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The Exponential-X Power Function (NEXPF) Distribution and its Applications to Modelling Reliability

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ABSTRACT

When lifetime models have non-monotone hazard functions, the best fitting leads to accurate estimates and predictions in reliability studies; This is the purpose of the new Exponential-X Power function (NEXPF) distribution. This belongs to the new exponential-X (NEX) family proposed to be a better fitting model for some reliability models; it beats competitive distributions like the Power function distribution. Therefore, we concentrated our efforts on introducing a new model. Our goal is to study the properties of its statistical measures throughout this research of the NEXPF distribution, the reliability function estimation method MLE and MPS; The numerical simulation is detailed to evaluate the proposed estimation methods.

Introduction

Real-life events and natural processes are being modelled using probability distributions in statistics and probability, where these processes are complex and risky. Due to these reasons, statisticians have worked on developing probability distributions since proven probability distributions cannot accurately describe data obtained from natural events. This allows generalized probability distributions to be expanded and modified. The availability of additional parameters led to the emergence of generalized probability distributions. It has been shown that data obtained from natural phenomena and the tail form of the distribution are more accurate when a particular parameter is added to the probability functions already existing.

In the past few years, various research works have been undertaken to create new distributions by adding new shape parameters to classical distributions to create new families and classes. Within the literature, there are a wide variety of distribution classes. See references for examples [2,4,5,11,13,14,15,19].

The exponential distribution is one of the best known lifetime distributions in statistics, which statisticians have therefore given considerable attention. An exponential-X (NEX) family of distributions is derived from exponential distribution , Introduced by Huo et al. [10]. It can represent data in various forms for risk and reliability functions. The distributive function is defined by Cumulative density function (CDF) and probability density function (PDF) of the NEX family with the following formula

$$G(x;\eta) = 1 - \frac{1-F(x;\xi)}{e^{\lambda F(x;\xi)}} \quad ,x>0,\lambda>0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$g(x;\eta) = \frac{f(x;\xi)}{e^{\lambda F(x;\xi)}}.\left[1+\lambda\big(1-F(x;\xi)\big)\right] \quad ,x>0\,,\lambda>0 \quad \ldots\ldots \quad (2)$$

where the vector of the parameters is denoted by $\boldsymbol{\eta}$ of the family. It consists of ($\boldsymbol{\xi}$), the vectors of parameters for the classical distribution, and (λ), the additional shape parameter for the family.

The power function distribution is one of the important and widely used distributions in modelling data related to failure and survival times of electrical devices and equipment and representing life or death data for living organisms.[1][12]. let x_i a random variable that follows The power function distribution, if its CDF and PDF are written as follows:

$$G(x) = \left(\frac{x}{\alpha}\right)^{\beta} \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (3)$$

$$g(x) = \frac{\beta\, x^{\beta-1}}{\alpha^{\beta}} \qquad 0 < x < \alpha, \alpha, \beta > 0 \quad \dots\dots\dots\dots\dots\dots \quad (4)$$

The power function distribution has received the attention of a large number of authors, such as M. H. Tahir[17], where the power function distribution is generalized, So are the author's Bursa et al. [7], He proposed the power function of the Coomaraswamy exponent and its properties were studied.[9] Amal et al., Provide a probability distribution model consisting of four parameters called(OGEPF) Using the generative family odd generalized exponential.[6] Monje SAMUH et al., Introduce the cubic transformed power function distribution (CTPFD). [18] Zaka et al., generalized power function distribution (EGPFD) is proposed. [8] Ekum et al. proposed a new distribution of four parameters called Normal-Power {logistic} distribution.

This article aims to introduce the new Exponential-X Power function (NEXPF) distribution that is a good fit for age models with increasing and decreasing rates. We have studied its mathematical properties such as central and eccentric moments, quantitative function, generation function and characteristic function. Moreover, approaches to estimating the reliability function such as maximum likelihood and a maximum spacing between products using accurate data are discussed. The Monte Carlo algorithm (MCMC) was used. An extensive simulation study was performed. A comparison was made between the proposed estimation techniques. Finally, the NEXPF distribution demonstrated its efficiency in fitting real-world data more than the power function distribution concerning real-world applications.

1- The New Exponential-X Power function distribution(NEXPF)

Based on the NEX family with a baseline distribution and a power function, we obtained a NEXPF distribution according to three parameters. We can easily derive the CDF and PDF of the NEXPF distribution with the help of the power function distribution and both equations (1) and (2), which can be given as:

$$G_{next}(x;\alpha,\beta,\lambda) = 1 - \frac{\left(1-\left(\frac{x}{\alpha}\right)^{\beta}\right)}{e^{\lambda\left(\frac{x}{\alpha}\right)^{\beta}}} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (5)$$

$$g_{nexpf}(x;\alpha,\beta,\lambda) = \frac{\beta\, x^{\beta-1}\left[1+\lambda\left(1-\left(\frac{x}{\alpha}\right)^{\beta}\right)\right]}{\alpha^{\beta} e^{\lambda\left(\frac{x}{\alpha}\right)^{\beta}}} \quad , \; 0 < x < \alpha, \alpha, \beta, \lambda > 0 \quad \dots\dots (6)$$

We can say that a random variable X is distributed according to the distribution of NEXPF with PDF as in equation (6) by $X \sim NEXPF(\alpha, \beta, \lambda)$.

The risk rate (HR) function for the NEXPF distribution is given by:

$$h(t;\eta) = \frac{\beta\, t^{\beta-1}\left\{1+\lambda\left(1-\left(\frac{t}{\alpha}\right)^{\beta}\right)\right\}}{\alpha^{\beta}\left(1-\left(\frac{t}{\alpha}\right)^{\beta}\right)} \quad \dots\dots\dots\dots\dots\dots\dots \quad (7)$$

The reliability function of the NEXPF distribution is provided by the following:

$$R(t;\eta) = e^{-\lambda\left(\frac{t}{\alpha}\right)^{\beta}}\left(1-\left(\frac{t}{\alpha}\right)^{\beta}\right) \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (8)$$

In order to make a reasonable study of the distribution, We chose different values for its parameters and graphs of its PDF and HR functions as well as the reliability function are shown in Figures 1 and 2and 3.

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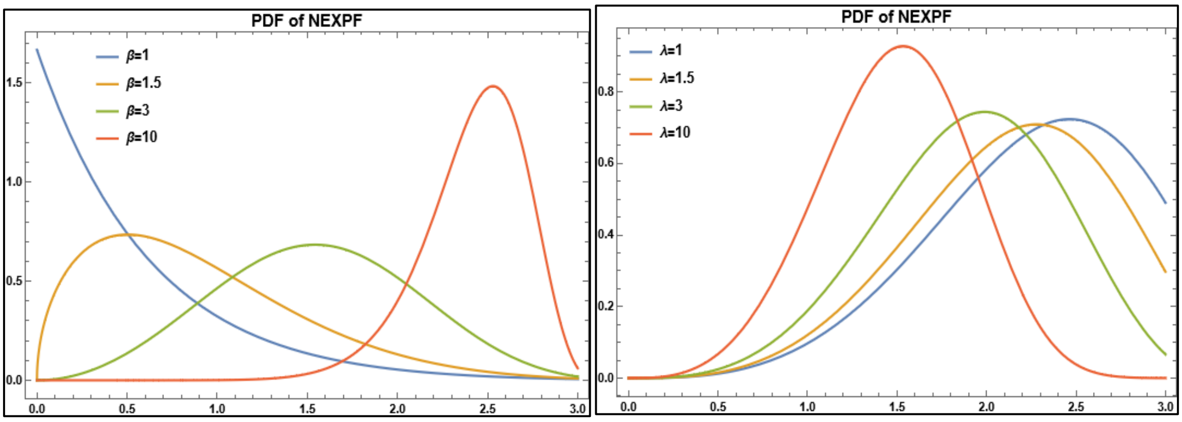

Figure 1_ PDF plots of NEXPF distribution

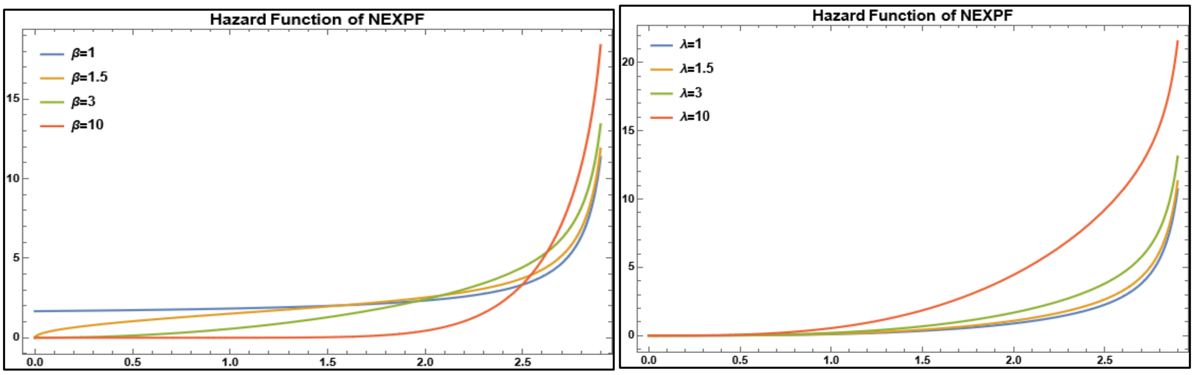

Figure 2_HR plots of NEXPF distribution.

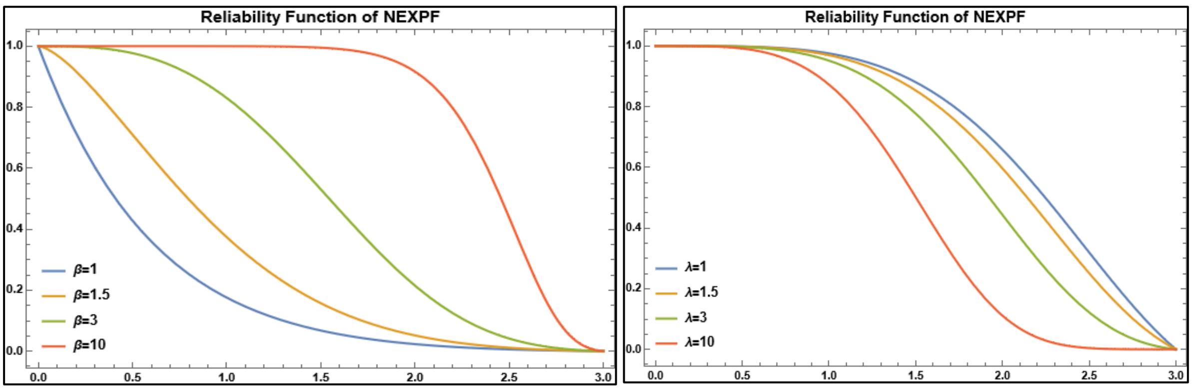

Figure 3_ the reliability function plots of NEXPF distribution.

2- Mathematical Properties

2-1: Non-central r^{th} moment

$$\mu_r' = E(x^r) = \int_0^{\alpha} x^r g_{nexpf}(x;\alpha,\beta,\lambda)\ dx$$

$$\mu_r' = E(x^r) = \frac{\alpha^r}{\lambda^{\frac{r}{\beta}}}\left(\frac{\Gamma\left(\frac{r+\beta}{\beta}\right) - \Gamma\left(\frac{r+\beta}{\beta},\lambda\right) - \Gamma\left(\frac{r+2\beta}{\beta}\right) + \Gamma\left(\frac{r+2\beta}{\beta},\lambda\right)}{\lambda} + \Gamma\left(\frac{r+\beta}{\beta}\right) - \Gamma\left(\frac{r+\beta}{\beta},\lambda\right)\right)$$

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$$\dots..(9)$$

$r = 1,2,3, \dots \dots$

2-2 : Central Moments r^{th}

$$\mu_r = E(x-\mu)^r = \int_0^\alpha (x-\mu)^r g_{nexpf}(x,\alpha,\beta,\lambda)\,dx$$

$$\mu_r = \sum_{k=0}^{r} \binom{r}{k} (-\mu)^{r-k} \frac{\alpha^k}{\lambda^{\frac{k}{\beta}}} \left\{ \begin{array}{c} \dfrac{\boldsymbol{\gamma}\left(\frac{k+\beta}{\beta},\boldsymbol{\lambda}\right) - \boldsymbol{\gamma}\left(\frac{k+2\beta}{\beta},\boldsymbol{\lambda}\right)}{\lambda} \\ +\boldsymbol{\gamma}\left(\frac{k+\beta}{\beta},\boldsymbol{\lambda}\right) \end{array} \right\} \quad \dots (10)$$

2-3: MomentGeneratingFunction(MGF)

$$M_x^{(t)} = E(e^{tx}) = \int_0^\alpha e^{tx}\, g_{nexpf}(x;\alpha,\beta,\lambda)\,dx$$

$$M_x^{(t)} = \sum_{k=0}^{\infty} \frac{(t)^k \alpha^k}{k!\ \lambda^{\frac{k}{\beta}}} \left\{ \frac{\boldsymbol{\gamma}\left(\frac{k+\beta}{\beta},\boldsymbol{\lambda}\right) - \boldsymbol{\gamma}\left(\frac{k+2\beta}{\beta},\boldsymbol{\lambda}\right)}{\lambda} + \boldsymbol{\gamma}\left(\frac{k+\beta}{\beta},\boldsymbol{\lambda}\right) \right\} \dots \dots (11)$$

2-4: Characteristic function φ(t)

$$\phi\,(t) = E(e^{itx}) = \int_0^\alpha e^{its}\, g_{nexpf}(x,\alpha,\beta,\lambda)\,dx$$

$$\phi\,(t) = \sum_{k=0}^{\infty} \frac{(it)^k \alpha^k}{k!\ \lambda^{\frac{k}{\beta}}} \left\{ \frac{\boldsymbol{\gamma}\left(\frac{k+\beta}{\beta},\boldsymbol{\lambda}\right) - \boldsymbol{\gamma}\left(\frac{k+2\beta}{\beta},\boldsymbol{\lambda}\right)}{\lambda} + \boldsymbol{\gamma}\left(\frac{k+\beta}{\beta},\boldsymbol{\lambda}\right) \right\} \dots \dots (12)$$

2-5: Quantile Function QF

We can express and determine the QF of the NEXPF distribution as the CDF inverse of equation (5), which is as follows:

$$Q(q) = e^{\frac{\ln\left(-\frac{\text{Lambert W}\left(\frac{\lambda\,(q-1)}{e^{\lambda}}\right)+\lambda}{\lambda}\right) + \beta\,\ln(\alpha)}{\beta}} \quad \dots\dots\dots\dots\dots \quad (13)$$

Where $W(.)$ is the Lambert function.

3- Estimation Methods

Estimation is an essential basis for statistical inference, and its importance lies in estimating the parameters of the community model based on the statistics resulting from the samples taken from it; two methods of estimation are addressed.

3-1: Maximum Likelihood Estimators

Suppose we have a sample order as follows such that $x_1, x_2, \dots \dots.., x_n$ It is a random sample from the NEXPF distribution. vector parameter $\eta = (\alpha, \beta, \lambda)$ Moreover, the likelihood function of the NEXPF distribution takes the form as follows:

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$$L(x_i\,;\eta) = \frac{\beta^n}{\alpha^{n\,\beta}}\,e^{-\frac{\lambda}{\alpha^\beta}\sum_{i=1}^{n}(x_i)^\beta}\prod_{i=1}^{n} x_i^{\beta-1}\left[1+\lambda\left(1-\left(\frac{x_i}{\alpha}\right)^\beta\right)\right] \;\ldots\ldots..(14)$$

$$\ln L\,(x_i\,;\eta) = n\ln(\beta) - n\,\beta\ln(\alpha) - \frac{\lambda}{\alpha^\beta}\sum_{i=1}^{n}(x_i)^\beta + (\beta-1)\sum_{i=1}^{n}\ln x_i + \sum_{i=1}^{n}\ln\left(1+\lambda\left(1-\left(\frac{x_i}{\alpha}\right)^\beta\right)\right) \;\ldots\ldots\ldots..(15)$$

By differentiating the equation (15), we get the following three equations:

$$\frac{\partial \ln L}{\partial\,\hat{\alpha}} = \left\{-\frac{n\,\beta}{\hat{\alpha}} + \frac{\lambda\,\beta\,\sum_{i=1}^{n}(x_i)^\beta}{\hat{\alpha}^{\beta+1}} - \sum_{i=1}^{n}\frac{\lambda\,\beta\,x_i^\beta}{\left(1+\lambda\left(1-\left(\frac{x_i}{\alpha}\right)^\beta\right)\right)\,\hat{\alpha}^{\,\beta+1}}\right\} = 0, \quad \ldots(16)$$

$$\frac{\partial \ln L}{\partial\,\hat{\beta}} = \left\{\begin{array}{c} \frac{n}{\hat{\beta}} - n\ln\alpha + \sum_{i=1}^{n}\ln x_i + \lambda\alpha^{-\hat{\beta}}\ln\alpha\sum_{i=1}^{n}(x_i)^{\hat{\beta}} \\ -\lambda\,\alpha^{-\hat{\beta}}\sum_{i=1}^{n}\ln x_i\;.(x_i)^{\hat{\beta}} - \sum_{i=1}^{n}\frac{\lambda\ln\left(\frac{x_i}{\alpha}\right).\;\left(\frac{x_i}{\alpha}\right)^{\hat{\beta}}}{\left(1+\lambda\left(1-\left(\frac{x_i}{\alpha}\right)^{\hat{\beta}}\right)\right)} \end{array}\right\} = 0, \ldots\ldots(17)$$

$$\frac{\partial \ln L}{\partial\,\hat{\lambda}} = \left\{-\frac{1}{\alpha^\beta}\sum_{i=1}^{n}(x_i)^\beta + \sum_{i=1}^{n}\frac{\left(1-\left(\frac{x_i}{\alpha}\right)^\beta\right)}{\left(1+\hat{\lambda}\left(1-\left(\frac{x_i}{\alpha}\right)^{\hat{\beta}}\right)\right)}\right\} = 0, \ldots\ldots\ldots\ldots\ldots\ldots(18)$$

Since the equations in (16)–(18) are not solved analytically, numerical approaches will be used to solve these equations, such as the Newton–Raphson method.

3-2: Maximum product of spacing estimation method

It is a powerful alternative to the MLE method for estimating non-existent parameters of a distribution. By referring to Ronneby et al. [16], and Almetwally et al. [3]. The log-MPSEs of the NEXPF distribution take the form as follows:

$$\ln H(\alpha,\beta,\lambda) = \frac{1}{n+1} * \left\{\begin{array}{c} \ln\left[e^{\lambda\left(\frac{x_1}{\alpha}\right)^\beta} - 1 + \left(\frac{x_1}{\alpha}\right)^\beta\right] - \lambda\left(\frac{x_1}{\alpha}\right)^\beta + \ln\left[1-\left(\frac{x_n}{\alpha}\right)^\beta\right] - \lambda\left(\frac{x_n}{\alpha}\right)^\beta + \\ \sum_{i=2}^{n}\ln\left[e^{\lambda\left(\frac{x_i}{\alpha}\right)^\beta}.\left(1-\left(\frac{x_{i-1}}{\alpha}\right)^\beta\right) - e^{\lambda\left(\frac{x_{i-1}}{\alpha}\right)^\beta}.\left(1-\left(\frac{x_i}{\alpha}\right)^\beta\right)\right] - \lambda\left(\frac{x_{i-1}}{\alpha}\right)^\beta - \lambda\left(\frac{x_i}{\alpha}\right)^\beta \end{array}\right\}$$

………(19)

The MPS of the distribution parameters can be derived through the following steps :

1-We take the partial derivative of the equation (19) concerning the parameters of the distribution.

2-We all know that solving these equations is very difficult, so use Newton's Raphson algorithm to solve the three equations.

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4-Simulation Study

As of now, for any distribution, we must evaluate its performance using different values of its parameters; Monte Carlo simulations are performed in this section to compare the performance of the methods used in the paper and determine the parameter behaviours using MLEs and MPS estimates of NEXPF parameters, based on the MATHEMATICA-12.2 program. A random sample was generated from the NEXPF distribution according to the following parameter sets.

As shown in the following tables, we used different parameter values with different sample sizes to estimate the reliability function. The comparison of score results by points is based on IMSE values. Tables 2-4 show the different simulation results of the estimation methods proposed in this paper.

Table 1: Initial default values for the proposed parameters and models

Models	α	β	λ
(1)	2	1	0.05
(2)	1	1.5	1
(3)	3	3	0.5

Table 2: It shows the actual values of the reliability function and its estimators in two ways: MLE, MPS, mean squared error, MSE, and mean integrated square error, IMSE. for the first model

n	t_i	R_real	R_MLE	MSE	R_MPS	MSE
25	0.361325	0.81197	0.81818	0.004307	0.728895	0.042576
	0.728456	0.624298	0.613952	0.007348	0.537489	0.039822
	0.794027	0.591135	0.577349	0.007504	0.506192	0.037218
	0.872652	0.55151	0.533673	0.007554	0.469178	0.034102
	1.10687	0.434377	0.405852	0.006958	0.362104	0.025081
	1.18494	0.395636	0.36427	0.006565	0.327439	0.022237
	1.43531	0.272394	0.235323	0.004898	0.219564	0.014036
	1.66488	0.16073	0.123879	0.003183	0.12811	0.007088
	1.71136	0.138274	0.102172	0.002861	0.111153	0.005691
	1.7231	0.132614	0.09674	0.002783	0.106894	0.00537
IMSE				0.005396		0.023322
Rank				1		2
n	t_i	R_real	R_MLE	MSE	R_MPS	MSE
50	0.361325	0.81197	0.820126	0.002205	0.811843	0.003048
	0.728456	0.624298	0.62138	0.003233	0.62005	0.004476
	0.794027	0.591135	0.585743	0.003287	0.586002	0.004543
	0.872652	0.55151	0.543153	0.003316	0.545354	0.004559
	1.10687	0.434377	0.417835	0.003202	0.425739	0.004232
	1.18494	0.395636	0.376764	0.003102	0.386452	0.004011
	1.43531	0.272394	0.248044	0.00259	0.262728	0.003027
	1.66488	0.16073	0.134609	0.001954	0.152583	0.001971
	1.71136	0.138274	0.112217	0.001826	0.13068	0.001772
	1.7231	0.132614	0.106597	0.001795	0.125174	0.001725
IMSE				0.002651		0.003336
Rank				1		2
n	t_i	R_real	R_MLE	MSE	R_MPS	MSE

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n	t_i	R_real	R_MLE	MSE	R_MPS	MSE
75	0.361325	0.81197	0.810009	0.002126	0.806192	0.002557
	0.728456	0.624298	0.614022	0.002945	0.614284	0.003276
	0.794027	0.591135	0.579294	0.002934	0.580438	0.003234
	0.872652	0.55151	0.537872	0.002876	0.540096	0.003136
	1.10687	0.434377	0.416285	0.002482	0.421748	0.002613
	1.18494	0.395636	0.376471	0.002293	0.382987	0.002382
	1.43531	0.272394	0.251588	0.001569	0.261272	0.001545
	1.66488	0.16073	0.141203	0.000864	0.153391	0.000802
	1.71136	0.138274	0.119359	0.000735	0.131997	0.000677
	1.7231	0.132614	0.113874	0.000704	0.126621	0.000648
IMSE				0.001953		0.002087
Rank				1		2
n	t_i	R_real	R_MLE	MSE	R_MPS	MSE
100	0.361325	0.81197	0.810742	0.000843	0.802072	0.00117103
	0.728456	0.624298	0.616431	0.001283	0.611651	0.00155636
	0.794027	0.591135	0.582144	0.001318	0.578277	0.00156835
	0.872652	0.55151	0.541269	0.001342	0.538516	0.00156508
	1.10687	0.434377	0.421241	0.001293	0.421755	0.00144402
	1.18494	0.395636	0.381868	0.001235	0.383405	0.00136689
	1.43531	0.272394	0.257909	0.000918	0.262336	0.00101159
	1.66488	0.16073	0.147494	0.000521	0.15388	0.00061442
	1.71136	0.138274	0.125523	0.000441	0.13221	0.000539334
	1.7231	0.132614	0.119999	0.000421	0.126756	0.000521219
IMSE				0.000961		0.001135829
Rank				1		2

Table 3: It shows the actual values of the reliability function and its estimators in two ways: MLE, MPS, mean squared error, MSE, and mean integrated square error, IMSE. for the second model

n	t_i	R_real	R_MLE	MSE	R_MPS	MSE
25	0.19959	0.833131	0.874104	0.00884622	0.814386	0.00493621
	0.376245	0.610689	0.684444	0.0335406	0.612309	0.00773169
	0.469995	0.49109	0.570523	0.0456923	0.503679	0.0079984
	0.494276	0.460961	0.540833	0.0468726	0.475885	0.00793322
	0.527531	0.42051	0.501206	0.0459047	0.43818	0.00775912
	0.588196	0.349599	0.426887	0.0413478	0.370726	0.00720977
	0.645141	0.286972	0.355131	0.034138	0.309307	0.00646875
	0.678001	0.252756	0.313052	0.0291368	0.274828	0.00597099
	0.736953	0.195134	0.236777	0.0201293	0.214913	0.00502205
	0.834462	0.110925	0.116101	0.0110377	0.12203	0.00358996
IMSE				0.031664602		0.006462016
Rank				2		1
n	t_i	R_real	R_MLE	MSE	R_MPS	MSE
50	0.19959	0.833131	0.83717	0.00252419	0.791061	0.00557607
	0.376245	0.610689	0.623296	0.00738579	0.57772	0.00600214
	0.469995	0.49109	0.50582	0.0104854	0.467174	0.00526799
	0.494276	0.460961	0.475963	0.0113108	0.43935	0.0050623

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n	t_i	R_real	R_MLE	MSE	R_MPS	MSE
	0.527531	0.42051	0.435725	0.0124029	0.401905	0.00478509
	0.588196	0.349599	0.364769	0.0140512	0.335793	0.00430603
	0.645141	0.286972	0.301608	0.014756	0.276592	0.00389075
	0.678001	0.252756	0.266832	0.0145939	0.243782	0.00366368
	0.736953	0.195134	0.207587	0.0130285	0.187518	0.00327489
	0.834462	0.110925	0.118348	0.00737154	0.104249	0.00206131
IMSE				0.010791022		0.004389025
Rank				2		1
n	t_i	R_real	R_MLE	MSE	R_MPS	MSE
75	0.19959	0.833131	0.84285	0.00303234	0.80991	0.00380564
	0.376245	0.610689	0.604806	0.0187316	0.599836	0.00397414
	0.469995	0.49109	0.492351	0.0163664	0.48846	0.00353114
	0.494276	0.460961	0.46366	0.0157806	0.460223	0.00340363
	0.527531	0.42051	0.424861	0.0149369	0.422104	0.00322501
	0.588196	0.349599	0.355928	0.0131418	0.354493	0.00288687
	0.645141	0.286972	0.293833	0.0110115	0.293636	0.00254916
	0.678001	0.252756	0.259299	0.00956718	0.259789	0.00234327
	0.736953	0.195134	0.19991	0.00667905	0.201563	0.00195661
	0.834462	0.110925	0.10937	0.00208282	0.112687	0.00132765
IMSE				0.011133019		0.002900312
Rank				2		1
n	t_i	R_real	R_MLE	MSE	R_MPS	MSE
100	0.19959	0.833131	0.841701	0.00231014	0.812547	0.00226743
	0.376245	0.610689	0.625448	0.00671071	0.597046	0.00290629
	0.469995	0.49109	0.50601	0.00798776	0.483325	0.0027154
	0.494276	0.460961	0.475525	0.0080673	0.454626	0.00263705
	0.527531	0.42051	0.434315	0.00798647	0.415987	0.00251668
	0.588196	0.349599	0.361203	0.00728362	0.347794	0.00226673
	0.645141	0.286972	0.29555	0.0060805	0.28685	0.00200535
	0.678001	0.252756	0.259164	0.00524313	0.253163	0.00184625
	0.736953	0.195134	0.196882	0.00375495	0.195611	0.00155429
	0.834462	0.110925	0.105778	0.00179801	0.10893	0.00110508
IMSE				0.005722259		0.002182055
Rank				2		1

Table 4: It shows the actual values of the reliability function and its estimators in two ways: MLE, MPS, mean squared error, MSE, and mean integrated square error, IMSE. for the third model

n	t_i	R_real	R_MLE	MSE	R_MPS	MSE
25	1.56953	0.797597	0.806924	0.00509445	0.678604	0.0784473
	1.57331	0.796218	0.805597	0.00512342	0.677405	0.0783684
	1.60066	0.786084	0.795824	0.00532913	0.668613	0.0777692
	2.272	0.455207	0.456261	0.00557683	0.391813	0.0391114
	2.43752	0.354548	0.346971	0.00449056	0.30659	0.026073
	2.444	0.350514	0.342562	0.00444909	0.303119	0.0255995
	2.49732	0.317133	0.30602	0.00411692	0.274228	0.0218361
	2.58358	0.262519	0.246115	0.00361716	0.22631	0.0162932

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	2.7313	0.168234	0.142825	0.0028716	0.141598	0.00867192
	2.80338	0.12237	0.0928329	0.0025883	0.101006	0.00553893
IMSE				0.004325746		0.037770895
Rank				1		2
n	t_i	R_real	R_MLE	MSE	R_MPS	MSE
50	1.56953	0.797597	0.803909	0.00323833	0.773613	0.0091395
	1.57331	0.796218	0.802544	0.00325953	0.772256	0.00916633
	1.60066	0.786084	0.792492	0.00341239	0.762288	0.00935357
	2.272	0.455207	0.454967	0.00490461	0.4376	0.00928073
	2.43752	0.354548	0.350177	0.00407678	0.338102	0.00803571
	2.444	0.350514	0.345969	0.00403392	0.334106	0.00798009
	2.49732	0.317133	0.31113	0.0036557	0.301098	0.00745438
	2.58358	0.262519	0.254084	0.00296428	0.248301	0.00582039
	2.7313	0.168234	0.155561	0.00170611	0.15674	0.00345271
	2.80338	0.12237	0.107648	0.00117371	0.113459	0.0021667
IMSE				0.003242536		0.007185011
Rank				1		2
n	t_i	R_real	R_MLE	MSE	R_MPS	MSE
75	1.56953	0.797597	0.80965	0.00136444	0.80172	0.00153044
	1.57331	0.796218	0.808282	0.00136949	0.800353	0.00153329
	1.60066	0.786084	0.798199	0.00140438	0.790292	0.00155203
	2.272	0.455207	0.456113	0.00137091	0.454527	0.00142345
	2.43752	0.354548	0.350399	0.00122115	0.352091	0.00132507
	2.444	0.350514	0.346175	0.00121401	0.348004	0.001319
	2.49732	0.317133	0.311279	0.00114948	0.314261	0.00125942
	2.58358	0.262519	0.254484	0.00101764	0.259381	0.00112078
	2.7313	0.168234	0.157668	0.000695928	0.165815	0.000760628
	2.80338	0.12237	0.111316	0.00050553	0.120943	0.000558973
IMSE				0.001131296		0.001238308
Rank				1		2
n	t_i	R_real	R_MLE	MSE	R_MPS	MSE
100	1.56953	0.797597	0.79749	0.000972937	0.787325	0.00153265
	1.57331	0.796218	0.796129	0.000979417	0.785978	0.00154093
	1.60066	0.786084	0.786126	0.00102662	0.776079	0.00159912
	2.272	0.455207	0.456906	0.00127954	0.454075	0.00294892
	2.43752	0.354548	0.355263	0.00172479	0.355	0.00062526
	2.444	0.350514	0.35117	0.00111348	0.351007	0.0006078
	2.49732	0.317133	0.317243	0.00110506	0.317896	0.00114955
	2.58358	0.262519	0.261477	0.00037397	0.2634	0.00013886
	2.7313	0.168234	0.164322	0.000263504	0.168153	0.00048519
	2.80338	0.12237	0.116577	0.000507586	0.121149	0.00015545
IMSE				0.00093469		0.001078373
Rank				1		2

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4-1: Concluding Remarks Conducted from the Simulation Study

The simulation results can be summarized in the following points:

1- In Table 2, the MLE method ranked first by obtaining the most negligible value from the mean IMSE for all sample sizes.
2- The MPS method got a first-place through the most negligible value of the mean integral error squares IMSE. This is what we notice in Table 3.
3- Table 4 notes that the MLE method ranked first for all sample sizes.
4- We conclude from the results of the simulation experiment conducted on three models that the best method for estimating the reliability function of the NEXPF distribution is the MLE method.

5- Real Data Set Application:

This section includes data analysis to demonstrate distribution performance. The data were collected by taking the working times until the failure of the continuous positive air pressure (CPAP) device, and they were in months, where the sample size was n = 96.

Some statistical measures were found for the data, as shown in Table 5, and the power function distribution was compared with the NEXPF distribution by applying three differentiation measures (AIC, AIC_C, BIC), which proved the NEXPF distribution to be effective in modelling accurate data, such as shown in Table 6,7 is similar to Figure 4.

Table 5: Shows statistical measures of accurate data

mean	3.36319
Variance	0.641053
skuness	-0.63475
kurtosis	3.2761
median	3.46667
Standard Deviation	0.800658

Table 6: It shows the estimation of the NEXPF distribution parameters and the power function distribution by the MLE method.

Dist	α	β	λ
NEXPF	4.8338	4.4592	2.0621
PF	0.206897	2.51116	_

Table 7: AIC, AIC_c and BIC of the distributions.

dist	AIC	AICc	BIC
NEXPF	226.125	226.386	233.818
PF	245.261	245.39	250.39

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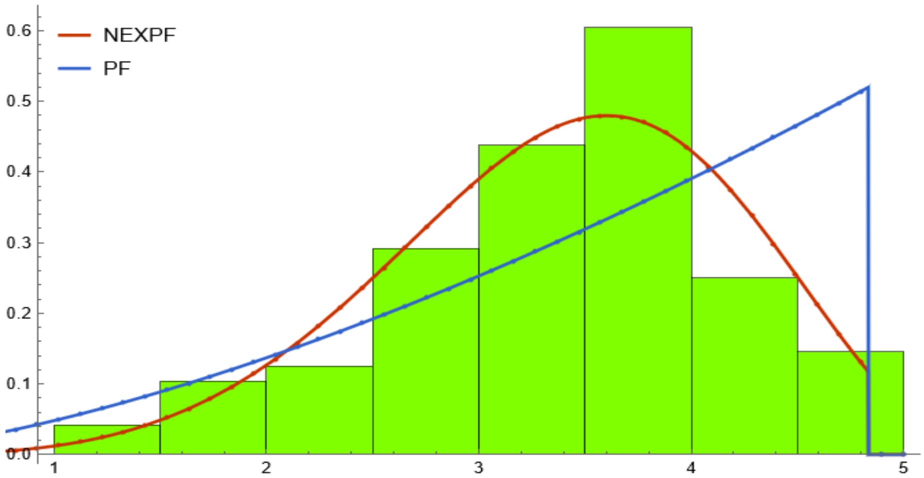

Figure 4: Shows the comparison between the NEXPF distribution and the power function distribution for accurate data

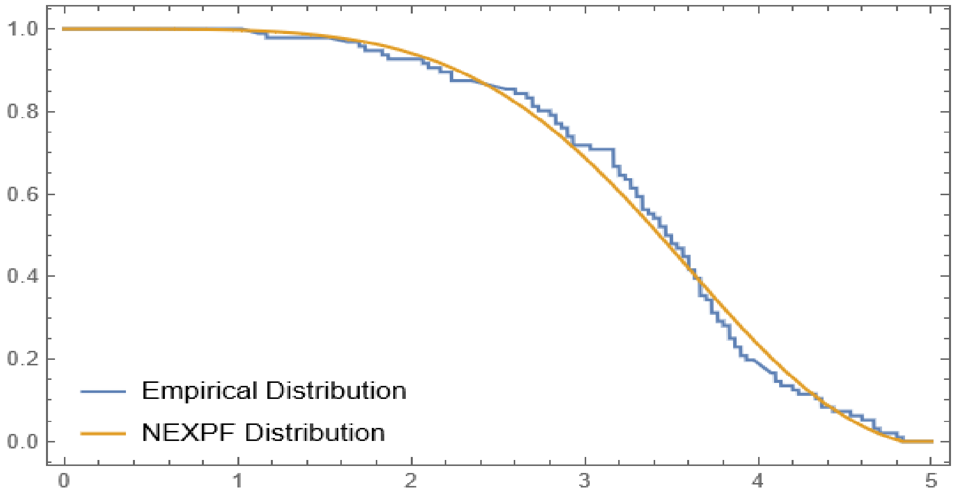

Figure 5: Shows the comparison between the reliability function for the raw data and the reliability function for the NEXPF distribution

6- Conclusions and Remarks:

In this paper we propose a model tert features called NEXPF, as an extension of the distribution function of power .model ,The new highly flexible and able to represent data at rates different failures, to describe the behavior of others unstable failure and common in reliability studies and biological studies , We provide mathematical treatment for the distribution including moments, generating function of moments, characteristic function, We discuss estimates of the maximum probability and estimates of the maximum output capabilities spacing function reliability, A set of real data to show the ability distribution NEXPF, analysis results indicate performance is best distribution (NEXPF) compare with distribution of (PF) , we hope to attract expanded model , The proposed new wider applications in the analysis of survival, and its application to controlled data.

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