JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 723 - 729
https://publishoa.com
ISSN: 1309-3452

Fuzzy Fixed-Point Theorems for $\gamma -$ Weak Contractive Mappings in 3 Metric Spaces

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ABSTRACT

In this paper, we introduce some new concepts of contractions called $\gamma -$ weak contractions in 2 metric spaces and 3 metric spaces using fuzzy fixed point theorems. We correlate some fixed point theorems for mappings providing $\gamma -$ weak contractions which approach in new way for getting a better result compare to the previous results. Also, we provide a few examples to justify the better results obtained in this paper.

Keywords: fixed point, complete metric space, non-Archimedean, fuzzy set theory

AMS Mathematics Subject Classification (2000): 47H10, 54E50, 46S10, 03E72

I. Introduction

The fixed point theory is very important chamber in mathematics. In 1922, Banach created a famous result called Banach contraction principle in the concept of the fixed point theory (see [2]). On the way , most of the authors delivers many works regarding the fixed point theoryin various spaces. The concept of a fuzzy metric space was introduced in different ways by some authors (see [5,13]). Gregori and Sapena (see[1]) introduced the notion of fuzzy contractive mappings and gave some fixed point theorems for complete fuzzy metric spaces in the sense of George and Veeramani, and also for Kramosil and Michalek's fuzzy metric spaces which are complete in Grabiec's sense. Mihet provided the top notch of fuzzy contractive mappings of Gregori and Sapena, considered these mappings in non-Archimedean fuzzy metric spaces in the sense of Kramosil and Michalek, and obtained a fixed point theorem for fuzzy contractive mappings. At the same time, there are lots of different types of fixed point theorems given by many researchers on the basis of the Banach's results (see[6 – 11,16,17,18]).

In this work, using a mapping: $[0,1) \rightarrow R$ we introduce some new types of contractions called $\gamma -$ weak contractions in 2 metric spaces and 3 metric spaces using fuzzy fixed point theorems. Also, we prove some fixed point theorems for mappings providing $\gamma -$ weak contractions in 2 metric spaces and 3 metric spaces. Some examples are provided here to justify the uniqueness of our results. Our main approach to give a another way for mappings in 2 & 3 metric spaces and compared to the results which is already given.

II. Preliminaries

Definition 2.1: (see[15,4]) The 3-tuple $(X, \mathbb{M}, *)$ is called a fuzzy metric space if X is an arbitrary (non-empty) set, * is a continuous t-norm and $\mathbb{M}$ is a fuzzy set on $X^2 \times [0,\infty)$ satisfying the following conditions, for all $a, b, c \in X$, each s and $t > 0$

(i) $\mathbb{M}(a, b, 0) > 0$
(ii) $\mathbb{M}(a, b, t) = 1$ if and only if $a = b$,
(iii) $\mathbb{M}(a, b, t) = \mathbb{M}(b, a, t)$,
(iv) $\mathbb{M}(a, b, t) * \mathbb{M}(b, c, u) \leq \mathbb{M}(a, b, t + u)$,
(v) $\mathbb{M}(a, b, \diamond) : (0, \infty) \rightarrow [0,1]$ is left continuous.

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Volume 13, No. 2, 2022, p. 723 - 729
https://publishoa.com
ISSN: 1309-3452

If, in the above definition, the triangular inequality (iv) is replaced by
$\mathbb{M}(a,b,t) * \mathbb{M}(b,c,u) \leq \mathbb{M}(a,b,\max\{t,u\})$ for all a, $b,c \in X$, $t,u > 0$, or equivalently,

$$\mathbb{M}(a,b,t) * \mathbb{M}(b,c,t) \leq \mathbb{M}(a,c,t)$$

and then the triple $(X,\mathbb{M},*)$ is called a non-Archimediean fuzzy metric space (see [12]).

Lemma 2.1: (see [10]) $\mathbb{M}(a,b,c,.)$ is non-decreasing for all a, $b,c \in X$.

Definition 2.2 : (see [3]) Let $(X,\mathbb{M},*)$ is a fuzzy metric space.

(i) A sequence $\{a_n\}$ in X is said to be convergent to a point $a \in X$ and denoted by $\lim_{n\to\infty} a_n = a$ if $\lim_{n\to\infty} \mathbb{M}(a_n,a,t) = 1$, for all $t > 0$.

(ii) A sequence $\{a_n\}$ in X is said to be X Cauchy sequence if for all $t > 0$ and $p > 0$, $\lim_{n\to\infty} \mathbb{M}(a_{n+m},a_n,t) = 1$.

(iii) If every Cauchy sequence in a fuzzy metric space is convergent then it is complete.

Definition 2.3: A function $\mathbb{M}$ is continuous in fuzzy metric space if and only if whenever, $a_n \to a$, $b_n \to b$ then $\lim_{n\to\infty} \mathbb{M}(a_n,b_n,t) = \mathbb{M}(a,b,t)$, for each $t > 0$.

Definition 2.4: A function $\mathbb{M}$ is continuous in fuzzy 2 - metric space if and only if whenever, $a_n \to a$, $b_n \to b$ then $\lim_{n\to\infty} \mathbb{M}(a_n,b_n,s,t) = \mathbb{M}(a,b,s,t)$, for all $s \in X$ and $t > 0$.

Definition 2.5: A function $\mathbb{M}$ is continuous in fuzzy 3 - metric space if and only if whenever, $a_n \to a$, $b_n \to b$ then $\lim_{n\to\infty} \mathbb{M}(a_n,b_n,s,u,t) = \mathbb{M}(a,b,s,u,t)$, for all $s,u \in X$ and $t > 0$.

Definition 2.6: (see [14]) Let $\gamma: [0,1) \to \mathcal{R}$ be strictly increasing, continous mapping and for each sequence $\{u_n\}_{n\in\mathcal{N}}$ of positive numbers $\lim_{n\to\infty} u_n = 1$ iff $\lim_{n\to\infty} \gamma(u_n) = +\infty$. Let Γ be the family of all γ functions.

A mapping $\mathcal{T}: \mathbb{X} \to \mathbb{X}$ is said to be a $\gamma -$ contraction if there exists a $\sigma \in (0,1)$ such that

$$\mathbb{M}(Sa,Sb,t) < 1 \Rightarrow \gamma\big(\mathbb{M}(Sa,Sb,t)\big) \geq \gamma\big(\mathbb{M}(a,b,t)\big) + \sigma$$

For all $a,b \in \mathbb{X}$ and $\gamma \in \Gamma$.

III. Main Theorem

Definition 3.1:

Let $(\mathbb{X},\mathbb{M},*)$ be a non-Archimedean fuzzy 2 - metric space. A mapping $\mathcal{T}: \mathbb{X} \to \mathbb{X}$ is said to be a $\gamma -$ weak contraction if there exists $\sigma \in (0,1)$ such that $\mathbb{M}(Sa,Sb,s,qt) < 1$

$\Rightarrow \gamma\big(\mathbb{M}(Sa,Sb,s,qt)\big) \geq \gamma(\min\{\mathbb{M}(a,b,s,t),\mathbb{M}(a,Sb,s,t),\mathbb{M}(b,Sb,s,t)\}) + \sigma$ for all $a,b \in \mathbb{X}$ and $\gamma \in \Gamma$.

Remark 1: $\gamma -$ contraction is a $\gamma -$ weak contraction. But the converse of the theorem is not true.

Theorem 3.2: Let $(\mathbb{X},\mathbb{M},*)$ be a non-Archimedean fuzzy 2 - metric space and let $\mathcal{T}: \mathbb{X} \to \mathbb{X}$ be a $\gamma -$ weak contraction. Then $\mathcal{T}$ has a unique fixed point in $\mathbb{X}$.

Proof:

Let $u_0 \in \mathbb{X}$ be arbitrary and fixed. Then we define the sequence $\{u_n\}$ by $Su_n = u_{n+1}$ for all $n \in \mathcal{N}$.

If $u_n = u_{n+1}$ then u_{n+1} is a fixed point of $\mathcal{T}$; then the proof is enough. Suppose that $u_n \neq u_{n+1}$ for all $n \in \mathcal{N}$. Therefore by definition (3.1) , we get from

$$\gamma\big(\mathbb{M}(Su_{n-1},Su_n,s,qt)\big) \geq \gamma(\min\{\mathbb{M}(u_{n-1},u_n,s,t),\mathbb{M}(u_{n-1},Su_{n-1},s,t),\mathbb{M}(u_n,Su_n,s,t)\}) + \sigma$$

$= \gamma(\min\{\mathbb{M}(u_{n-1},u_n,s,t),\mathbb{M}(u_{n-1},u_n,s,t),\mathbb{M}(u_n,u_{n+1},s,t)\}) + \sigma$ --------- (1)

$$= \gamma(\min\{\mathbb{M}(u_{n-1},u_n,s,t),\mathbb{M}(u_n,u_{n+1},s,t)\}) + \sigma$$

If there exists $n \in \mathcal{N}$ such that,

$$\min\{\mathbb{M}(u_{n-1},u_n,s,t),\mathbb{M}(u_n,u_{n+1},s,t)\} = \mathbb{M}(u_n,u_{n+1},s,t)$$

From equation (1) becomes,

$$\gamma(\mathbb{M}(Su_n,Su_{n+1},s,t)) = \mathbb{M}(u_n,u_{n+1},s,t)$$
$$\geq \gamma(\mathbb{M}(u_n,u_{n+1},s,t)) + \sigma$$

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Volume 13, No. 2, 2022, p. 723 - 729
https://publishoa.com
ISSN: 1309-3452

$$> \mathbb{M}(u_n, u_{n+1}, s, t)$$

Which is contradiction. Therefore we go for,

$\min\{\mathbb{M}(u_{n-1}, u_n, s, t), \mathbb{M}(u_n, u_{n+1}, s, t)\} = \mathbb{M}(u_{n-1}, u_n, s, t)$ --------------- (2)

For all $n \in \mathcal{N}$. that is property of γ equation (1) and equation (2), we get

$$\mathbb{M}(u_n, u_{n+1}, s, t) > M(u_{n-1}, u_n, s, t)$$

Thus from equation (1) we have,

$$\gamma\big(\mathbb{M}(u_n, u_{n+1}, s, t)\big) \geq \gamma\big(\mathbb{M}(u_{n-1}, u_n, s, t)\big) + \sigma$$

For all $n \in \mathcal{N}$. this implies that

$\gamma\big(\mathbb{M}(u_n, u_{n+1}, s, t)\big) \geq \gamma\big(\mathbb{M}(u_{n-1}, u_n, s, t)\big) + n\sigma$ ---------------------------- (3)

By taking $n \to \infty$ in equation (3) we get,

$$\lim_{n\to\infty} \gamma\big(\mathbb{M}(u_n, u_{n+1}, s, t)\big) = \lim_{n\to\infty} \gamma\big(\mathbb{M}(Su_{n-1}, Su_n, s, t)\big)$$
$$= +\infty$$

Then, we have

$\lim_{n\to\infty} \gamma\big(\mathbb{M}(Su_{n-1}, Su_n, s, t)\big) = 1$ -- (4)

Already we know that let $(\mathbb{X}, \mathbb{M}, *)$ be a non-Archimedean fuzzy metric space and let $\mathcal{T}: \mathbb{X} \to \mathbb{X}$ be a $\gamma -$ contraction. Then $\mathcal{T}$ has a unique fixed point in $\mathbb{X}$ (see [14]).

Therefore the proof that $\{u_n\}$ is a Cauchy sequence can be shown as the completeness of $(\mathbb{X}, \mathbb{M}, *)$ there exists $p \in \mathbb{X}$ such that,

$\lim_{n\to\infty} u_n = p$ -- (5)

Now we show that p is a fixed point of $\mathcal{T}$. since γ is continuous, there are two type of cases exists.

Case 1. For each $n \in \mathcal{N}$, there exists $i_n \in \mathcal{N}$ such that $i_{n+1} = Sp$ and $i_n > i_{n-1}$, where $i_0 = 1$. then we get,

$$p = \lim_{n\to\infty} \mathrm{u}i_{n+1} = \lim_{n\to\infty} Sp = Sp$$

This proves that p is a fixed point of $\mathcal{T}$.

Case 2. There exists $n_0 \in \mathcal{N}$ such that $u_{n+1} \neq Sp$ for all $n \geq n_0$. That is, $\mathbb{M}(Su_n, Su, s, t) < 1$for all $n \geq n_0$. it follows from definition 3.1 property of the $\gamma -$ contraction,

$$\gamma\big(\mathbb{M}(u_{n+1}, Sp, s, t)\big) = \gamma\big(\mathbb{M}(Su_n, Sp, s, t)\big)$$
$$\geq \gamma(\min\{\mathbb{M}(u_n, p, s, t), \mathbb{M}(u_n, Su_n, s, t), \mathbb{M}(p.Sp, s, t)\}) + \sigma \text{ -------------------- (6)}$$
$$= \gamma(\min\{\mathbb{M}(u_n, p, s, t), \mathbb{M}(u_n, Su_{n+1}, s, t), \mathbb{M}(p.Sp, s, t)\}) + \sigma$$

If $\mathbb{M}(p.Sp, s, t) < 1$, then we have

$$\underset{n\to\infty}{lim} \mathbb{M}(u_n, p, s, t) = 1$$

and there exists $n_1 \in \mathcal{N}$ such that for all $n \geq n_1$, we get

$$\min\{\mathbb{M}(u_n, p, s, t), \mathbb{M}(u_n, u_{n+1}, s, t), \mathbb{M}(p, Sp, s, t)\} = \mathbb{M}(p, Sp, s, t)$$

From the equation (6) we get,

$\gamma\big(\mathbb{M}(u_{n+1}, Sp, s, t)\big) \geq \gamma\big(\mathbb{M}(p, Sp, s, t)\big) + \sigma$ ---------------------------- (7)

For all $n \geq \max\{n_0, n_1\}$. Since γ is continuous and we take the limit as $n \to \infty$ in equation (7) then we find $\gamma\big(\mathbb{M}(p, Sp, s, t)\big) \geq \gamma\big(\mathbb{M}(p, Sp, s, t)\big) + \sigma$

Then it leads to contradiction. Therefore, $\mathbb{M}(p, Sp, s, t) = 1$; that is, p is a fixed point of $\mathcal{T}$.

Next, we justify that the fixed point of $\mathcal{T}$ is unique. Let p_1, p_2 be two fixed points of $\mathcal{T}$. Suppose that $p_1 \neq p_2$; then we have $Sp_1 \neq Sp_2$. On this way according to the definition of $\gamma -$ weak contraction we represent,

$$\gamma\big(\mathbb{M}(p_1, p_2, s, t)\big) = \gamma\big(\mathbb{M}(p_1, p_2, s, t)\big)$$
$$\geq \gamma(\min\{\mathbb{M}(p_1, p_2, s, t), \mathbb{M}(p_1, Sp_1, s, t), \mathbb{M}(p_2, Sp_2, s, t)\}) + \sigma$$

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Volume 13, No. 2, 2022, p. 723 - 729
https://publishoa.com
ISSN: 1309-3452

$$= \gamma(\min\{\mathbb{M}(p_1, p_2, s, t), \mathbb{M}(p_1, p_1, s, t), \mathbb{M}(p_2, p_2, s, t)\}) + \sigma$$
$$= \gamma\big(\mathbb{M}(p_1, p_2, s, t)\big) + \sigma$$

leads to contradiction. Then, $\mathbb{M}(p_1, p_2, s, t) = 1$, that is $p_1 = p_2$. Therefore, the fixed point of $\mathcal{T}$ is unique.

Example 3.3:

Let $(\mathbb{X}, \mathbb{M}, *)$ be the non-Archimedian fuzzy 2 - metric space and let $\mathcal{T}: \mathbb{X} \to \mathbb{X}$ by

$$\mathcal{T}(a) = \begin{cases} \frac{1}{10}, & a \in A_1. \\ \frac{1}{2}, & a \in A_2. \end{cases}$$

Let $\mathbb{X} = A_1 \cup A_2$ where $A_1 = \{{}^1/_{10}, {}^1/_2, 1,2,3\}$, $A_2 = \{4,6\}$. $u * v = \min\{u, v\}$ and $\mathbb{M}(a, b, s, t) = \min\{a, b\} \big/ \max\{a, b\}$ for all $s, t > 0$. And let $\gamma: [0,1) \to \mathbb{R}$ such that $\gamma(a) = {}^1/_{(1 - a^2)}$ for all $a \in [0,1)$. s

Case 1: Let $a = 1$ and $b \in A_2$,

$\mathbb{M}(Sa, Sb, s, t) = \frac{1}{5} > \frac{1}{2b} = \min\{\frac{a}{b}, \frac{1}{10a}, \frac{1}{2b}\}$
$= \min\{\mathbb{M}(a, b, s, t), \mathbb{M}(a, Sa, s, t), \mathbb{M}(b, Sb, s, t)\}$

Then we have,

$\gamma\left(\frac{1}{1-\left(\frac{1}{5}\right)^2}\right) > \gamma\left(\frac{1}{1-\left(\frac{1}{2b}\right)^2}\right)$. So there exists $\sigma \in (0,1)$ such that,

$$\gamma(\mathbb{M}(Sa, Sb, s, t)) \geq \gamma(\min\{\mathbb{M}(a, b, s, t), \mathbb{M}(a, Sa, s, t), \mathbb{M}(b, Sb, s, t)\}) + \sigma$$

Case 2: Let $a \in \{2,3\}$ and $b \in A_2$,

$\mathbb{M}(Sa, Sb, s, t) = \frac{1}{5} > \frac{1}{10a} = \min\{\frac{a}{b}, \frac{1}{10a}, \frac{1}{2b}\}$
$= \min\{\mathbb{M}(a, b, s, t), \mathbb{M}(a, Sa, s, t), \mathbb{M}(b, Sb, s, t)\}$

Then we have,

$\gamma\left(\frac{1}{1-\left(\frac{1}{5}\right)^2}\right) > \gamma\left(\frac{1}{1-\left(\frac{1}{10a}\right)^2}\right)$. So there exists $\sigma \in (0,1)$ such that,

$$\gamma(\mathbb{M}(Sa, Sb, s, t)) \geq \gamma(\min\{\mathbb{M}(a, b, s, t), \mathbb{M}(a, Sa, s, t), \mathbb{M}(b, Sb, s, t)\}) + \sigma$$

Case 3: Let $a \in \{{}^1/_{10}, {}^1/_2\}$ and $b \in A_2$,

$\mathbb{M}(Sa, Sb, s, t) = \frac{1}{5} > \frac{a}{b} = \min\{\frac{a}{b}, \frac{1}{10a}, \frac{1}{2b}\}$
$= \min\{\mathbb{M}(a, b, s, t), \mathbb{M}(a, Sa, s, t), \mathbb{M}(b, Sb, s, t)\}$

Then we have,

$\gamma\left(\frac{1}{1-\left(\frac{1}{5}\right)^2}\right) > \gamma\left(\frac{1}{1-\left(\frac{a}{b}\right)^2}\right)$. So there exists $\sigma \in (0,1)$ such that,

$$\gamma(\mathbb{M}(Sa, Sb, s, t)) \geq \gamma(\min\{\mathbb{M}(a, b, s, t), \mathbb{M}(a, Sa, s, t), \mathbb{M}(b, Sb, s, t)\}) + \sigma$$

Therefore, $\mathcal{T}$ is a $\gamma -$ weak contraction and unique fixed point of $\mathcal{T}$ is $1/10$.

Theorem 3.4: Let $(\mathbb{X}, \mathbb{M}, *)$ be a non-Archimedean fuzzy 3 - metric space and let $\mathcal{T}: \mathbb{X} \to \mathbb{X}$ be a $\gamma -$ weak contraction. Then $\mathcal{T}$ has a unique fixed point in $\mathbb{X}$.

Proof:

Let $u_0 \in \mathbb{X}$ be arbitrary and fixed. Then we define the sequence $\{u_n\}$ by $Su_n = u_{n+1}$ for all $n \in \mathcal{N}$.

If $u_n = u_{n+1}$ then u_{n+1} is a fixed point of $\mathcal{T}$; then the proof is enough. Suppose that $u_n \neq u_{n+1}$ for all $n \in \mathcal{N}$. Therefore by definition (3.1), we get from

$$\gamma\big(\mathbb{M}(Su_{n-1}, Su_n, s, r, qt)\big) \geq \gamma(\min\{\mathbb{M}(u_{n-1}, u_n, s, r, t), \mathbb{M}(u_{n-1}, Su_{n-1}, s, r, t), \mathbb{M}(u_n, Su_n, s, r, t)\}) + \sigma$$
$$= \gamma(\min\{\mathbb{M}(u_{n-1}, u_n, s, r, t), \mathbb{M}(u_{n-1}, u_n, s, r, t), \mathbb{M}(u_n, u_{n+1}, s, r, t)\}) + \sigma$$

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Volume 13, No. 2, 2022, p. 723 - 729
https://publishoa.com
ISSN: 1309-3452

$= \gamma(\min\{\mathbb{M}(u_{n-1}, u_n, s, r, t), \mathbb{M}(u_n, u_{n+1}, s, r, t)\}) + \sigma$ --------------- (8)

If there exists $n \in \mathcal{N}$ such that,

$$\min\{\mathbb{M}(u_{n-1}, u_n, s, r, t), \mathbb{M}(u_n, u_{n+1}, s, r, t)\} = \mathbb{M}(u_n, u_{n+1}, s, r, t)$$

From equation (8) becomes,

$$\begin{aligned} \gamma(\mathbb{M}(Su_n, Su_{n+1}, s, r, t)) &= \mathbb{M}(u_n, u_{n+1}, s, r, t) \\ &\geq \gamma(\mathbb{M}(u_n, u_{n+1}, s, r, t)) + \sigma \\ &> \mathbb{M}(u_n, u_{n+1}, s, r, t) \end{aligned}$$

Which is contradiction. Therefore,

$\min\{\mathbb{M}(u_{n-1}, u_n, s, r, t), \mathbb{M}(u_n, u_{n+1}, s, r, t)\} = \mathbb{M}(u_{n-1}, u_n, s, r, t)$ ---------- (9)

For all $n \in \mathcal{N}$. that is property of γ equation (8) and equation (9) ,we get

$$\mathbb{M}(u_n, u_{n+1}, s, r, t) > M(u_{n-1}, u_n, s, r, t)$$

Thus from equation (8) we have,

$$\gamma\big(\mathbb{M}(u_n, u_{n+1}, s, r, t)\big) \geq \gamma\big(\mathbb{M}(u_{n-1}, u_n, s, r, t)\big) + \sigma$$

For all $n \in \mathcal{N}$. this implies that

$\gamma\big(\mathbb{M}(u_n, u_{n+1}, s, r, t)\big) \geq \gamma\big(\mathbb{M}(u_{n-1}, u_n, s, r, t)\big) + n\sigma$ ------------------------- (10)

By taking $n \to \infty$ in equation (10) we get

$$\begin{aligned} \lim_{n\to\infty} \gamma\big(\mathbb{M}(u_n, u_{n+1}, s, r, t)\big) &= \lim_{n\to\infty} \gamma\big(\mathbb{M}(Su_{n-1}, Su_n, s, r, t)\big) \\ &= +\infty \end{aligned}$$

Then, we have

$$\lim_{n\to\infty} \gamma\big(\mathbb{M}(Su_{n-1}, Su_n, s, r, t)\big) = 1$$

Already we know that let $(\mathbb{X}, \mathbb{M}, *)$ be a non-Archimedean fuzzy metric space and let $\mathcal{T}: \mathbb{X} \to \mathbb{X}$ be a $\gamma -$ contraction. Then $\mathcal{T}$ has a unique fixed point in $\mathbb{X}$ (see [14]).

Therefore the proof that $\{u_n\}$ is a Cauchy sequence can be shown as the completeness of $(\mathbb{X}, \mathbb{M}, *)$ there exists $p \in \mathbb{X}$ such that,

$$\lim_{n\to\infty} u_n = p$$

Now we show that p is a fixed point of $\mathcal{T}$. since γ is continuous, there are two type of cases exists.

Case 1. For each $n \in \mathcal{N}$, there exists $i_n \in \mathcal{N}$ such that $i_{n+1} = Sp$ and $i_n > i_{n-1}$, where $i_0 = 1$. then we get,

$$p = \lim_{n\to\infty} \mathrm{u}i_{n+1} = \lim_{n\to\infty} Sp = Sp$$

This proves that p is a fixed point of $\mathcal{T}$.

Case 2. There exists $n_0 \in \mathcal{N}$ such that $u_{n+1} \neq Sp$ for all $n \geq n_0$. That is, $\mathbb{M}(Su_n, Su, s, r, t) < 1$for all $n \geq n_0$. it follows from definition 3.1 have the property of the $\gamma -$ contraction,

$$\gamma\big(\mathbb{M}(u_{n+1}, Sp, s, r, t)\big) = \gamma\big(\mathbb{M}(Su_n, Sp, s, r, t)\big)$$

$\geq \gamma(\min\{\mathbb{M}(u_n, p, s, r, t), \mathbb{M}(u_n, Su_n, s, r, t), \mathbb{M}(p.Sp, s, r, t)\}) + \sigma$ -------- (11)

$$= \gamma(\min\{\mathbb{M}(u_n, p, s, r, t), \mathbb{M}(u_n, Su_{n+1}, s, r, t), \mathbb{M}(p.Sp, s, r, t)\}) + \sigma$$

If $\mathbb{M}(p.Sp, s, r, t) < 1$, then we have

$$\lim_{n\to\infty} \mathbb{M}(u_n, p, s, r, t) = 1$$

and there exists $n_1 \in \mathcal{N}$ such that for all $n \geq n_1$, we get

$$\min\{\mathbb{M}(u_n, p, s, r, t), \mathbb{M}(u_n, u_{n+1}, s, r, t), \mathbb{M}(p, Sp, s, r, t)\} = \mathbb{M}(p, Sp, s, r, t)$$

From the equation (11) we get,

$\gamma\big(\mathbb{M}(u_{n+1}, Sp, s, r, t)\big) \geq \gamma\big(\mathbb{M}(p, Sp, s, r, t)\big) + \sigma$ ---------------------- (12)

For all $n \geq \max\{n_0, n_1\}$, since γ is continuous and we take the limit as $n \to \infty$ in equation (12) then we find $\gamma\big(\mathbb{M}(p, Sp, s, r, t)\big) \geq \gamma\big(\mathbb{M}(p, Sp, s, r, t)\big) + \sigma$

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Volume 13, No. 2, 2022, p. 723 - 729
https://publishoa.com
ISSN: 1309-3452

Then which is contradiction. Therefore, $\mathbb{M}(p, Sp, s, r, t) = 1$; that is, p is a fixed point of $\mathcal{T}$.

Next, we prove that the fixed point of $\mathcal{T}$ is unique. Let p_1, p_2 be two fixed points of $\mathcal{T}$. Suppose that $p_1 \neq p_2$; then we have $Sp_1 \neq Sp_2$.

It follows by the definition of $\gamma -$ weak contraction we have,

$$\gamma\big(\mathbb{M}(p_1, p_2, s, r, t)\big) = \gamma\big(\mathbb{M}(p_1, p_2, s, r, t)\big) \geq \gamma(\min\{\mathbb{M}(p_1, p_2, s, r, t), \mathbb{M}(p_1, Sp_1, s, r, t), \mathbb{M}(p_2, Sp_2, s, r, t)\}) + \sigma$$
$$= \gamma(\min\{\mathbb{M}(p_1, p_2, s, r, t), \mathbb{M}(p_1, p_1, s, r, t), \mathbb{M}(p_2, p_2, s, r, t)\}) + \sigma \ = \gamma\big(\mathbb{M}(p_1, p_2, s, r, t)\big) + \sigma$$

which is contradiction. Then, $\mathbb{M}(p_1, p_2, s, r, t) = 1$, that is $p_1 = p_2$. Therefore, the fixed point of $\mathcal{T}$ is unique.

IV.Conclusion

In this paper, we found out the new contraction types in non-Archimedean fuzzy metric spaces and presented new fixed point results on the basis of our new contraction approach. Our results can be explored to given better results in solutions to new problems can be produced in this way. Also, a new way of contraction can be achieved or common fixed point theorems for a class of mappings can be obtained using $\gamma-$ weak contractions and also our result provided better results to various spaces.

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