n+1}{2}\right\rfloor-1\\ n-i+1, & i=2,4,6,\dots,2\left\lceil\frac{n-1}{2}\right\rceil\end{cases}$$

$$\chi(v_{2i}v'_{2i+2})=\begin{cases} n+i, & i=1,3,5,\dots,2\left\lceil\frac{n-1}{2}\right\rceil-1\\ n-i+1, & i=2,4,6,\dots,2\left\lfloor\frac{n-1}{2}\right\rfloor\end{cases}$$

$$\chi(v'_{2i}v_{2i+2})=\begin{cases} n-i+1, & i=1,3,5,\dots,2\left\lceil\frac{n-1}{2}\right\rceil-1\\ n+i, & i=2,4,6,\dots,2\left\lfloor\frac{n-1}{2}\right\rfloor\end{cases}$$

$\chi(v_2v'_2)=n$

THEOREM 2.7: Total edge irregularity strength $tes\big(EDG(L_n)\big)$ of extended duplicate graph of ladder graph is $\left\lceil\frac{6n-1}{3}\right\rceil$

Proof: By algorithm 2.6 , $\left\lceil\frac{6n-1}{3}\right\rceil$ is the maximum value of label for the vertex v_{2n} *or* v'_{2n} according as *n* is odd or even.

The induced edge sums (weights) of the edges are computed using

$\chi^*(uv)=\chi(u)+\chi(uv)+\chi(v)$ as

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$$\chi^*(v_{2i-1}v'_{2i+1}) = \begin{cases} n-\left(\frac{2i-1-1}{2}\right)+n-i+n-\left(\frac{2i+1-1}{2}\right)=3n-3i+1, & i=1,3,5,\dots,2\left\lceil\frac{n-1}{2}\right\rceil-1 \\ n+\left(\frac{2i-1-1}{2}\right)+n+i+n+\left(\frac{2i+1+1}{2}\right)=3n+3i, & i=2,4,6,\dots,2\left\lfloor\frac{n-1}{2}\right\rfloor \end{cases}$$

$$\chi^*(v_{2i-1}v'_{2i}) = \begin{cases} n-\left(\frac{2i-1-1}{2}\right)+n-i+1+n-\left(\frac{2i-2}{2}\right)=3n-3i+3, & i=1,3,5,\dots,2\left\lceil\frac{n+1}{2}\right\rceil-1 \\ n+\left(\frac{2i-1-1}{2}\right)+n+i+n+\left(\frac{2i}{2}\right)=3n+3i-1, & i=2,4,6,\dots,2\left\lceil\frac{n-1}{2}\right\rceil \end{cases}$$

$$\chi^*(v'_{2i-1}v_{2i+1}) = \begin{cases} n+\left(\frac{2i-1+1}{2}\right)+n+i+n+\left(\frac{2i+1-1}{2}\right)=3n+3i, & i=1,3,5,\dots,2\left\lceil\frac{n-1}{2}\right\rceil-1 \\ n-\left(\frac{2i-1-1}{2}\right)+n-i+n-\left(\frac{2i+1-1}{2}\right)=3n-3i+1, & i=2,4,6,\dots,2\left\lfloor\frac{n-1}{2}\right\rfloor \end{cases}$$

$$\chi^*(v'_{2i-1}v_{2i}) = \begin{cases} n+\left(\frac{2i-1+1}{2}\right)+n+i-1+n+\left(\frac{2i}{2}\right)=3n+3i-1, & i=1,3,5,\dots,2\left\lfloor\frac{n+1}{2}\right\rfloor-1 \\ n-\left(\frac{2i-1-1}{2}\right)+n-i+1+n-\left(\frac{2i-2}{2}\right)=3n-3i+3, & i=2,4,6,\dots,2\left\lceil\frac{n-1}{2}\right\rceil \end{cases}$$

$$\chi^*(v_{2i}v'_{2i+2}) = \begin{cases} n+\left(\frac{2i}{2}\right)+n+i+n+\left(\frac{2i+1+1}{2}\right)=3n+3i+1, & i=1,3,5,\dots,2\left\lceil\frac{n-1}{2}\right\rceil-1 \\ n-\left(\frac{2i-2}{2}\right)+n-i+1+n-\left(\frac{2i+2-2}{2}\right)=3n-3i+2, & i=2,4,6,\dots,2\left\lfloor\frac{n-1}{2}\right\rfloor \end{cases}$$

$$\chi^*(v'_{2i}v_{2i+2}) = \begin{cases} n-\left(\frac{2i-2}{2}\right)+n-i+1+n-\left(\frac{2i}{2}\right)=3n-3i+2, & i=1,3,5,\dots,2\left\lceil\frac{n-1}{2}\right\rceil-1 \\ n+\left(\frac{2i}{2}\right)+n+i+n+\left(\frac{2i+2}{2}\right)=3n+3i+1, & i=2,4,6,\dots,2\left\lfloor\frac{n-1}{2}\right\rfloor \end{cases}$$

$\chi^*(v_2v'_2) = n+1+n+n = 3n+1$

All the induced edge sums (weights) of the edges are distinct. For, if for some i,j

$3n+3i+1 = 3n+3j+2$ then $i-j=\frac{1}{3}$ which contradicts the fact that i,j are integers . Hence, the extended duplicate graph of ladder graph has total edge irregularity strength $tes(EDG(L_n)) = \left\lceil\frac{6n-1}{3}\right\rceil$. An illustration of edge irregular total 6-labeling of $EDG(L_3)$ is given in Figure 2.□

Algorithm 2.8: Assignment of labels to the vertices and edges of extended duplicate graph of comb graph

Case(i) $n \equiv 0 \ mod\ 3$

Take $\alpha = \left\lceil\frac{2n+3}{3}\right\rceil$

The vertices of $EDG(CB_n)$ are labeled as follows:

$$\chi(v_{2i-1}) = \begin{cases} \alpha-\left(\frac{i+1}{2}\right)-\left\lfloor\frac{i-1}{6}\right\rfloor, & i=1,3,5,\dots,2\left\lceil\frac{n}{2}\right\rceil-1 \\ \alpha+\left(\frac{i}{2}\right)+\left\lfloor\frac{i-2}{6}\right\rfloor, & i=2,4,6,\dots,2\left\lfloor\frac{n}{2}\right\rfloor \end{cases}$$

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$$\chi(v_{2i}) = \begin{cases} \alpha + \left(\frac{i-1}{2}\right) + \left\lceil\frac{i-1}{6}\right\rceil, & i = 1,3,5,..,2\left\lceil\frac{n}{2}\right\rceil - 1 \\ \alpha - \left(\frac{i}{2}\right) - \left\lfloor\frac{i}{6}\right\rfloor, & i = 2,4,6,\dots,2\left\lfloor\frac{n}{2}\right\rfloor \end{cases}$$

$$\chi(v'_{2i-1}) = \begin{cases} \alpha + \left(\frac{i-1}{2}\right) + \left\lfloor\frac{i+1}{6}\right\rfloor, & i = 1,3,5,..,2\left\lceil\frac{n}{2}\right\rceil - 1 \\ \alpha - \left(\frac{i}{2}\right) - \left\lceil\frac{i-2}{6}\right\rceil, & i = 2,4,6,\dots,2\left\lfloor\frac{n}{2}\right\rfloor \end{cases}$$

$$\chi(v'_{2i}) = \begin{cases} \alpha - \left(\frac{i-1}{2}\right) - \left\lceil\frac{i-1}{6}\right\rceil, & i = 1,3,5,..,2\left\lceil\frac{n}{2}\right\rceil - 1 \\ \alpha + \left(\frac{i}{2}\right) + \left\lfloor\frac{i}{6}\right\rfloor, & i = 2,4,6,\dots,2\left\lfloor\frac{n}{2}\right\rfloor \end{cases}$$

The edges of $EDG(CB_n)$ are labeled as follows:

$$\chi(v_{2i-1}v'_{2i}) = \begin{cases} \alpha - \left(\frac{i+1}{2}\right) - \left\lfloor\frac{i+1}{6}\right\rfloor, & i = 1,3,5,..,2\left\lceil\frac{n}{2}\right\rceil - 1 \\ \alpha + \left(\frac{i-2}{2}\right) + \left\lceil\frac{i-2}{6}\right\rceil, & i = 2,4,6,\dots,2\left\lfloor\frac{n}{2}\right\rfloor \end{cases}$$

$$\chi(v_{2i-1}v'_{2i+1}) = \begin{cases} \alpha - \left(\frac{i+1}{2}\right) - \left\lfloor\frac{i+1}{6}\right\rfloor, & i = 1,3,5,..,2\left\lfloor\frac{n}{2}\right\rfloor - 1 \\ \alpha + \left(\frac{i}{2}\right) + \left\lfloor\frac{i}{6}\right\rfloor, & i = 2,4,6,\dots,2\left\lceil\frac{n}{2}\right\rceil - 2 \end{cases}$$

$$\chi(v'_{2i-1}v_{2i+1}) = \begin{cases} \alpha + \left(\frac{i-1}{2}\right) + \left\lceil\frac{i-1}{6}\right\rceil, & i = 1,3,5,..,2\left\lfloor\frac{n}{2}\right\rfloor - 1 \\ \alpha - \left(\frac{i+2}{2}\right) - \left\lceil\frac{i-2}{6}\right\rceil, & i = 2,4,6,\dots,2\left\lceil\frac{n}{2}\right\rceil - 2 \end{cases}$$

$$\chi(v'_{2i-1}v_{2i}) = \begin{cases} \alpha + \left(\frac{i-1}{2}\right) + \left\lfloor\frac{i-1}{6}\right\rfloor, & i = 1,3,5,..,2\left\lceil\frac{n}{2}\right\rceil - 1 \\ \alpha - \left(\frac{i+2}{2}\right) - \left\lfloor\frac{i-2}{6}\right\rfloor, & i = 2,4,6,\dots,2\left\lfloor\frac{n}{2}\right\rfloor \end{cases}$$

$$\chi(v_2v'_2) = \alpha - 1$$

Case(ii) $n \equiv 1\ mod\ 3$

Take $\alpha = \left\lceil\frac{2n+1}{3}\right\rceil$

The vertices of $EDG(CB_n)$ are labeled as follows:

$$\chi(v_{2i-1}) = \begin{cases} \alpha - \left(\frac{i-1}{2}\right) - \left\lfloor\frac{i+1}{6}\right\rfloor, & i = 1,3,5,..,2\left\lceil\frac{n}{2}\right\rceil - 1 \\ \alpha + \left(\frac{i}{2}\right) + \left\lceil\frac{i-2}{6}\right\rceil, & i = 2,4,6,\dots,2\left\lfloor\frac{n}{2}\right\rfloor \end{cases}$$

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$$\chi(v_{2i}) = \begin{cases} \alpha + \left(\frac{i+1}{2}\right) + \left\lfloor \frac{i+1}{6} \right\rfloor, & i = 1,3,5,..,2\left\lceil \frac{n}{2} \right\rceil - 1 \\ \alpha - \left(\frac{i-2}{2}\right) - \left\lfloor \frac{i+2}{6} \right\rfloor, & i = 2,4,6,\dots,2\left\lfloor \frac{n}{2} \right\rfloor \end{cases}$$

$$\chi(v'_{2i-1}) = \begin{cases} \alpha + \left(\frac{i+1}{2}\right) + \left\lfloor \frac{i-1}{6} \right\rfloor, & i = 1,3,5,..,2\left\lceil \frac{n}{2} \right\rceil - 1 \\ \alpha - \left(\frac{i}{2}\right) - \left\lfloor \frac{i-2}{6} \right\rfloor, & i = 2,4,6,\dots,2\left\lfloor \frac{n}{2} \right\rfloor \end{cases}$$

$$\chi(v'_{2i}) = \begin{cases} \alpha - \left(\frac{i-1}{2}\right) - \left\lfloor \frac{i-1}{6} \right\rfloor, & i = 1,3,5,..,2\left\lceil \frac{n}{2} \right\rceil - 1 \\ \alpha + \left(\frac{i}{2}\right) + \left\lceil \frac{i}{6} \right\rceil, & i = 2,4,6,\dots,2\left\lfloor \frac{n}{2} \right\rfloor \end{cases}$$

The edges of $EDG(CB_n)$ are labeled as follows:

$$\chi(v_{2i-1}v'_{2i}) = \begin{cases} \alpha - \left(\frac{i-1}{2}\right) - \left\lceil \frac{i-1}{6} \right\rceil, & i = 1,3,5,..,2\left\lceil \frac{n}{2} \right\rceil - 1 \\ \alpha + \left(\frac{i}{2}\right) + \left\lfloor \frac{i}{6} \right\rfloor, & i = 2,4,6,\dots,2\left\lfloor \frac{n}{2} \right\rfloor \end{cases}$$

$$\chi(v_{2i-1}v'_{2i+1}) = \begin{cases} \alpha - \left(\frac{i-1}{2}\right) - \left\lceil \frac{i-1}{6} \right\rceil, & i = 1,3,5,..,2\left\lfloor \frac{n}{2} \right\rfloor - 1 \\ \alpha + \left(\frac{i}{2}\right) + \left\lceil \frac{i}{6} \right\rceil, & i = 2,4,6,\dots,2\left\lceil \frac{n}{2} \right\rceil - 2 \end{cases}$$

$$\chi(v'_{2i-1}v_{2i}) = \begin{cases} \alpha + \left(\frac{i-1}{2}\right) + \left\lceil \frac{i-1}{6} \right\rceil, & i = 1,3,5,..,2\left\lceil \frac{n}{2} \right\rceil - 1 \\ \alpha - \left(\frac{i}{2}\right) - \left\lfloor \frac{i}{6} \right\rfloor, & i = 2,4,6,\dots,2\left\lfloor \frac{n}{2} \right\rfloor \end{cases}$$

$$\chi(v'_{2i-1}v_{2i+1}) = \begin{cases} \alpha + \left(\frac{i+1}{2}\right) + \left\lfloor \frac{i+1}{6} \right\rfloor, & i = 1,3,5,..,2\left\lfloor \frac{n}{2} \right\rfloor - 1 \\ \alpha - \left(\frac{i}{2}\right) - \left\lfloor \frac{i}{6} \right\rfloor, & i = 2,4,6,\dots,2\left\lceil \frac{n}{2} \right\rceil - 2 \end{cases}$$

$\chi(v_2v'_2) = \alpha$

Case(iii) $n \equiv 2 \ mod\ 3$

Take $\alpha = \left\lceil \frac{2n+1}{3} \right\rceil$

The vertices of $EDG(CB_n)$ are labeled as follows:

$$\chi(v_{2i-1}) = \begin{cases} \alpha - \left(\frac{i-1}{2}\right) - \left\lceil \frac{i-1}{6} \right\rceil, & i = 1,3,5,..,2\left\lceil \frac{n}{2} \right\rceil - 1 \\ \alpha + \left(\frac{i}{2}\right) + \left\lfloor \frac{i}{6} \right\rfloor, & i = 2,4,6,\dots,2\left\lfloor \frac{n}{2} \right\rfloor \end{cases}$$

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$$\chi(v_{2i}) = \begin{cases} \alpha, & i = 1 \\ \alpha + \left(\frac{i-1}{2}\right) + \left\lceil \frac{i+1}{6} \right\rceil, & i = 3,5,..,2\left\lceil \frac{n}{2} \right\rceil - 1 \\ \alpha - \left(\frac{i}{2}\right) - \left\lfloor \frac{i-2}{6} \right\rfloor, & i = 2,4,6,\ldots,2\left\lfloor \frac{n}{2} \right\rfloor \end{cases}$$

$$\chi(v'_{2i-1}) = \begin{cases} \alpha + 1, & i = 1 \\ \alpha + \left(\frac{i-1}{2}\right) + \left\lceil \frac{i-1}{6} \right\rceil, & i = 3,5,..,2\left\lceil \frac{n}{2} \right\rceil - 1 \\ \alpha - \left(\frac{i}{2}\right) - \left\lfloor \frac{i}{6} \right\rfloor, & i = 2,4,6,\ldots,2\left\lfloor \frac{n}{2} \right\rfloor \end{cases}$$

$$\chi(v'_{2i}) = \begin{cases} \alpha - \left(\frac{i-1}{2}\right) - \left\lfloor \frac{i+1}{6} \right\rfloor, & i = 1,3,5,..,2\left\lceil \frac{n}{2} \right\rceil - 1 \\ \alpha + \left(\frac{i}{2}\right) + \left\lceil \frac{i-2}{6} \right\rceil, & i = 2,4,6,\ldots,2\left\lfloor \frac{n}{2} \right\rfloor \end{cases}$$

The edges of $EDG(CB_n)$ are labeled as follows:

$$\chi(v_{2i-1}v'_{2i}) = \begin{cases} \alpha - \left(\frac{i+1}{2}\right) - \left\lfloor \frac{i-1}{6} \right\rfloor, & i = 1,3,5,..,2\left\lceil \frac{n}{2} \right\rceil - 1 \\ \alpha + \left(\frac{i}{2}\right) + \left\lfloor \frac{i-2}{6} \right\rfloor, & i = 2,4,6,\ldots,2\left\lfloor \frac{n}{2} \right\rfloor \end{cases}$$

$$\chi(v_{2i-1}v'_{2i+1}) = \begin{cases} \alpha - \left(\frac{i+1}{2}\right) - \left\lfloor \frac{i-1}{6} \right\rfloor, & i = 1,3,5,..,2\left\lfloor \frac{n}{2} \right\rfloor - 1 \\ \alpha + \left(\frac{i}{2}\right) + \left\lceil \frac{i-2}{6} \right\rceil, & i = 2,4,6,\ldots,2\left\lceil \frac{n}{2} \right\rceil - 2 \end{cases}$$

$$\chi(v'_{2i-1}v_{2i}) = \begin{cases} \alpha + \left(\frac{i-1}{2}\right) + \left\lfloor \frac{i+1}{6} \right\rfloor, & i = 1,3,5,..,2\left\lceil \frac{n}{2} \right\rceil - 1 \\ \alpha - \left(\frac{i}{2}\right) - \left\lceil \frac{i-2}{6} \right\rceil, & i = 2,4,6,\ldots,2\left\lfloor \frac{n}{2} \right\rfloor \end{cases}$$

$$\chi(v'_{2i-1}v_{2i+1}) = \begin{cases} \alpha, & i = 1 \\ \alpha + \left(\frac{i-1}{2}\right) + \left\lceil \frac{i+1}{6} \right\rceil, & i = 3,5,..,2\left\lfloor \frac{n}{2} \right\rfloor - 1 \ldots \\ \alpha - \left(\frac{i}{2}\right) - \left\lceil \frac{i-2}{6} \right\rceil, & i = 2,4,6,\ldots,2\left\lceil \frac{n}{2} \right\rceil - 2 \end{cases}$$

$\chi(v_2v'_2) = \alpha$

Theorem 2.9: Total edge irregularity strength $tes\big(EDG(CB_n)\big)$ of extended duplicate graph of comb graph is $\left\lceil \frac{4n+3}{3} \right\rceil$

Proof: By algorithm 3.8 , $\left\lceil \frac{4n+3}{3} \right\rceil$ is the maximum value of label for the vertex v'_{2n}.

The induced edge sums (weights) of the edges are computed using

$\chi^*(uv) = \chi(u) + \chi(uv) + \chi(v)$ as

Case(i) $n \equiv 0 \bmod 3$

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For $i = 1,3,5,..,2\left\lceil\frac{n}{2}\right\rceil - 1$

$$\chi^*(v_{2i-1}v'_{2i}) = \alpha - \left(\frac{i+1}{2}\right) - \left\lfloor\frac{i-1}{6}\right\rfloor + \alpha - \left(\frac{i-1}{2}\right) - \left\lceil\frac{i-1}{6}\right\rceil + \alpha - \left(\frac{i+1}{2}\right) - \left\lfloor\frac{i+1}{6}\right\rfloor = 3\alpha - 2i$$

$$\chi^*(v'_{2i-1}v_{2i}) = \alpha + \left(\frac{i-1}{2}\right) + \left\lfloor\frac{i+1}{6}\right\rfloor + \alpha + \left(\frac{i-1}{2}\right) + \left\lceil\frac{i-1}{6}\right\rceil + \alpha + \left(\frac{i-1}{2}\right) - \left\lfloor\frac{i-1}{6}\right\rfloor = 3\alpha + 2i - 2$$

For $i = 1,3,5,..,2\left\lfloor\frac{n}{2}\right\rfloor - 1$

$$\chi^*(v_{2i-1}v'_{2i+1}) = \alpha - \left(\frac{i+1}{2}\right) - \left\lfloor\frac{i-1}{6}\right\rfloor + \alpha - \left(\frac{i+1}{2}\right) - \left\lceil\frac{i-1}{6}\right\rceil + \alpha - \left(\frac{i+1}{2}\right) - \left\lfloor\frac{i+1}{6}\right\rfloor = 3\alpha - 2i - 1$$

$$\chi^*(v'_{2i-1}v_{2i+1}) = \alpha + \left(\frac{i-1}{2}\right) + \left\lfloor\frac{i+1}{6}\right\rfloor + \alpha + \left(\frac{i-1}{2}\right) + \left\lceil\frac{i-1}{6}\right\rceil + \alpha + \left(\frac{i+1}{2}\right) + \left\lfloor\frac{i-1}{6}\right\rfloor = 3\alpha + 2i - 1$$

For $i = 2,4,6, \dots ,2\left\lceil\frac{n}{2}\right\rceil$

$$\chi^*(v_{2i-1}v'_{2i}) = \alpha + \left(\frac{i}{2}\right) + \left\lfloor\frac{i-2}{6}\right\rfloor + \alpha + \left(\frac{i}{2}\right) + \left\lfloor\frac{i}{6}\right\rfloor + \alpha + \left(\frac{i-2}{2}\right) + \left\lceil\frac{i-2}{6}\right\rceil = 3\alpha + 2i - 2$$

$$\chi^*(v'_{2i-1}v_{2i}) = \alpha - \left(\frac{i}{2}\right) - \left\lceil\frac{i-2}{6}\right\rceil + \alpha - \left(\frac{i+2}{2}\right) - \left\lfloor\frac{i-2}{6}\right\rfloor + \alpha - \left(\frac{i}{2}\right) - \left\lfloor\frac{i}{6}\right\rfloor = 3\alpha - 2i$$

For $i = 2,4,6, \dots ,2\left\lceil\frac{n}{2}\right\rceil - 2$

$$\chi^*(v_{2i-1}v'_{2i+1}) = \alpha + \left(\frac{i}{2}\right) + \left\lfloor\frac{i-2}{6}\right\rfloor + \alpha + \left(\frac{i}{2}\right) + \left\lceil\frac{i+2}{6}\right\rceil + \alpha + \left(\frac{i}{2}\right) + \left\lfloor\frac{i}{6}\right\rfloor = 3\alpha + 2i - 1$$

$$\chi^*(v'_{2i-1}v_{2i+1}) = \alpha - \left(\frac{i}{2}\right) - \left\lceil\frac{i-2}{6}\right\rceil + \alpha - \left(\frac{i+2}{2}\right) - \left\lfloor\frac{i-2}{6}\right\rfloor + \alpha - \left(\frac{i+2}{2}\right) - \left\lfloor\frac{i}{6}\right\rfloor = 3\alpha - 2i - 1$$

$$\chi(v_2v'_2) = \alpha + \alpha - 1 + \alpha = 3\alpha - 1$$

Case(ii) $\boldsymbol{n \equiv 1 \bmod 3}$

For $i = 1,3,5,..,2\left\lceil\frac{n}{2}\right\rceil - 1$

$$\chi^*(v_{2i-1}v'_{2i}) = \alpha - \left(\frac{i-1}{2}\right) - \left\lfloor\frac{i+1}{6}\right\rfloor + \alpha - \left(\frac{i-1}{2}\right) - \left\lceil\frac{i-1}{6}\right\rceil + \alpha - \left(\frac{i-1}{2}\right) - \left\lfloor\frac{i-1}{6}\right\rfloor = 3\alpha - 2i + 2$$

$$\chi^*(v'_{2i-1}v_{2i}) = \alpha + \left(\frac{i+1}{2}\right) + \left\lfloor\frac{i-1}{6}\right\rfloor + \alpha + \left(\frac{i+1}{2}\right) + \left\lfloor\frac{i+1}{6}\right\rfloor + \alpha + \left(\frac{i-1}{2}\right) + \left\lceil\frac{i-1}{6}\right\rceil = 3\alpha + 2i$$

For $i = 1,3,5,..,2\left\lfloor\frac{n}{2}\right\rfloor - 1$

$$\chi^*(v_{2i-1}v'_{2i+1}) = \alpha - \left(\frac{i-1}{2}\right) - \left\lfloor\frac{i+1}{6}\right\rfloor + \alpha - \left(\frac{i+1}{2}\right) - \left\lfloor\frac{i-1}{6}\right\rfloor + \alpha - \left(\frac{i-1}{2}\right) - \left\lceil\frac{i-1}{6}\right\rceil = 3\alpha - 2i + 1$$

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$$\chi^*(v'_{2i-1}v_{2i+1}) = \alpha + \left(\frac{i+1}{2}\right) + \left\lfloor\frac{i-1}{6}\right\rfloor + \alpha + \left(\frac{i+1}{2}\right) + \left\lceil\frac{i-1}{6}\right\rceil + \alpha + \left(\frac{i+1}{2}\right) + \left\lfloor\frac{i+1}{6}\right\rfloor = 3\alpha + 2i + 1$$

For $i = 2,4,6, \dots ,2\left\lceil\frac{n}{2}\right\rceil$

$$\chi^*(v_{2i-1}v'_{2i}) = \alpha + \left(\frac{i}{2}\right) + \left\lceil\frac{i-2}{6}\right\rceil + \alpha + \left(\frac{i}{2}\right) + \left\lceil\frac{i}{6}\right\rceil + \alpha + \left(\frac{i}{2}\right) + \left\lfloor\frac{i}{6}\right\rfloor = 3\alpha + 2i$$

$$\chi^*(v'_{2i-1}v_{2i}) = \alpha - \left(\frac{i}{2}\right) - \left\lfloor\frac{i-2}{6}\right\rfloor + \alpha - \left(\frac{i-2}{2}\right) - \left\lfloor\frac{i+2}{6}\right\rfloor + \alpha - \left(\frac{i}{2}\right) - \left\lfloor\frac{i}{6}\right\rfloor = 3\alpha - 2i + 2$$

For $i = 2,4,6, \dots ,2\left\lceil\frac{n}{2}\right\rceil - 2$

$$\chi^*(v_{2i-1}v'_{2i+1}) = \alpha + \left(\frac{i}{2}\right) + \left\lceil\frac{i-2}{6}\right\rceil + \alpha + \left(\frac{i+2}{2}\right) + \left\lfloor\frac{i}{6}\right\rfloor + \alpha + \left(\frac{i}{2}\right) + \left\lceil\frac{i}{6}\right\rceil = 3\alpha + 2i + 1$$

$$\chi^*(v'_{2i-1}v_{2i+1}) = \alpha - \left(\frac{i}{2}\right) - \left\lfloor\frac{i}{6}\right\rfloor + \alpha - \left(\frac{i}{2}\right) - \left\lfloor\frac{i-2}{6}\right\rfloor + \alpha - \left(\frac{i}{2}\right) - \left\lfloor\frac{i+2}{6}\right\rfloor = 3\alpha - 2i + 1$$

$$\chi(v_2v'_2) = \alpha + 1 + \alpha + \alpha = 3\alpha + 1$$

Case(iii) $\boldsymbol{n \equiv 2 \bmod 3}$

For $i = 1,3,5, \dots ,2\left\lceil\frac{n}{2}\right\rceil - 1$

$$\chi^*(v_{2i-1}v'_{2i}) = \alpha - \left(\frac{i-1}{2}\right) - \left\lceil\frac{i-1}{6}\right\rceil + \alpha - \left(\frac{i+1}{2}\right) - \left\lfloor\frac{i-1}{6}\right\rfloor + \alpha - \left(\frac{i-1}{2}\right) - \left\lfloor\frac{i+1}{6}\right\rfloor = 3\alpha - 2i + 1$$

$\chi^*(v'_1v_2) = \alpha + 1 + \alpha + \alpha = 3\alpha + 1$ For $i = 3,5, \dots ,2\left\lceil\frac{n}{2}\right\rceil - 1$

$$\chi^*(v'_{2i-1}v_{2i}) = \alpha + \left(\frac{i-1}{2}\right) + \left\lfloor\frac{i-1}{6}\right\rfloor + \alpha + \left(\frac{i-1}{2}\right) + \left\lfloor\frac{i+1}{6}\right\rfloor + \alpha + \left(\frac{i-1}{2}\right) + \left\lceil\frac{i+1}{6}\right\rceil = 3\alpha + 2i - 1$$

For $i = 1,3,5, \dots ,2\left\lfloor\frac{n}{2}\right\rfloor - 1$

$$\chi^*(v_{2i-1}v'_{2i+1}) = \alpha - \left(\frac{i-1}{2}\right) - \left\lceil\frac{i-1}{6}\right\rceil + \alpha - \left(\frac{i+1}{2}\right) - \left\lceil\frac{i-1}{6}\right\rceil + \alpha - \left(\frac{i+1}{2}\right) - \left\lfloor\frac{i+1}{6}\right\rfloor = 3\alpha - 2i$$

$$\chi^*(v'_1v_3) = \alpha + 1 + \alpha + \alpha + 1 = 3\alpha + 2$$

For $i = 3,5, \dots ,2\left\lfloor\frac{n}{2}\right\rfloor - 1$

$$\chi^*(v'_{2i-1}v_{2i+1}) = \alpha + \left(\frac{i-1}{2}\right) + \left\lceil\frac{i-1}{6}\right\rceil + \alpha + \left(\frac{i-1}{2}\right) + \left\lceil\frac{i+1}{6}\right\rceil + \alpha + \left(\frac{i+1}{2}\right) + \left\lfloor\frac{i+1}{6}\right\rfloor = 3\alpha + 2i$$

For $i = 2,4,6, \dots ,2\left\lceil\frac{n}{2}\right\rceil$

$$\chi^*(v_{2i-1}v'_{2i}) = \alpha + \left(\frac{i}{2}\right) + \left\lfloor\frac{i}{6}\right\rfloor + \alpha + \left(\frac{i}{2}\right) + \left\lfloor\frac{i-2}{6}\right\rfloor + \alpha + \left(\frac{i}{2}\right) + \left\lceil\frac{i-2}{6}\right\rceil = 3\alpha + 2i - 1$$

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$$\chi^*(v'_{2i-1}v_{2i}) = \alpha - \left(\frac{i}{2}\right) - \left\lfloor\frac{i}{6}\right\rfloor + \alpha - \left(\frac{i}{2}\right) - \left\lceil\frac{i-2}{6}\right\rceil + \alpha - \left(\frac{i}{2}\right) - \left\lfloor\frac{i-2}{6}\right\rfloor = 3\alpha - 2i + 1$$

For $i = 2,4,6, \dots, 2\left\lceil\frac{n}{2}\right\rceil - 2$

$$\chi^*(v_{2i-1}v'_{2i+1}) = \alpha + \left(\frac{i}{2}\right) + \left\lfloor\frac{i}{6}\right\rfloor + \alpha + \left(\frac{i}{2}\right) + \left\lceil\frac{i-2}{6}\right\rceil + \alpha + \left(\frac{i}{2}\right) + \left\lfloor\frac{i}{6}\right\rfloor = 3\alpha + 2i$$

$$\chi^*(v'_{2i-1}v_{2i+1}) = \alpha - \left(\frac{i}{2}\right) - \left\lfloor\frac{i}{6}\right\rfloor + \alpha - \left(\frac{i}{2}\right) - \left\lceil\frac{i-2}{6}\right\rceil + \alpha - \left(\frac{i}{2}\right) - \left\lfloor\frac{i}{6}\right\rfloor = 3\alpha - 2i$$

$$\chi(v_2v'_2) = \alpha + \alpha + \alpha = 3\alpha$$

All the induced edge sums (weights) of the edges are distinct in all the above three cases. For if for some i, j $3\alpha + 2i + 1 = 3\alpha - 2j - 1$ then $i + j = -\frac{2}{3}$ which contradicts the fact that i, j are integers. Hence, the total edge irregularity strength of extended duplicate of comb graph is $tes(EDG(CB_n)) = \left\lceil\frac{4n+3}{3}\right\rceil$. Illustration of irregular 5 labeling of $EDG(CB_3)$ is given in Figure 3.

Algorithm 2.10 : Assignment of labels to the vertices and edges of extended duplicate graph of twig graph.

The vertices of $EDG(T_n)$ are labeled as below:

$$\alpha = \left\lceil\frac{6n+5}{3}\right\rceil$$

$\chi(v_1) = 1$ $\qquad$ $\chi(v'_1) = 2n + 2$

$$\chi(v_{3i}) = \begin{cases} i+1, i = 1,3,5, \dots, 2\left\lceil\frac{n}{2}\right\rceil - 1 \\ 2n+2-i, i = 2,4,6, \dots, 2\left\lfloor\frac{n}{2}\right\rfloor \end{cases}$$

$$\chi(v_{3i+1}) = \begin{cases} i+1, i = 1,3,5, \dots .2\left\lceil\frac{n}{2}\right\rceil - 1 \\ 2n+2-i+1, i = 2,4,6, \dots, 2\left\lfloor\frac{n}{2}\right\rfloor \end{cases}$$

$$\chi(v_{3i-1}) = \begin{cases} 2n+2-i, i = 1,3,5, \dots .2\left\lceil\frac{n}{2}\right\rceil - 1 \\ i+1, i = 2,4,6, \dots, 2\left\lfloor\frac{n}{2}\right\rfloor \end{cases}$$

$$\chi(v'_{3i}) = \begin{cases} i, \quad i = 2,4,6, \dots, 2\left\lfloor\frac{n}{2}\right\rfloor \\ 2n+2-i, i = 1,3,5, \dots .2\left\lceil\frac{n}{2}\right\rceil - 1 \end{cases}$$

$$\chi(v'_{3i-1}) = \begin{cases} i, i = 1,3,5, \dots .2\left\lceil\frac{n}{2}\right\rceil - 1 \\ 2n+2-i, i = 2,4,6, \dots, 2\left\lfloor\frac{n}{2}\right\rfloor \end{cases}$$

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$$\chi(v'_{3i+1}) = \begin{cases} i+1, & i = 2,4,6,\dots,2\left\lfloor\frac{n}{2}\right\rfloor \\ 2n+3-i, & i = 1,3,5,\dots\dots,2\left\lceil\frac{n}{2}\right\rceil - 1 \end{cases}$$

$$\chi(v_{3n+2}) = \begin{cases} n+2, & n \text{ is odd} \\ n+1, & n \text{ is even} \end{cases}$$

$$\chi(v'_{3n+2}) = n+1$$

The edges of $EDG(T_n)$ are labeled as below:

$\chi(v_1v'_2) = 1$ $\quad\chi(v'_1v_2) = 2n+2$ $\quad\chi(v_{3n-1}v'_{3n-1}) = \begin{cases} n+1, n \text{ is even} \\ n+2, n \text{ is odd} \end{cases}$

$$\chi(v_{3i-1}v'_{3i}) = \begin{cases} 2n+2-i, i = 1,3,5,,2\left\lceil\frac{n}{2}\right\rceil - 1 \\ i, i = 2,4,6,\dots,2\left\lfloor\frac{n}{2}\right\rfloor \end{cases}$$

$$\chi(v'_{3i-1}v_{3i}) = \begin{cases} i, i = 1,3,5,\dots,2\left\lceil\frac{n}{2}\right\rceil - 1 \\ 2n+2-i, i = 2,4,6,\dots,2\left\lfloor\frac{n}{2}\right\rfloor \end{cases}$$

$$\chi(v_{3i-1}v'_{3i+1}) = \begin{cases} 2n+2-i, i = 1,3,5,\dots,2\left\lceil\frac{n}{2}\right\rceil - 1 \\ i, & i = 2,4,6,\dots,2\left\lfloor\frac{n}{2}\right\rfloor \end{cases}$$

$$\chi(v'_{3i-1}v_{3i+1}) = \begin{cases} i+1\ , & i = 1,3,5\dots,\left\lceil\frac{n}{2}\right\rceil - 1 \\ 2n+3-i, & i = 2,4,6,\dots,2\left\lfloor\frac{n}{2}\right\rfloor \end{cases}$$

$$\chi(v_{3i-1}v'_{3i+2}) = \begin{cases} 2n+2-i, & i = 1,3,5,\dots,2\left\lceil\frac{n}{2}\right\rceil - 1 \\ i+1, & i = 2,4,6,\dots,2\left\lfloor\frac{n}{2}\right\rfloor \end{cases} \qquad \chi(v'_{3i-1}v_{3i+2}) = \begin{cases} i+1, & i = 1,3,5,\dots,2\left\lceil\frac{n}{2}\right\rceil - 1 \\ 2n+2-i, & i = 2,4,6,\dots,2\left\lfloor\frac{n}{2}\right\rfloor \end{cases}$$

Theorem 2.11 Total edge irregularity strength $tes(EDG(T_n)$ of extended duplicate graph of twig graph is $2n+2$

Proof: By algorithm 2.10 , $2n+2$ is the maximum value of label for the vertex v'_1.

The induced edge sums (weights) of the edges are computed using

$\chi^*(uv) = \chi(u) + \chi(uv) + \chi(v)$ as

$\chi^*(v_1v'_2) = 3$ $\quad\chi^*(v'_1v_2) = 6n+5$

For $i = 1,3,5,,2\left\lceil\frac{n}{2}\right\rceil - 1$

$$\chi^*(v_{3i-1}v'_{3i}) = 2n+2-i+2n+2-i+2n+2-i = 6n+6-3i$$

$$\chi^*(v'_{3i-1}v_{3i}) = i+i+i+1 = 3i+1$$

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$\chi^*(v_{3i-1}v'_{3i+1}) = 2n+2-i+2n+2-i+2n+3-i = 6n+7-3i$

$\chi^*(v'_{3i-1}v_{3i+1}) = i+i+1+i+1 = 3i+2$

$\chi^*(v_{3i-1}v'_{3i+2}) = 2n+2-i+2n+2-i+2n+2-i-1 = 6n+5-3i$

$\chi^*(v'_{3i-1}v_{3i+2}) = i+i+1+i+2 = 3i+3$

For $i = 2,4,6,\dots,2\left\lfloor\frac{n}{2}\right\rfloor$

$\chi^*(v_{3i-1}v'_{3i}) = i+1+i+i = 3i+1$

$\chi^*(v'_{3i-1}v_{3i}) = 2n+2-i+2n+2-i+2n+2-i = 6n+6-3i$

$\chi^*(v_{3i-1}v'_{3i+1}) = i+1+i+i+1 = 3i+2$

$\chi^*(v'_{3i-1}v_{3i+1}) = 2n+2-i+2n+2-i+2n+3-i = 6n+7-3i$

$\chi^*(v_{3i-1}v'_{3i+2}) = i+1+i+1+i+1 = 3i+3$

$\chi^*(v'_{3i-1}v_{3i+2}) = 2n+2-i+2n+2-i+2n+2-i-1 = 6n+5-3i$

$$\chi^*(v_{3n-1}v'_{3n-1}) = \begin{cases} 2n+2-n+n+2+n, & n \text{ is even} \\ n+1+n+1+2n+2-n, & n \text{ is odd} \end{cases}$$

$= 3n+4$

The induced edge sums (weights) are distinct. For if, for any $1 \le i \le n$, $3\alpha - 3i + 1 = 3n + 4$ then $i = n + 1$ which contradicts the choice of range of values taken by i. Hence, total edge irregularity strength of extended duplicate graph of twig graph is $tes(EDG(T_n))= 2n+2$. Illustration of irregular 6-labeling of $EDG(T_2)$ is given in Figure 4.

ILLUSTRATION

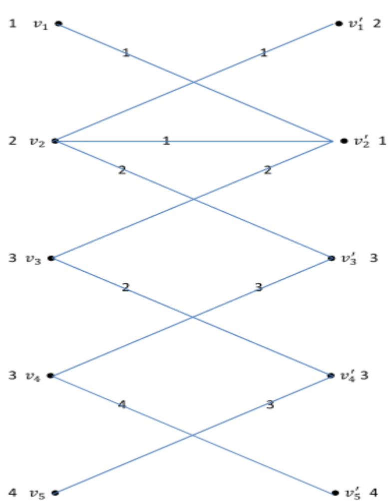

Figure 1

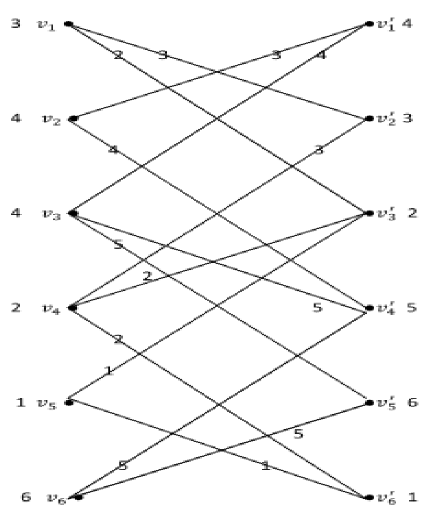

Figure 2

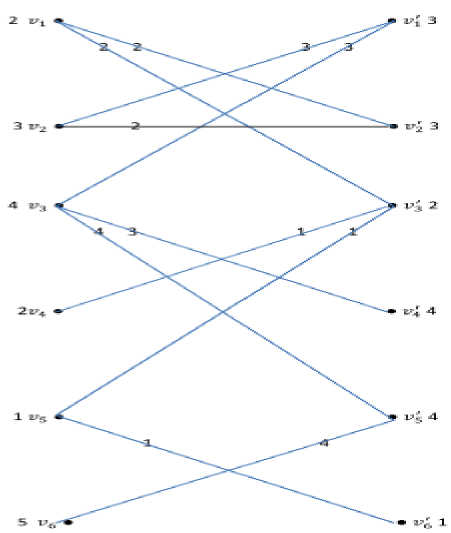

Figure 3

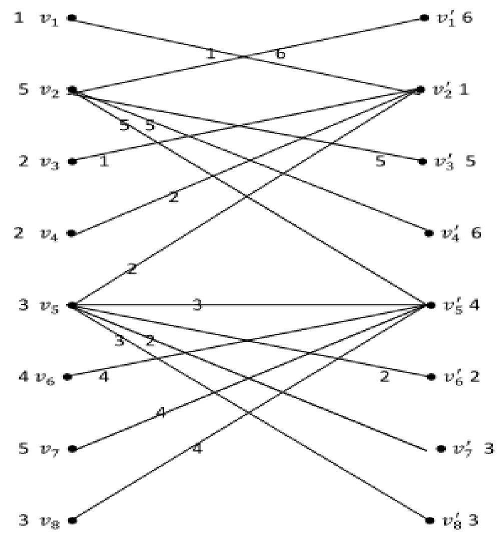

Figure 4

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